Tesis Doctoral

Transferencia de energía y cantidad de movimiento en magnetósferas inducidas

Romanelli, Norberto Julio
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Transferencia de energía y cantidad de movimiento en magnetosferas inducidas

Tesis presentada para optar por el título de
Doctor de la Universidad de Buenos Aires en el área Ciencias Físicas

por Lic. Norberto Julio Romanelli

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Resumen

La presente tesis propone, como objetivo general, contribuir al estudio de los plasmas astrofísicos presentes en los alrededores y dentro de diferentes magnetósferas inducidas en el sistema solar, más específicamente las de Marte, Titán, Venus y el cometa P/Halley. Utilizando mediciones provistas por distintas misiones espaciales conjuntamente con herramientas teóricas, se estudian propiedades fundamentales de las mismas y su relación con los distintos fenómenos de transferencia de energía y cantidad de movimiento entre cada uno de dichos objetos y el plasma que los rodea.

Especificamente, se determinan propiedades de ondas de plasma observadas en las cercanías de Marte y Venus por las sondas Mars Global Surveyor (MGS) y Venus Express (VEX) y su relación con inestabilidades microscópicas que derivan de la interacción de sus exósferas con el viento solar. A tal fin se utiliza como marco teórico general la descripción magnetohidrodinámica (MHD), aunque teniendo en cuenta también efectos adicionales tales como la corriente de Hall y otros derivados a partir de la teoría cinética. A partir de modelos teóricos advertimos que las ondas observadas proveen evidencia indirecta de la pérdida de neutros atmosféricos en ambos planetas.

Asimismo, a partir de las ecuaciones MHD también modelamos la estructura global de las magnetósferas inducidas y su dependencia con las variaciones del medio circundante, obteniendo resultados concordantes con observaciones de la morfología magnética provistas por MGS en el entorno marciano. La morfología magnética es también estudiada en el entorno del cometa Halley (en estado activo) por medio de observaciones provistas por la sonda Vega-1, permitiendo complementar estudios previos acerca de las discontinuidades situadas dentro de esta clase de magnetósferas. Gracias a observaciones provistas por la sonda Cassini, se investigaron también los procesos de aceleración de partículas cargadas provenientes de la atmósfera de Titán. Dichos estudios muestran la importancia de las fuerzas de tensión magnética en estos entornos y nos permiten derivar estimaciones del flujo de partículas que pierde dicho satélite.

En síntesis, el trabajo efectuado en esta tesis se ha centrado en la compleja interacción de los objetos atmosféricos mencionados con sus respectivos entornos de plasma magnetizados y ha permitido, entre otras cosas, evaluar distintos procesos de transferencia de energía y cantidad de movimiento.

Palabras clave: Plasmas astrofísicos, Magnetósferas inducidas, Topología magnética, Ondas de plasma de ultra baja frecuencia, Erosión atmosférica
Transfer of energy and linear momentum in induced magnetospheres

Abstract

The main goal of the present thesis is to contribute to the study of the astrophysical plasmas in the surroundings and within different induced magnetospheres (IMs) in the solar system, more specifically those of Mars, Titan, Venus and Halley’s comet. By making use of measurements provided by different space missions together with theoretical tools, we study fundamental properties of these environments and their relationship with different phenomena of transfer of energy and linear momentum between each of these objects and the plasma around them.

Specifically, we determine properties of the plasma waves observed by the Mars Global Surveyor (MGS) and Venus Express spacecrafts in the surroundings of Mars and Venus, and their relationship with microscopic instabilities that arise as a result of the interaction of their exospheres with the solar wind. To this end, we use the magnetohydrodynamic (MHD) description as a general theoretical framework, taking also into account additional effects such as the Hall current and other kinetic effects. From theoretical models, we find that these waves provide indirect evidence of the loss of neutral atmospheric particles in both planets.

Additionally, by making use of the MHD equations, we model the global structure of the IMs and its response to variabilities on the surrounding medium. Our results are in agreement with observations of the magnetic field morphology obtained by MGS in the Martian environment. The magnetic field morphology is also studied in the surroundings of Halley’s comet (in active state) by means of measurements provided by the Vega-1 spacecraft. This allows to complement previous works about the discontinuities located inside this kind of magnetospheres. Thanks to measurements provided by the Cassini spacecraft, we also investigate the acceleration processes of charged particles originated in the atmosphere of Titan. These studies show the importance of the magnetic tension forces in these environments and allow to derive estimates of the flux of particles that escape from this moon.

In summary, the work performed in this thesis has been focused on the complex interaction between the previously mentioned atmospheric objects and their respective magnetized plasma environments. Among other things, the present study has allowed to evaluate different processes of transfer of energy and linear momentum.

Keywords: Astrophysical plasmas, Induced magnetospheres, Magnetic field topology, Ultra-low frequency plasma waves, Atmospheric erosion
A mis padres
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A mi familia por estar siempre a mi lado apoyándome. Por toda la ayuda, el ánimo y la confianza que me han dado. Sin ellos no lo hubiese logrado. Voy a estar agradecido por siempre.

Finalmente a Maíra, por hacerme tan feliz, por cuidarme, por acompañarme incondicionalmente, brindándome todo su amor y haciendo que siempre siga adelante. Juntos por siempre.
If there's an answer to the questions we feel bound to ask...

_Innuendo_, by Queen

For what is a man, what has he got?

_My Way_, by Frank Sinatra

And treat those two impostors just the same...

_If_, by Rudyard Kipling
Publications in peer reviewed journals


List of Abbreviations

AB  Aerobraking
ASPERA  Analyzer of Space Plasma and Energetic Atoms
AU  Astronomical Unit
BS  Bow Shock
CA  Closest Approach
CAPS  Cassini Plasma Spectrometer
DOY  Day of Year
DRAP  Draping
E/C  Energy-per-Charge
ELS  Electron Spectrometer Sensor
ER  Electron Reflectometer
ESA  Electrostatic Analyzer
EUV  Extreme Ultra-violet
FGM  Fluxgate Magnetometer
FOV  Field of View
GCM  Global Climate Model
GZ  Giacobini-Zinner
HT  deHoffmann-Teller
IC  Ion Cyclotron
ICA  Ion Composition Analyzer
ICE  International Cometary Explorer
IM  Induced Magnetosphere
IMA  Ion Mass Analyzer
IMB  Induced Magnetospheric Boundary
IMF  Interplanetary Magnetic Field
IMS  Ion Mass Spectrometer
IN  Inner Negative
INMS  Ion and Neutral Mass Spectrometer
IP Inner Positive
IPRL Inverse Polarity Reversal Layer
KSO Kronocentric Solar Orbital
LEF Linear Electric Field
LH Left Hand
LMD Laboratoire de Météorologie Dynamique
LP Langmuir Probe
MAG Magnetometer
MAVEN Mars Atmospheric Volatile Evolution
MCP Microchannel Plate
MEX Mars Express
MGS Mars Global Surveyor
MHD Magnetohydrodynamics
MISCHA Magnetic field in Interplanetary Space during comet Halley’s approach
MOI Mars Orbit Insertion
MPB Magnetic Pile-up Boundary
MPR Magnetic Pile-up Region
MSH Magnetosheath
MSO Mars Solar Orbital
MVA Minimum variance Analysis
MVAB Minimum variance Analysis of Magnetic Field Vector
ON Outer Negative
OP Outer Positive
PCW Proton Cyclotron Waves
PH Perihelion
PRL Polarity Reversal Layer
PSD Power Spectral Density
PVO Pionner Venus Orbiter
RH Right Hand
RHS Right Hand Side
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPC</td>
<td>Rosetta Plasma Consortium</td>
</tr>
<tr>
<td>RPWS</td>
<td>Radio and Plasma Wave Spectrometer</td>
</tr>
<tr>
<td>SAE</td>
<td>Southern Hemisphere Autumn Equinox</td>
</tr>
<tr>
<td>SC</td>
<td>Spacecraft</td>
</tr>
<tr>
<td>SD</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>SEM</td>
<td>Solar Extreme Ultra-violet Monitor</td>
</tr>
<tr>
<td>SLT</td>
<td>Saturn Local Time</td>
</tr>
<tr>
<td>SNG</td>
<td>Singles</td>
</tr>
<tr>
<td>SOHO</td>
<td>Solar and Heliospheric Observatory</td>
</tr>
<tr>
<td>SPO</td>
<td>Science Phase Orbit</td>
</tr>
<tr>
<td>SSE</td>
<td>Southern Hemisphere Spring Equinox</td>
</tr>
<tr>
<td>SSS</td>
<td>Southern Hemisphere Summer Solstice</td>
</tr>
<tr>
<td>ST</td>
<td>Straight-Through</td>
</tr>
<tr>
<td>SW</td>
<td>Solar Wind</td>
</tr>
<tr>
<td>SWIA</td>
<td>Solar Wind Ion Analyzer</td>
</tr>
<tr>
<td>TIIS</td>
<td>Titan Titan Ionospheric Interaction System</td>
</tr>
<tr>
<td>TOF</td>
<td>Time of Flight</td>
</tr>
<tr>
<td>UCT</td>
<td>Coordinated Universal Time</td>
</tr>
<tr>
<td>ULF</td>
<td>Ultra Low Frequency</td>
</tr>
<tr>
<td>UM</td>
<td>Unstable mode</td>
</tr>
<tr>
<td>VEX</td>
<td>Venus Express</td>
</tr>
<tr>
<td>VHM</td>
<td>Vector Helium Magnetometer</td>
</tr>
<tr>
<td>VSO</td>
<td>Venus Solar Orbital</td>
</tr>
<tr>
<td>WH</td>
<td>Whistler</td>
</tr>
</tbody>
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A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behaviour [Chen, 1984]. Most of the space plasmas are also characterized by extremely low densities. In the particular case of our solar system, a plasma flow emanates from the Sun and it is known as the solar wind (SW). This space plasma consists almost exclusively of charged particles. Indeed, a good approximation is to consider the SW to be totally ionized. It is mainly composed of protons and electrons that surpass the gravitational attraction of the Sun as a result of the presence of a pressure gradient between the solar corona and the interstellar space.

In spite of its extremely low density (about 8 ions cm$^{-3}$ at the orbit of the Earth), the SW is an excellent electrical conductor as collisions between particles are extremely rare. For instance, the mean free path of a proton in the surroundings of our planet is on the order of $10^8$ km. As a result of its high conductivity, the SW convects the solar magnetic field lines, giving rise to what is known as the interplanetary magnetic field (IMF). On average, this charged gas can be considered neutral when the length scales of interest are larger than the Debye length ($L_D$). At the Earth’s orbit, the Debye length of the SW is of the order of 10 m.

In addition to being collisionless and magnetized (i.e. it carries the coronal magnetic field with it), the SW plasma is supersonic. As a result, the SW particles are not aware
of the presence of obstacles in the flow. Thus, a bow shock is generally formed in front of each of these bodies, allowing the flow to be diverted around them. The different obstacles that the SW finds in its path can be classified into three types: absorbers such as our Moon, in which the SW interacts directly with their surfaces, obstacles with intrinsic magnetic fields (such as the Earth), and objects with no intrinsic magnetic field and whose atmospheres directly interact with the SW. Such atmospheres are subject to several ionizing mechanisms and perturb the streaming magnetized plasma flow through different processes. These last environments are called induced magnetospheres (IM). In the solar system, the planets Mars and Venus, Saturn’s largest satellite Titan\(^1\) and also the active comets have associated IMs. Table 1 displays the magnetic moments of several objects in our solar system. As it can be seen, the magnetic moments of objects with and without intrinsic magnetic fields differ in at least four orders of magnitude.

<table>
<thead>
<tr>
<th>Object</th>
<th>M (G cm(^3))</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Moon</td>
<td>$&lt; 1.3 \times 10^{18}$</td>
<td>[Russell et al., 1974]</td>
</tr>
<tr>
<td>Venus</td>
<td>$&lt; 3.0 \times 10^{21}$</td>
<td>[Russell et al., 1980]</td>
</tr>
<tr>
<td>Mars</td>
<td>$&lt; 2.0 \times 10^{21}$</td>
<td>[Acuña et al., 1998]</td>
</tr>
<tr>
<td>Titan</td>
<td>$&lt; 1.4 \times 10^{20}$</td>
<td>[Wei et al., 2010]</td>
</tr>
<tr>
<td>The Earth</td>
<td>$\sim 7.9 \times 10^{25}$</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1.1**: Magnetic moments of different types of objects present in the solar system.

The magnetic field and plasma measurements used in this thesis were provided by several space missions around and within the IMs of Mars, Venus, Titan and Halley’s comet. As it will become apparent throughout the following chapters, fundamental properties of these IMs are mainly defined from the exchange of energy and momentum between the external streaming magnetized plasma flow and the corresponding atmospheric unmagnetized obstacle. Interestingly, in spite of their differences (atmospheric composition, size, and upstream magnetic field and flow conditions), the atmospheric unmagnetized obstacles perturb the streaming magnetized plasma in a similar way, thus suggesting that common physical processes are at work. Such similarities are partially the result of the presence of a ionosphere whose conductivity is very large at high altitudes. In addition, in the case of objects with extended exospheres, neutral particles can also be ionized in the external flow domain. As a result, the following effects are present (with varying degrees of effectiveness) when a magnetized plasma flow interacts with this kind of objects:

\(^{1}\)In the case of Titan, the streaming magnetized plasma is generally the Kronian plasma rather than the SW. This topic is studied in Chapter 6.
Introduction

- The electric field\(^2\) in the obstacle’s reference frame sets currents in the object’s ionosphere. Such currents, in turn, prevent the external streaming magnetic field to reach lower altitudes. Since the external plasma flow has a high conductivity, it slows down in front of the object. As shown in Chapter 4, such deceleration strongly defines the magnetic field morphology observed in the surroundings of these obstacles.

- When an exospheric neutral atom is ionized, a newborn exospheric ion is created. As soon as this occurs, these ions are accelerated by the electric and magnetic fields frozen into the plasma flow. As more and more newborn ions are incorporated to the external wind in this matter, the amount of energy taken from the external flow becomes significant, and it slows down. This effect is known as massloading. Furthermore, the picked-up newborn ions (by the external magnetized plasma) are also capable of producing plasma instabilities and waves (see Chapter 5), coupling physical properties of the two ion populations (external and exospheric).

- As counterpart of the linear momentum lost by the external wind, the newborn exospheric charged particles are accelerated and escape the atmospheric unmagnetized object (see Chapter 6), contributing to the erosion of its atmosphere, with important consequences for its climate evolution.

Additionally, it is important to stress that the presence of discontinuities within the IMs are pointing out towards additional interactions where a significant transfer of momentum and energy takes place. As an example of the different regions around and within an IM, Figure 1.1 displays a scheme for the case of Mars. In the following we describe each region and boundary beginning from the unperturbed SW and proceeding in a direction away from the Sun, from day to night.

- The upstream SW region exterior to the Martian bow shock (BS), contains mainly protons and electrons with a nominal speed of 400 km s\(^{-1}\) interacting with the interplanetary magnetic field whose typical intensity at these locations is 4 nT. Additionally, since the Martian exosphere extends up to these distances, newborn ions (produced by various ionization mechanisms) originate different plasma instabilities that give rise to ultra-low frequency waves (ULF waves). This region is dominated by the solar wind dynamic pressure, with the thermal and magnetic being significant smaller.

\(^2\)For example the convective electric field, associated with the convection of the external magnetic field.
- The foreshock is the upstream region magnetically connected to the BS. Solar wind particles in this region are reflected from the shock, giving rise to instabilities that also lead to ULF waves.

- The bow shock is the boundary at which the supersonic incident magnetized plasma becomes subsonic. Among several other properties it is worth noticing that it is located much closer to the planet than in objects with intrinsic magnetic fields [Brain et al., 2015]. For instance, the subsolar point in Mars is located at \( \sim 1.6 R_M \) from its center \( (R_M \text{ stands for radius of Mars, } 1R_M = 3400 \text{ km}) \) while near the terminator plane (plane that separates the illuminated day-side region from the night one) is located at \( \sim 2.6 R_M \) from the center of Mars.

- The magnetosheath (MSH), the region located between the BS and the induced magnetospheric boundary (IMB) and extending into the wake, is filled mainly with shocked SW plasma. The Martian MSH is dominated by the shocked SW thermal plasma pressure and it is characterized by a strong wave activity. The magnetic field strength increases throughout this region while the plasma velocity decreases as a result of mass loading.

- The IMB, also known as the magnetic pile-up boundary (MPB), separates the region dominated by SW plasma from the one dominated by the planetary plasma. It is characterized by changes in the ion composition, the energy electron spectra, the magnetic field strength and orientation (wrapping or draping) together with its oscillations. Its subsolar point is located at \( \sim 1.3 R_M \) from the center of Mars and near the terminator plane is located at \( \sim 2.1 R_M \) from the center of Mars.

- The induced magnetosphere, i.e., the region between the MPB and the ionopause, is composed of planetary plasma and dominated by the magnetic pressure. Usually there is less wave activity than in the magnetosheath. Concerning the magnetic field topology, the field lines are draped around the Martian ionosphere.

- The ionopause is the location below which the thermal plasma pressure becomes dominant. It is characterized by a rapid increase in the thermal electron densities as the altitude decreases. Under nominal conditions, it is located at 450 km of altitude in the dayside and at higher altitudes near the terminator.

- The ionosphere (located below the ionopause) is a region of cold thermal planetary plasma that results from the photoionization of the upper atmospheric neutrals. On the Martian night side it is very tenuous.
- The induced magnetotail, contained inside the downstream region of the MPB, is the nightside extension of the IM. It is filled with planetary plasma and consists of two magnetic lobes of opposite magnetic polarities.

- The current plasma sheet is an interface often located in the orbital plane of Mars (inside the induced magnetotail) which is filled with energetic planetary ions.

![Diagram of plasma boundaries and regions](image)

**Figure 1.1:** Plasma boundaries and regions found in the plasma environment surrounding Mars according to Brain et al. [2015].

- Finally, it is important to point out that although a global field is inexistent, Mars possesses crustal magnetic fields (remnant magnetic fields) that shield the atmosphere from the SW in some locations and create small-scale regions of access between the SW and atmosphere in others. These fields are however not strong enough to withstand the incoming magnetized plasma at a global scale [Brain et al., 2003].

In summary, the main objective of this PhD thesis is to study several physical phenomena occurring in the surroundings and inside IMs based on measurements provided
by various space missions and interpretations derived from different theoretical models. In particular, we have focused our analysis on four objects: Mars, Titan, Venus and the comet Halley. This thesis is structured as follows:

- In Chapter 2 we present several theoretical descriptions of plasmas to describe phenomena occurring in these environments. Among other topics, magnetohydrodynamics (MHD) and Hall-MHD are contained in this chapter.

- Chapter 3 is devoted to explain various techniques of data analysis required to interpret measurements obtained by different spacecrafts. The limitations and capabilities of the instruments onboard these missions are also discussed.

- Chapter 4 addresses the magnetic field topology of an induced magnetosphere. First, we have analyzed several magnetic field draping signatures by analytically solving the problem in which a perfectly conducting magnetized plasma flows around a spheric conducting body for several orientations of the background magnetic field. By considering a given fixed velocity field we determine the associated electric and magnetic field under a steady-state MHD description. Second, we investigate the magnetic field morphology in the environments of Mars and comet P/Halley based on measurements obtained by the magnetometers (MAG) onboard the Mars Global Surveyor (MGS) and Vega-1 spacecrafts, respectively.

- In Chapter 5 we study the properties and the generation mechanisms of ultra low frequency electromagnetic plasma waves (known as proton cyclotron waves or PCWs) observed by MGS and Venus Express (VEX) in the upstream regions of Mars and Venus, respectively. Particularly, we analyze data from the MAG and electron reflectometer (ER) onboard MGS and data from the MAG and the Analyzer of Space Plasma and Energetic Atoms (ASPERA) onboard VEX.

- Chapter 6 concerns the magnetized plasma environment surrounding Saturn’s largest satellite Titan and the acceleration processes of ionospheric plasma in its IM. For this particular task, we analyze data provided by the Cassini Plasma Spectrometer (CAPS) for electrons and ions, the magnetometer and the Radio and Plasma Wave Science (RPWS) onboard Cassini to interpret the observed changes in the kinetic energy of the cold plasma in Titan’s induced magnetotail.

- Finally, Chapter 7 presents the main conclusions of this thesis.
Resumen en castellano

El objetivo principal de la presente tesis doctoral es estudiar varios procesos físicos que tienen lugar en los alrededores y dentro de magnetósferas inducidas por medio de mediciones provistas por distintas sondas espaciales e interpretaciones derivadas a partir de distintos modelos teóricos. En particular, hemos focalizado nuestro análisis en cuatro objetos: Marte, Titan, Venus y el cometa Halley. Esta tesis está estructurada de la siguiente manera:

- En el Capítulo 2 presentamos varias descripciones teóricas de los plasmas que son utilizadas para describir distintos fenómenos que ocurren en estos entornos. Entre otros temas, la teoría magnetohidrodinámica (MHD) y Hall-MHD están contenidos en este capítulo.

- El Capítulo 3 está dedicado a desarrollar varias técnicas de análisis de datos necesarias para interpretar mediciones provistas por distintas sondas espaciales. Las limitaciones y capacidades de los instrumentos a bordo de estas misiones también son discutidas.

- El Capítulo 4 trata sobre la topología del campo magnético de una magnetósfera inducida. Primero, analizamos distintas características del “arropeamiento” del campo magnético resolviendo analíticamente el problema en el que un plasma magnetizado perfectamente conductor fluye alrededor de una esfera conductora para distintas orientaciones del campo magnético de fondo. Considerando un campo de velocidades fijo y predeterminado, calculamos los campos eléctricos y magnéticos asociados, bajo una descripción MHD estacionaria. Además, estudiamos la morfología magnética en los entornos de Marte y el cometa P/Halley basándonos en mediciones provistas por medio de los magnetómetros a bordo de las sondas Mars Global Surveyor (MGS) y Vega-1, respectivamente.

- En el Capítulo 5 estudiamos las propiedades y los mecanismos de generación de ondas electromagnéticas de plasma de ultra baja frecuencia (conocidas como ondas al ciclotrón del protón) observadas por MGS y Venus Express (VEX) en las regiones “aguas arriba” de Marte y Venus, respectivamente. En particular, analizamos datos provistos por el magnetómetro (MAG) y el reflectómetro de electrones a bordo de MGS, y del MAG y el Analizador de Plasma Espacial y Átomos Energéticos (ASPERA) a bordo de VEX.
- El Capítulo 6 está centrado en el entorno de plasma magnetizado que rodea a Titán (el satélite más grande de Saturno) y los procesos de aceleración de plasma ionosférico que tienen lugar dentro de su magnetósfera inducida. Para esta tarea particular, analizamos datos provistos por el Espectrómetro de Plasma para electrones y protones, el magnetómetro y el instrumento de Ondas de Plasma y Radio (RPWS) a bordo de Cassini e interpretamos los cambios observados en la energía cinética del plasma frío presente en la magnetocula inducida de Titán.

- Finalmente, en el Capítulo 7 presentamos las conclusiones principales de la presente tesis.
CHAPTER 2

Physical description of plasmas: theoretical models

Most of the space plasmas are extremely tenuous gases of ionized particles in which, on average, there is no net charge. As a result of their very low density (much lower than that of an extremely good laboratory vacuum), there are very few close encounters between charged particles and they mainly respond to the large-scale force fields in which they move. Determining these force fields can be difficult since, in the presence of Coulomb interactions, a specific particle “feels” the effects of even very remote particles. However, the other particles need to be considered only in an average sense, and therefore only the collective interactions of the particles are important. Moreover, because the collective interactions impose a long-range order, and since in a magnetized plasma the magnetic field creates links between distant spatial regions, it is also possible to study space plasmas in terms of a conducting fluid description. In this chapter we present fundamental theoretical concepts related to particle, kinetic and fluid descriptions of space plasmas, which will be used to interpret observations in subsequent chapters.
2.1. Single particle motion

A plasma is an electrically neutral gas (on average) composed mainly of charged particles. Hence the roles of electrical and magnetic forces are fundamental for understanding the behaviour of a plasma. In this section, we consider the dynamics of a charged particle in external electric \( E \) and magnetic \( B \) fields, which are negligibly modified by the moving charge itself. Most of the results are therefore applicable to a small minority of particles which do not affect the bulk of the plasma. The coupling between the magnetic field and the bulk plasma will be considered later in this chapter.

2.1.1. Motion of charges in constant and uniform fields

The equation of motion for a non-relativistic particle of charge \( q \) and velocity \( \mathbf{v} \) subjected to the fields \( \mathbf{E} \) and \( \mathbf{B} \) is (in CGS units)

\[
\frac{m \, d \mathbf{v}}{dt} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})
\]

**Uniform magnetic field**

In the case of \( \mathbf{E} = 0 \) and \( \mathbf{B} \) being constant, the Lorentz force reduces to \( q \frac{\mathbf{v}}{c} \times \mathbf{B} \). Since this force is perpendicular to the velocity vector, it produces a curvature of the particle path, but no change in the speed \(|\mathbf{v}|\). Because the force vanishes along \( \mathbf{B} \), \( v_\parallel \) is also constant. Hence, so is the angle between \( \mathbf{v} \) and \( \mathbf{B} \). This is the so-called pitch angle of the charged particle. In the plane perpendicular to \( \mathbf{B} \), the magnetic force produces a circular motion of radius

\[
r_L = \frac{mc}{|q|B} v_\perp : \text{Larmor radius or Gyroradius}
\]

whose angular frequency is

\[
\Omega_{ce} = \frac{|q|B}{mc} : \text{Cyclotron frequency or Gyrofrequency}
\]

The resulting trajectory is a helix of constant pitch angle around a magnetic field line. Thus, the Larmor radius and the gyrofrequency set scales below which the individual particle gyration play a significant role. For example, in the case of a magnetic field intensity of 7 nT (about the IMF in the surrounding of the Earth’s environment) solar wind protons have a typical Larmor radius of 70 km and a cyclotron frequency of about 0.67 rad s\(^{-1}\).
Electric field or applied force

When the electric field is not zero, the particle trajectory changes with respect to previous case. It is clear that if we add a force \( F_\parallel \) or electric field parallel to \( \mathbf{B} \), the particle will be accelerated in that direction with \( \mathbf{a} = F_\parallel /m \). Let us now then focus on an additional electric field strictly perpendicular to \( \mathbf{B} \) denoted as \( \mathbf{E}_\perp \mathbf{B} \). In this case, the trajectory of the charged particle can be derived in terms of the results derived above (when electric field was zero) by making use of the Lorentz transformations\(^1\).

Let \( \mathbf{E} \) and \( \mathbf{B} \) be the electric and magnetic fields in some reference frame R, respectively. Consider now a reference frame \( R' \) moving with respect to R with a velocity \( \mathbf{v} \). Since we are considering non-relativistic particles (i.e. \( v/c \ll 1 \)), we will consider all expressions to be correct to first order in \( v/c \). The electric and magnetic fields in the \( R' \) reference frame are given by the Lorentz transformations which, to first order in \( v/c \), are:

\[
\mathbf{E}' = \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \tag{2.4}
\]
\[
\mathbf{B}' = \mathbf{B} - \frac{\mathbf{v}}{c} \times \mathbf{E} \tag{2.5}
\]

Also note that if \( \mathbf{E}' = 0 \), then \( \mathbf{E} = -\frac{\mathbf{v}}{c} \times \mathbf{B} \), and thus \( \mathbf{B}' = \mathbf{B} \) to first order in \( v/c \).

By making use of these transformations, a charged particle that is subjected to a magnetic \( \mathbf{B} \) and electric \( \mathbf{E}_\perp \mathbf{B} \) fields seen from the R frame will not be affected by a component of the electric field perpendicular to \( \mathbf{B} \) seen from the \( R' \) frame that moves with a velocity \( \mathbf{v} = \mathbf{V}_D \) (with respect to R):

\[
\mathbf{V}_D = c \frac{\mathbf{E} \times \mathbf{B}}{\mathbf{B}^2} \tag{2.6}
\]

In other words, in the latter reference frame \( R' \) is \( \mathbf{E}'_\perp \mathbf{B} = \mathbf{E}_\perp \mathbf{B} + \frac{\mathbf{V}_D}{c} \times \mathbf{B} = 0 \). Hence, in this reference frame, the motion in the plane perpendicular to \( \mathbf{B} \) reduces to the gyration found above. Going back to R, the motion in the plane perpendicular to \( \mathbf{B} \) is then the superposition of the gyrating motion plus a drift of velocity \( \mathbf{V}_D \). Note that this velocity drift is the same for all charged particles, making the plasma move as a whole. This result can be applied to other forces by simply replacing the term \( q\mathbf{E} \) in Equation 2.6 by a general force \( \mathbf{F} \), which then produces a drift \( \mathbf{V}_D = (c\mathbf{F}/q) \times \mathbf{B}/\mathbf{B}^2 \).

2.1.2. Non-uniform magnetic field

The magnetic field is generally non-uniform. In cases where the non-uniformity is weak, i.e. when the field does not change much over a distance equal to the gyroradius

\(^1\)Note the component \( \mathbf{E}_\perp \mathbf{B} \) is perpendicular to \( \mathbf{B} \) and not to \( \mathbf{v} \).
(or during a time equal to the inverse of the gyrofrequency), the motion can be approximated by the gyration described above, around a point which is moving slowly. The instantaneous centre of gyration is called the guiding-centre of the particle. When this condition is fulfilled, the charged particles gyrate around the magnetic field lines, keeping their magnetic moment invariant. A weak longitudinal increase in the magnetic field intensity acts as a “magnetic mirror”. In addition, a weak transverse magnetic gradient produces a small transverse drift. The reader interested in these effects is referred to Meyer-Vernet [2007].

2.2. Collections of particles

In the previous section, we have briefly described the motion of individual particles. Plasmas consist of collections of charged and neutral particles. In classical mechanics, the state of this system is defined by the position \( r \) and velocity \( v \) of each particle at time \( t \), and the evolution is determined by the equation of motion for each particle. In a kinetic description of a large collection of particles, all particles of a given species are considered indistinguishable. The statistical information of how many particles are in a given position with a given velocity is stored in the velocity distribution function for each particle species. This approach amounts to replace the equation of motion for each particle by a differential equation for the velocity distribution. The velocity distribution function for species \( s \), \( f_s(r, v, t) \), is defined so that the number of particles in the volume element \([r, r + d^3r]\), and with velocities in the range \([v, v + d^3v]\) at time \( t \) is

\[
d^6N_s = f_s(r, v, t)d^3r d^3v \tag{2.7}
\]

In a fluidistic description, the state of the plasma is given by fields (i.e. functions of \( r \) and \( t \)) and deals with averages such as the number density, the average velocity and the kinetic energy which are the lower order moments of \( f_s \) in velocity space. The number density \( n_s(r,t) \) of type \( s \) particles is:

\[
n_s(r,t) = \int_{V^3} f_s(r,v,t) \, d^3v \tag{2.8}
\]

The average velocity \( v_s \) is:

\[
v_s(r,t) = \frac{1}{n_s(r,t)} \int_{V^3} f_s(r,v,t) \, v \, d^3v \tag{2.9}
\]

The kinetic energy of the plasma in the reference frame moving the fluid velocity \( v_s(r,t) \) is:

\[
< \frac{1}{2} m_s(v - v_s)^2 >= \frac{1}{n_s(r,t)} \int_{V^3} f_s(r,v,t) \frac{1}{2} m_s(v - v_s)^2 \, d^3v \tag{2.10}
\]
The hydrostatic partial pressure of particles of species $s$ is related to their average random energy by

$$
\frac{p_s}{n_s} = \frac{2}{3} < \frac{1}{2} m_s (v - v_s)^2 >
$$

In the case of thermodynamic equilibrium at temperature $T$, the velocity distribution function is a gaussian function along each coordinate in velocity space, i.e. the phase-space distribution is the Maxwell-Boltzmann distribution:

$$
f_s(r, v) = n_s \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left[ -\frac{m_s(v - v_s)^2}{2kBT_s} \right]
$$

where $k_B$ is the Boltzmann constant. In the case of the Maxwell-Boltzmann distribution, the thermal speed is:

$$
v_{th,s} = \sqrt{\frac{2kBT_s}{m_s}}
$$

On the other hand, the mean square (thermal) speed is $< v^2 > = 3k_BT_s/m_s$. Thus, the average kinetic energy per particle in the reference frame moving with velocity $v_s$ is

$$
W = < \frac{1}{2} n_s (v - v_s)^2 > = \frac{3k_BT_s}{2}
$$

### 2.3. The plasma state

The definitions presented in the preceding section (regarding $f_s$ and their moments) are applicable to collections of particles quite generally. In this section we consider features that distinguish a plasma from other gases. As previously stated, a plasma is a gas containing charged particles (and in some cases also neutral ones) which is quasi-neutral and exhibits collective behaviour.

For an assembly of charged particles to qualify as a gas, the particles must move freely. In other words, random motions should largely overrun mutual interactions. The plasma thus behaves as a gas if the energy of the Coulomb interaction between two particles separated by the average interparticle distance $< r >$ is much smaller that the average kinetic energy per particle,

$$
\frac{e^2}{< r >} \ll k_BT
$$

Since $< r > \sim n^{-1/3}$, the coupling parameter $\Gamma$, defined as the ratio of the average energy of the interaction to $k_BT$,

$$
\Gamma \equiv \frac{n^{-1/3} e^2}{k_BT}
$$
must be much smaller than unity for the plasma to qualify as a gas\(^2\). In the case of the solar wind plasma, \(\Gamma\) varies between \(10^{-8}\) and \(10^{-7}\) at 1 AU, and varies weakly with heliocentric distance.

The Coulomb force between charged particles helps to achieve electric neutrality. Random agitation, however, mixes the particles, jeopardizing this neutrality. The “competition” between both effects might give rise to small regions that are not exactly neutral. Detailed calculations show that the electrostatic potential at a distance \(r\) from a charge \(q\) in an equilibrium electron-ion plasma is

\[
\Phi(r) = \frac{q}{r} e^{-r/L_D}
\]

where \(L_D\), the Debye length, is

\[
L_D = \sqrt{\frac{k_B T}{4\pi n e^2}}
\]

At distances \(r \ll L_D\), the electric potential around the charge \(q\) is nearly the Coulomb one, whereas at \(r \gg L_D\), the charge is completely shielded by the charges of the ambient plasma, and the potential almost vanishes. Therefore the plasma is quasi-neutral at scales larger than \(L_D\). The solar wind at 1 AU from the Sun is characterized by a numerical density \(n \sim 5 \times 10^6\ \text{m}^{-3}\) and \(T \sim 10^5\ \text{K}\). As a result, the Debye length of the solar wind at such location is about 10 m. It is worth noticing that the Debye shielding requires that a region of size \(L_D\) contains a large number of particles, i.e., \(nL_D^3 \gg 1\). Taking into account the definition of \(\Gamma\) and \(L_D\), the latter condition also reads as \((4\pi \Gamma)^{3/2} \ll 1\).

When the plasma initially at equilibrium is perturbed (for instance by an additional charged particle or an incident electromagnetic wave) the charges will redistribute themselves in such a way to restore again the Debye shielding. Since electrons are faster than ions, they set the timescale in which the shielding take place. Such time scale is

\[
T = 2\pi \sqrt{\frac{m_e}{4\pi n e^2}} = \frac{2\pi}{\omega_{pe}}
\]

where \(\omega_{pe}\) is the electron plasma frequency. With a number density of \(n \sim 5 \times 10^6\ \text{m}^{-3}\), the solar wind electrons (at 1 AU) require a time \(T \sim 5 \times 10^{-5}\ \text{s}\) to restore the Debye shielding.

This collective behaviour constitutes a significant difference between a neutral gas and a plasma. In a neutral gas, particles interact only during a collision, i.e., when two gas atoms are affected by the short-range van der Waals force, which decays with the interparticle distance as \(r^{-6}\). Therefore, during most of the time, the gas atoms move

\(^2\)To have a well defined number density \(n\), we need to consider spatial scales \(L\) larger than the average distance between particles. In the case of the SW in the vicinity of the Earth is \(L > 0.01\ \text{m}\).
on straight trajectories, unaffected by the other atoms. The notion of collision is quite different in a plasma. In this case the interactions between particles are electromagnetic and thus they are long-range ($\propto r^{-2}$). This means that each plasma particle interacts with a large number of other particles at all times. However, even though each particle "feels" all the other ones, it interacts with them mainly via their collective (large scale) electromagnetic field. This collective electromagnetic field depends on the statistical distribution of all the source charged particles, but it does not depend on their precise positions. In contrast with the neutral gas case, the notion of a collision in a totally ionized plasma is a succession of small deviations that diffuse the charged particles from their "ideal" trajectory, i.e., the one that depends only on the collective electromagnetic fields [Meyer-Vernet, 2007]. When a totally ionized plasma is then called collisional, this means that each particle deviates from the path set by the collective field. This is done, not by strong episodic deviations, but by a succession of small deviations. The cumulative effect of these numerous small deviations determines the mean free path of each charged particle. The electron mean free path for collisions is:

$$\lambda_{mfp} = \left[ \frac{4\pi}{3} nr^2 ln(1/\Gamma) \right]^{-1}$$  \hspace{1cm} (2.20)

where $r_l$ is the so-called Landau radius and is equal to:

$$r_l = \frac{e^2}{k_B T}$$  \hspace{1cm} (2.21)

Collisionless fully ionized plasmas are then the ones where each particle trajectory is only determined by the collective electromagnetic field\footnote{For example the Debye length is the characteristic length scale over which the collective electrostatic field varies.}. More specifically, these plasmas satisfy that the Knudsen number, $K_n = \lambda_{mfp}/L$ is much larger than unity, with $L$ being the characteristic length scale of the phenomena under study.

### 2.4. Plasma kinetic description

Most plasma microphenomena\footnote{Following Gary [1993], these are phenomena related to departures of the plasma velocity distribution function from the thermodynamic equilibrium. An example are plasma instabilities that occur only when non-maxwellian distributions are present.} are thought to be well described by Maxwell’s equations coupled with a kinetic equation, the latter one determining the time evolution of $f_s(r, v, t)$, for the distribution function for the $s$ species. On the other hand, plasma macrophenomena (involving larger spatial scales) are frequently described by the field
equations coupled with a set of fluid equations that may, under certain assumptions, be derived from a kinetic equation. When comparing these two broad categories, one of the most important plasma processes are the wave-particle interactions. This is the capability of waves with appropriate phase speeds or group velocities to exchange energy with plasma particles. Examples of these energy-exchange processes are the Landau and cyclotron damping/resonance and the plasma instabilities which are driven by non-Maxwellian properties of the velocity distribution function.

For nonrelativistic plasmas, the general form of the kinetic Boltzmann equation is:

\[
\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{r}} + \frac{q_s}{m_s} (\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \frac{\partial f_s}{\partial t} \bigg|_{c} \tag{2.22}
\]

where \(q_s\) is the charge of the species \(s\) particle, \(m_s\) is its mass, and the right-hand side represents the effects of short-range collisions.

Based on the previous estimations for the solar wind at 1 AU from the Sun, and using Equation 2.20 we obtain that the mean free path of the SW at those locations is about 1 AU. Compared to planetary scales, collisions are indeed very rare in the solar wind. In this case, the right hand side (hereafter RHS) of Equation 2.22 is negligible, leading to the Vlasov equation:

\[
\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{r}} + \frac{q_s}{m_s} (\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0 \tag{2.23}
\]

This is the kinetic equation considered throughout this thesis. In the fluidistic approach, Equation 2.22 is integrated in velocity space. In the next section we briefly describe the Magnetohydrodynamics fluid theory for plasmas.

### 2.5. Magnetohydrodynamics (MHD)

The MHD approach assumes that the plasma is a continuous medium in local thermodynamic equilibrium. The MHD equations describe the plasma as a conductive fluid that experiences electric and magnetic forces. Such a fluid is characterized by averaged properties over a collection of particles. These averaged quantities follow the laws of mass, momentum and energy conservation of the fluid, which can be derived from Equation 2.22.

Let us consider a plasma composed of fully ionized hydrogen. This plasma then contains electrons and protons of masses \(m_e\) and \(m_p\), respectively. The equations of continuity for each of these two species (assuming a non-relativistic description) are the following:

\[
\frac{\partial n_{i,e}}{\partial t} + \nabla \cdot (n_{i,e} \mathbf{v}_{i,e}) = 0 \tag{2.24}
\]
Physical description of plasmas: theoretical models

These equations take this form by assuming that electrons and protons are not created or destroyed in any fluid element. Instead, they simply flow through the boundaries of any fixed volume. On the other hand, the corresponding equations representing the balance of linear momentum are:

\[ m_i n_i \left[ \frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i \cdot \nabla \mathbf{v}_i \right] = q_i n_i (\mathbf{E} + \frac{\mathbf{v}_i}{c} \times \mathbf{B}) - \nabla p_i + \mathbf{R}_{ie} \]

\[ m_e n_e \left[ \frac{\partial \mathbf{v}_e}{\partial t} + \mathbf{v}_e \cdot \nabla \mathbf{v}_e \right] = q_e n_e (\mathbf{E} + \frac{\mathbf{v}_e}{c} \times \mathbf{B}) - \nabla p_e + \mathbf{R}_{ei} \]

where \( \mathbf{R}_{jk} = -m_j n_j \nu_{jk} (\mathbf{v}_j - \mathbf{v}_k) \) is the force associated with collisions between particles of the species j and k, and \( \nu_{jk} \) is the collision frequency between such species. Note also that \( \mathbf{R}_{jk} = -\mathbf{R}_{kj} \). Let us also assume that this plasma satisfies the quasineutrality condition, from which:

\[ n_i = n_e = n \]  \hspace{1cm} (2.27)

Therefore, by making use of this assumption, we obtain that the electric current density in a plasma of electrons and protons is:

\[ \mathbf{J} = e n (\mathbf{v}_i - \mathbf{v}_e) \]  \hspace{1cm} (2.28)

Even though it is clear that the hydrogen plasma is made of two species, to first approximation we describe the plasma in terms of only one fluid. This fluid is characterized by a mass density \( \rho \):

\[ \rho = m_e n_e + m_i n_i \approx m_i n \]  \hspace{1cm} (2.29)

a velocity field \( \mathbf{v} \):

\[ \mathbf{v} = \frac{m_e n_e \mathbf{v}_e + m_i n_i \mathbf{v}_i}{m_e n_e + m_i n_i} \approx \mathbf{v}_i \]  \hspace{1cm} (2.30)

and a total pressure (assuming isotropic pressure):

\[ p = p_e + p_i \]  \hspace{1cm} (2.31)

where the approximated values are obtained taking into account that \( m_e << m_i \). From the linear combination of the continuity equations for protons and electrons (Equation B.1) we derive the mass continuity equation, that is:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]  \hspace{1cm} (2.32)

Also, by adding the equation of motion for both species, we obtain:

\[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{\nabla p}{\rho} + \frac{\mathbf{J} \times \mathbf{B}}{\rho c} \]  \hspace{1cm} (2.33)
Additionally, Maxwell’s equations, in the CGS system are:

\[
\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} \\
\nabla \cdot \mathbf{B} = 0 \\
\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{E} = 4\pi \rho_c
\]

(2.34) \hspace{1cm} (2.35) \hspace{1cm} (2.36) \hspace{1cm} (2.37)

where \(\rho_c\) is the charge density.

**Generalized Ohm’s law**

Ohm’s law for the hydrogen plasma is simply derived from the equation of motion of its electrons, that is:

\[
m_en \left[ \frac{\partial \mathbf{v}_e}{\partial t} + \mathbf{v}_e \cdot \nabla \mathbf{v}_e \right] = -e n(\mathbf{E} + \frac{\mathbf{v}_e}{c} \times \mathbf{B}) - \nabla p_e + \mathbf{R}_{ei}
\]

(2.38)

In the limit of massless electrons, a balance between all the forces (see RHS of Equation 2.38) is instantaneously achieved. This implies that for sufficiently long temporal scales (slow processes), the electrons will be in equilibrium at all times. Therefore, neglecting electron inertia,

\[
0 = -e n(\mathbf{E} + \frac{\mathbf{v}_e}{c} \times \mathbf{B}) - \nabla p_e + \mathbf{R}_{ei}
\]

(2.39)

By making use of equations 2.28 and 2.30, we substitute the mean velocity of the electrons by

\[
\mathbf{v}_e = \mathbf{v} - \frac{1}{en} \mathbf{J}
\]

(2.40)

from which we obtain that:

\[
\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} = \frac{1}{en} \left( \frac{1}{c} \mathbf{J} \times \mathbf{B} - \nabla p_e \right) + \frac{1}{\sigma} \mathbf{J}
\]

(2.41)

where \(\sigma = ne^2/m_e \nu_{ei}\) is the electric conductivity. Equation 2.41 is the generalized Ohm’s law. The first two terms of the RHS of this equation describe the Hall and electronic pressure effects, respectively. These effects are also known as finite Larmor radius effects. We can neglect these terms whenever the Larmor radius of the protons is much smaller that the typical spatial scales associated with the problem under study. The latter term of the RHS is the Joule dissipation, and can be neglected when the electrical conductivity is large. In these cases, the generalized Ohm’s law reduces to:

\[
\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} = 0
\]

(2.42)
Note that \( \mathbf{E} + \frac{\nabla}{c} \times \mathbf{B} \) is the electric field in the reference frame of the plasma fluid.

We remind the reader that Equation 2.42 is consistent with a non-relativistic approach \((v << c)\), according to which we also neglected the term \( \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \) in equation 2.34 (since it is of order \( v^2/c^2 \) compared to the rest of the terms). In this limit, the equation describing charge continuity reduces to \( \nabla \cdot \mathbf{j} = 0 \). As a consequence, the lines of current are closed and the local accumulation of charges in the plasma are negligible.

By taking the curl of the generalized Ohm’s law and making use of equation 2.36, we obtain the magnetic induction equation:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[ \left( \mathbf{v} - \frac{1}{en} \mathbf{J} \right) \times \mathbf{B} - \frac{4\pi \eta}{c} \mathbf{J} \right]
\]  

(2.43)

where \( \eta = c^2/(4\pi \sigma) \) is the electrical resistivity, and we have assumed that the electrons satisfy a polytropic equation \((i.e. p_e = p_e(\rho))\), and therefore, \( \nabla \times \left( \frac{1}{\eta} \nabla p_e \right) = 0 \).

Note that for a given velocity field, Equations 2.35 and 2.43, completely determine the behaviour of the magnetic field. The electric field and the current density can be consistently derived from equations 2.34 and 2.41.

In cases where the Larmor radius effects are negligible, the magnetic field induction equation is:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}
\]  

(2.44)

As it can be seen, in this simplified version of Ohm’s law the magnetic field lines are convected by the flow and partially diffuse through it. The relative importance of these two effects can be estimated by means of the magnetic Reynolds number, defined as

\[
R_m = \frac{L_0 v_o}{\eta}
\]  

(2.45)

where \( L_0 \) y \( v_o \) are, respectively, a characteristic length and velocity of the fluid. When the plasma has a high conductivity \( R_m >> 1 \), and the diffusive effect is negligible compared to the convective one. In this limit the magnetic field induction equation can be further simplified:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})
\]  

(2.46)

It can be shown that the latter equation implies that the magnetic field lines move together with the plasma. This theorem due to Hannes Alfvén shows that, under these conditions, the magnetic field lines are frozen to the fluid. The fluid can move freely along magnetic field lines. When its motion is perpendicular to them, the magnetic field lines might be dragged by the fluid or the fluid might be slowed down by the magnetic field lines.
Finally, fluid mechanics also contains another equation that represents the conservation of energy. In the next section we will make use of the adiabatic approximation. In summary, the following conditions are needed to derive the MHD equations:

- The plasma is considered to be a non-relativistic continuous medium in local thermodynamic equilibrium.
- The plasma is assumed to be neutral on spatial scales larger that $L_D$ and time scales larger that $w_{pe}^{-1}$.
- The characteristic time of the process under study must also exceed the time between collisions and the gyropreiods: $(T \gg T_{coll}, 2\pi \Omega_{ci}^{-1}, 2\pi \Omega_{ce}^{-1})$. In this limit, the velocity distribution function, and consequently the pressure are isotropic.
- If the spatial scales under study are much larger than the Larmor radius of the ions, the Hall effect and the electronic pressure effect can be neglected, leading to the MHD approximation. When this condition is not fulfilled, this approach leads to Hall-MHD.
- Consistent with the nonrelativistic plasma approximation, the term $\frac{1}{c} \frac{\partial E}{\partial t}$ must be neglected in Ampere’s law.
- The electric conductivity $\sigma$ has to be determined based on the electron-ion collision frequency. When $\sigma \rightarrow \infty$ (electrons in equilibrium at all times), then $E = -\frac{\nabla \times B}{c}$, leading to the so-called ideal MHD.
- If the mean free path is much larger than $L$, then the plasma is considered to be collisionless.

Finally, in some cases the equation representing the energy conservation might be approximated by one of the following special equations of state depending on the problem under study:

- incompressible: $\rho = $ constant, implying $\nabla \cdot \mathbf{v} = 0$
- isothermal: $\rho \propto p$
- adiabatic: $p \rho^{-5/3} = $ constant

In the next section we seek for the normal modes of oscillation of a plasma in the ideal MHD approximation.

---

\footnote{With Alfvénic velocities.}
2.5.1. Magnetohydrodynamic waves

The study of the properties of the normal modes propagating in a continuous medium constitutes an important diagnostic tool for the medium itself. To compute the normal modes propagating in a magnetofluid, we follow the standard procedure: first identify an equilibrium state of the plasma under study, then perturb it slightly such that the deviations of each of its physical variables are much smaller than their equilibrium values.

Let us consider a uniform and static equilibrium given by:

\[ \rho = \rho_0 = cte, \quad p = p_0 = cte, \quad \mathbf{v} = 0, \quad \mathbf{E} = 0, \quad \mathbf{B} = B_0 \hat{z} \quad (2.47) \]

Let us now consider small deviations in each of these variables, i.e. \( f = f_0 + \delta f \), where \( \delta f << f_0 \) (\( f \) being any of the above physical variables).

We assume that the viscosity as well as the electrical resistivity of the plasma are sufficiently small so that, to a first order approximation, the resulting energy dissipation can be neglected when analyzing the propagation of waves. In addition, we also assume that the normal mode frequency is large enough to adopt the adiabatic approximation, i.e.

\[ p \rho^{-\gamma} = cte, \quad \gamma = 5/3 \quad (2.48) \]

Note that the plasma state described by Equation 2.47 is an equilibrium solution to Equations 2.32, 2.33, 2.35, 2.42, 2.44, 2.48. Next we linearly perturb these equations to describe the evolution of the perturbations. These equations are the following:

\[ \partial_t \delta \rho = -\rho_0 \nabla \cdot \delta \mathbf{v} \quad (2.49) \]

\[ \rho_0 \partial_t \delta \mathbf{v} = -\nabla \delta p + \frac{1}{4\pi} (\nabla \times \delta \mathbf{B}) \times \mathbf{B}_0 \quad (2.50) \]

\[ \delta \mathbf{E} = -\frac{1}{c} (\delta \mathbf{v} \times \mathbf{B}_0) \quad (2.51) \]

\[ \partial_t \delta \mathbf{B} = \nabla \times (\delta \mathbf{v} \times \mathbf{B}_0) \quad (2.52) \]

\[ \nabla \cdot \delta \mathbf{B} = 0 \quad (2.53) \]

\[ \delta p = c_s^2 \delta \rho, \quad c_s^2 = \frac{\gamma p_0}{\rho_0} \quad (2.54) \]

This set of equations is homogeneous and linear in the unknown deviations \((\delta \rho, \delta \mathbf{u}, \delta \mathbf{E}, \delta \mathbf{B}, \delta p)\). Also, the coefficients are constant since we have assumed a uniform equilibrium. As a consequence, the linear deviations will take the form:

\[ \delta f = f \exp[i(\mathbf{k} \cdot \mathbf{x} - wt)] \quad (2.55) \]
Without loss of generality we can choose the $\hat{x}$ axis such that $k = k\sin(\theta)\hat{x} + k\cos(\theta)\hat{z}$. Note that the equilibrium is anisotropic since $B_0$ is parallel to the $\hat{z}$ axis. After imposing the functional form shown in Equation 2.55 to each of the deviations present in the Equations 2.49-2.54, we obtain the following dispersion relations:

$$w^2 = k^2 v_A^2 \cos^2(\theta), \quad v_A = \frac{B_0}{\sqrt{4\pi \rho_o}} \quad (2.56)$$

$$w^2 = k^2 \left[ \frac{v_A^2 + c_s^2}{2} \pm \sqrt{\frac{(v_A^2 + c_s^2)^2}{4} - v_A^2 c_s^2 \cos^2(\theta)} \right] \quad (2.57)$$

Equation 2.56 is the dispersion relation for the so-called Alfvén mode. Equation 2.57 shows the dispersion relation for both the fast magnetosonic mode (positive sign) and the slow magnetosonic mode (negative sign). Interestingly, the normal modes of a plasma under the MHD description are three, in contrast with a classic neutral gas where only one mode (sound waves) is present.

Next, we determine the associated eigenvectors by replacing the dispersion relations in Equations 2.49-2.54. Hereafter we denote the phase velocity of each of the three normal modes as $v_\phi$, being $v_\phi = wk^{-1}$. Figure 2.1 shows a polar plot of the phase velocity of the Alfvén mode (A) and the fast (F) and slow (S) magnetosonic modes. The distance between each point in these curves and the origin of coordinates shows the phase velocity of each mode associated with the propagation angle $\theta$. This angle is measured from the direction of the $B_o = B_0\hat{z}$.

The following equations are derived:

$$\begin{bmatrix} \delta B_x \\ \delta B_y \\ \delta B_z \end{bmatrix} = \begin{bmatrix} -B_0^{-1} \sin(\theta) & 0 & 0 \\ 0 & -B_0^{-1} \sin(\theta) & 0 \\ B_0^{-1} \sin(\theta) & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta v_x \\ \delta v_y \\ \delta v_z \end{bmatrix} \quad (2.58)$$

$$\begin{bmatrix} v_\phi^2 - v_A^2 - c_s^2 \sin^2(\theta) & 0 & -c_s^2 \sin(\theta) \cos(\theta) \\ 0 & v_\phi^2 - v_A^2 \cos^2(\theta) & 0 \\ -c_s^2 \sin(\theta) \cos(\theta) & 0 & v_\phi^2 - c_s^2 \cos^2(\theta) \end{bmatrix} \begin{bmatrix} \delta v_x \\ \delta v_y \\ \delta v_z \end{bmatrix} = 0 \quad (2.59)$$

Equation 2.58 shows that the component of the velocity perturbation along the magnetic field $B_0$ ($\delta v_z$) does not have any effect on the magnetic field perturbation $\delta B$. Additionally, Equation 2.59 shows that $\delta v_y$ is not coupled to $\delta v_x$ and $\delta v_z$.

As it can be seen from Equation 2.56, the group velocity for the Alfvén modes is $(0, 0, \pm v_A)$. In other words, this normal mode has a velocity that is parallel to the
**Figure 2.1**: Phase velocity of each of the three MHD normal modes, normalized to the Alfvén velocity, (slow (S), Alfvén (A) and fast (F)) when \( c_A^2 = 0.25 \, v_A^2 \).

equilibrium magnetic field and an intensity equal to \( v_A \), regardless of the direction of propagation. The eigenvectors of this mode can be expressed as:

\[
\delta \mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \delta \mathbf{B} = -Bv_A^{-1} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}
\]

Hence, \( \delta \mathbf{v} \) and \( \delta \mathbf{B} \) are perpendicular to both the magnetic field \( \mathbf{B}_0 \) and the wave-vector \( \mathbf{k} \). Moreover, \( \delta \mathbf{v} \) is parallel to \( \delta \mathbf{B} \).

The eigenvectors for the remaining normal modes can be expressed as follows:

\[
\delta \mathbf{v} = \begin{bmatrix} v_A^2 - c_A^2 \cos^2(\theta) \\ 0 \\ c_A^2 \cos(\theta) \sin(\theta) \end{bmatrix}, \quad \delta \mathbf{B} = Bv_A^{-1} \left( v_A^2 - c_A^2 \cos^2(\theta) \right) \begin{bmatrix} -\cos(\theta) \\ 0 \\ \sin(\theta) \end{bmatrix}
\]

In this case, the magnetic field perturbation is perpendicular to the wave vector as well, as expected from Equation 2.35.
Based on their eigenvectors, the following fundamental properties of each of the MHD modes can be derived:

- The Alfvén waves are transverse to the equilibrium magnetic field and incompressible. In this mode, the restoring force is the tension associated with the magnetic field lines.
- In contrast with the Alfvénic mode, the fast magnetoacoustic waves are compressible ($\delta \rho \neq 0$). They also display magnetic fluctuations parallel to $B_o$. For this mode, the variation in the magnetic field strength and the plasma pressure are in phase. These waves propagate faster than the Alfvén mode.
- The slow magnetoacoustic waves also present variations in the intensity of the magnetic field and in the plasma pressure. In this mode, however, these perturbations are out of phase and the waves propagate slower than the Alfvénic ones.

### 2.6. Two-fluid MHD: MHD waves with Hall-effect

This section is devoted to the study of a plasma of fully ionized hydrogen when the Hall effect is not negligible. The equations describing its dynamics are known as the Hall-MHD equations.

To derive the Hall-MHD equations, we consider that the hydrogen plasma is composed of two ideal fluids with the same number density of protons and electrons. Equations 2.32, 2.33 and 2.40 are therefore satisfied. We also assume that both species satisfy a polytropic relation, this is:

$$p_{i,e} = p_{i,e}^0 \left( \frac{n}{n_o} \right) ^\gamma$$  \hspace{1cm} (2.62)

Once again, we also take into account Maxwell’s equations and the generalized Ohm’s law with $\sigma \to \infty$ (see Equation 2.41).

In the following we show a dimensionless version of the ideal Hall-MHD equations:

$$\frac{d\mathbf{v}}{dt} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \beta \nabla (n^\gamma)$$  \hspace{1cm} (2.63)

$$\mathbf{E} = -(\mathbf{v} - \frac{\varepsilon}{n} \nabla \times \mathbf{B}) \times \mathbf{B} - \frac{\varepsilon}{n} \beta \mathbf{e} \mathbf{B} \times (n^\gamma)$$  \hspace{1cm} (2.64)

$$\mathbf{v}_e = \mathbf{v} - \frac{\varepsilon}{n} \nabla \times \mathbf{B}$$  \hspace{1cm} (2.65)

where the associated units are $n_o$, $p_{i,e}^0$, $L_o$, $B_o$, $v_o = v_A = \frac{B_o}{\sqrt{4\pi n_o m_i}}$, and where we define

$$\beta = \frac{p_{i,e}^0 + p_{e}^0}{\gamma m_i n_o v_A^2}, \hspace{0.5cm} \beta^e = \frac{p_{e}^0}{\gamma m_e n_o v_A^2},$$

and

$$\varepsilon = \frac{c}{w_{p}\ell_o} = \frac{v_A}{\Omega_{ci} L_o}$$  \hspace{1cm} (2.66)
The $\varepsilon$ parameter is the so-called Hall parameter and measures the relative importance of this effect. In the limit of $\varepsilon \rightarrow 0$ we recover the MHD equations analyzed in the previous section.

**Linear wave modes**

As sketched in the previous section, we slightly perturb a uniform equilibrium of the plasma. In dimensionless units, this equilibrium is given by $n_o = 1$, $p_o = 1$, $B_o = \hat{z}$ and $\mathbf{v}_o = 0$. The plasma variables can then be written as (in a first order approximation): $n = 1 + \delta n$, $\mathbf{v} = \delta \mathbf{v}$, $B = \hat{z} + \delta B$ and $p = 1 + \gamma \delta n$.

The resulting set of equations is homogeneous and linear in the perturbations $\delta n$, $\delta \mathbf{v}$, $\delta B$. Also, the coefficients in these equations are constant, since we have assumed a uniform equilibrium. As a result, the solutions to these equations will have the functional form shown in Equation 2.55, and lead to:

\[
w \delta n = \mathbf{k} \cdot \delta \mathbf{v} \tag{2.67}
\]

\[
k \cdot \delta \mathbf{B} = 0 \tag{2.68}
\]

\[
w \delta \mathbf{v} = \gamma \beta k \delta n - (\mathbf{k} \times \delta \mathbf{B}) \times \hat{z} \tag{2.69}
\]

\[
w \delta \mathbf{B} = -k \times [(\delta \mathbf{v} - i\varepsilon \mathbf{k} \times B) \times \hat{z}] \tag{2.70}
\]

We choose the $\hat{x}$ axis such that $k = k \sin(\theta) \hat{x} + k \cos(\theta) \hat{z}$, where $B_o$ is parallel to the $\hat{z}$ axis.

The magnetic field perturbation $\delta \mathbf{B}$ can be written as:

\[
\delta \mathbf{B} = (\delta B_\perp \cos(\theta), \delta B_y, -\delta B_\perp \sin(\theta)) \tag{2.71}
\]

Such expression automatically satisfies Equation 2.68. On the other hand, the velocity field perturbation is represented in the following way:

\[
\delta \mathbf{v} = (\delta v_\perp \cos(\theta) + \delta v_{\parallel} \sin(\theta), \delta v_y, -\delta v_\perp \sin(\theta) + \delta v_{\parallel} \cos(\theta)) \tag{2.72}
\]

From Equations 2.67-2.72, we obtain:

\[
\begin{bmatrix}
v_{\phi} & -1 & 0 & 0 & 0 & 0 \\
\gamma \beta & v_{\phi} & 0 & 0 & 0 & \sin(\theta) \\
0 & 0 & v_{\phi} & \cos(\theta) & 0 & 0 \\
0 & 0 & \cos(\theta) & v_{\phi} & 0 & -i\mu \\
0 & 0 & 0 & 0 & v_{\phi} & \cos(\theta) \\
0 & \sin(\theta) & 0 & i\mu & \cos(\theta) & v_{\phi}
\end{bmatrix}
\begin{bmatrix}
\delta n \\
\delta v_{\parallel} \\
\delta v_y \\
\delta v_\perp \\
\delta B_{\parallel} \\
\delta B_y
\end{bmatrix} = 0 \tag{2.73}
\]
where $\mu = \varepsilon k \cos(\theta)$ and $v_\phi = w k^{-1}$.

We highlight the following points:

- When $\mu = 0$ (absence of the Hall effect) we recover the normal modes of MHD.

- The central block ($\delta v_y, \delta B_y$) corresponds to linearly-polarized Alfvén waves. The upper block is an acoustic mode, while the lower one is another branch of the linearly polarized Alfvén waves. These two blocks are decoupled only in the parallel propagation case ($\sin(\theta) = 0$). In the general case of oblique propagation, these two blocks are coupled to one another and generate the fast and slow magnetosonic waves.

- In the case of propagation perpendicular to $B_\alpha$, there is no Hall effect since $\cos(\theta) = 0$.

- Interestingly, the Hall effect couples the two components of the magnetic field perturbation vector ($\delta B_y, \delta B_\perp$), leading to the appearance of circularly polarized waves.

After taking the determinant of the matrix in Equation 2.73, the following dispersion relation is obtained:

$$v_\phi^2 - v_\phi^4 (1 + \gamma \beta + \mu^2 + \cos^2(\theta)) + v_\phi^2 (\mu^2 \gamma \beta + (1 + 2 \gamma \beta) \cos^2(\theta)) - \gamma \beta \cos^4(\theta) = 0 \quad (2.74)$$

The parallel propagation case

Next, we develop the case of parallel propagation, i.e., $k \parallel \hat{z}$ (and $\cos(\theta) = 1$) a bit further. In this case, the acoustic mode is decoupled from the other ones and is characterized by a dimensionless phase velocity $v_\phi^\pm = \pm \sqrt{(\gamma \beta)}$. Therefore, the phase velocity is simply the speed of sound, i.e. $v_\phi^\pm = \pm \sqrt{(\gamma \beta \frac{m_i e_o}{m_e})}$.

On the other hand, the perpendicular modes correspond to whistlers (fast magnetosonic branch) and shear ion-cyclotron waves (slow magnetosonic branch). The phase velocity of each of these normal modes is:

$$v_\phi^{\sigma \sigma'} = \sigma \frac{\mu}{2} + \sigma' \sqrt{\frac{\mu^2}{4} + 1} \quad (2.75)$$

where $\sigma = \pm 1$, $\sigma' = \pm 1$. Since we are dealing with the case of parallel propagation, $\mu = \varepsilon k$. Also, these modes are circularly polarized and are dispersive.
Shear ion-cyclotron waves

The shear ion-cyclotron wave frequency \( \sigma' = -\sigma \) asymptotically goes to the ion (proton) gyrofrequency when \( k \rightarrow \infty \). Indeed:

\[
v_{IC}^* = \frac{w}{k} = \sigma'\left[-\frac{\varepsilon k}{2} + \varepsilon k \sqrt{1 + \frac{4}{\varepsilon^2 k^2}}\right] \sim \left[-\frac{\varepsilon k}{2} + \frac{\varepsilon k}{2} \left(1 + \frac{2}{\varepsilon^2 k^2}\right)\right] = \pm \frac{1}{\varepsilon k}
\]

(2.76)

Hence \( |w(k \rightarrow \infty)| \sim \varepsilon^{-1} \). Therefore, the frequency of these waves in this limit (with the corresponding dimensions) is:

\[
|w(k \rightarrow \infty)| \sim \frac{v_A w_p}{c} = \Omega_{ci}
\]

(2.77)

for both the parallel and antiparallel propagation cases. We focus now on the polarization of the normal mode when \( k \rightarrow \infty \), this is, when \( v_{IC}^* \rightarrow 0 \). In this limit we can approximate the dispersion relation by

\[
v_{\phi}^2 (\mu^2 \gamma \beta + (1 + 2 \gamma \beta)) \sim \gamma \beta
\]

(2.78)

where the following result is derived:

\[
v_{\phi}^* \sim \frac{\sigma}{k \varepsilon}, \quad \sigma = \pm 1
\]

(2.79)

By replacing this expression for the phase velocity in the matrix of Equation 2.73, we obtain:

\[
\delta B = (1, -i \sigma, 0)
\]

(2.80)

\[
\delta v = (1, -i \sigma, 0)
\]

(2.81)

where components are given in the coordinate system \((\hat{\perp} k, \hat{y}, \hat{k})\). By taking into account Equations 2.79 and 2.80, it can be seen that if \( w > 0 \) (\( \sigma = 1 \)), \( \delta B = (1, -i, 0) \) rotates in the same sense of a positive ion around the mean magnetic field. By the same token, if \( w < 0 \) (\( \sigma = -1 \)), \( \delta B = (1, i, 0) \) which is the same sense of rotation as before. Therefore, both branches correspond to a circularly polarized wave that rotates around \( B_0 \) with the same sense as a positive ion. These waves are also known as left-hand polarized waves. If \( w > 0 \) the wave propagates parallel to \( \hat{z} \), if \( w < 0 \) the wave propagates antiparallel to \( \hat{z} \).

Whistlers

Let us now consider the whistler mode \( \sigma' = \sigma \) in the limit of \( k \rightarrow \infty \). The phase velocity of this mode in this limit can be approximated as:

\[
v_{\phi}^{WH} = \frac{w}{k} = \sigma \left[\frac{\varepsilon k}{2} + \frac{\varepsilon k}{2}\right] = \sigma \varepsilon k
\]

(2.82)
Physical description of plasmas: theoretical models

Hence \( w(k \rightarrow \infty) \sim \sigma \varepsilon k^2 \). Therefore the frequency of this wave (with its corresponding dimensions) is:

\[
w(k \rightarrow \infty) \sim \frac{\nu_A^2 k^2}{\Omega_{ci}}
\]

(2.83)

We now study the polarization of this mode when \( k \rightarrow \infty \), this is, when \( \nu_{\phi}^{WH} \rightarrow \sigma \varepsilon k \).

Replacing this expression for the phase velocity in the matrix of Equation 2.73, we obtain that:

\[
\delta \mathbf{B} = (1, i \sigma, 0)
\]

(2.84)

\[
\delta \mathbf{v} = (1, i \sigma, 0)
\]

(2.85)

If \( w > 0 \) (\( \sigma = 1 \)), \( \delta \mathbf{B} = (1, i, 0) \) rotates in the same sense as an electron around the mean magnetic field. On the other hand, if \( w < 0 \) (\( \sigma = -1 \)), \( \delta \mathbf{B} = (1, -i, 0) \) rotates in the same sense. Therefore, both branches correspond to a circularly polarized wave that rotates around \( \mathbf{B}_o \) in the same sense as an electron. These waves are also known as right-hand polarized waves. If \( w > 0 \) the wave propagates parallel to \( \hat{z} \), if \( w < 0 \) the wave propagates antiparallel to \( \hat{z} \).

In summary, the magnetic field perturbations corresponding to the ion-cyclotron and whistler modes can be represented by means of the real functions as:

\[
\delta \mathbf{B}_{IC/LH} = [\cos(kz - wt), \sigma \sin(kz - wt), 0]
\]

(2.86)

\[
\delta \mathbf{B}_{WH/RH} = [\cos(kz - wt), -\sigma \sin(kz - wt), 0]
\]

(2.87)

where \( \delta \mathbf{B}_{IC/LH} \) denotes the ion cyclotron waves (or left hand polarized waves) and \( \delta \mathbf{B}_{WH/RH} \) denotes the whistler waves (or right hand polarized waves) and \( \sigma = 1 \) for \( w > 0 \) and \( \sigma = -1 \) for \( w < 0 \).

Figure 2.2 displays the dispersion relation for both modes for the parallel propagation case and assuming an incompressible regime.

2.7. Wave-particle resonances in Hall MHD

Up to this point all wave modes are stable since \( w \in \mathbb{R} \) for all the cases. As we will see in Chapter 5, plasma instabilities can occur under suitable space plasma conditions. Indeed, plasma waves known as proton-cyclotron waves grow and are observed to propagate in the extended exospheres of Mars and Venus\(^6\). In these cases, the largest wave growth often is presented under wave-particle resonant conditions. In this section, we briefly present the basic notions on this topic.

\(^6\)These waves are referred to as proton cyclotron waves only because their frequency, in the reference frame of the spacecraft that observed them, is close to the local proton cyclotron frequency.
**Physical description of plasmas: theoretical models**

![Graph](image)

**Figure 2.2:** Dispersion relation for the ion-cyclotron and the whistler modes in the parallel propagation and incompressible case.

**Normal resonance**

Wave-particle resonant conditions are arbitrarily classified in two categories: normal and anomalous [Gary, 1993]. In the so-called normal cyclotron resonance, waves and charged particles propagate against each other. Left hand circularly polarized waves interact with positively charged ions, and correspondingly, right hand polarized waves interact with negative ions.

As an example of the normal resonance, let us consider the interaction between a positively charged ion (whose velocity parallel to $\mathbf{B}_0$ is negative) and an ion cyclotron wave propagating parallel to $\mathbf{B}_0$. Following the notation presented in the end of the previous section, the polarization state of the wave can be expressed as:

$$\delta \mathbf{B}_{LH} = [\cos(kz - wt), \sin(kz - wt), 0]$$  \hspace{1cm} (2.88)
with \( w > 0 \). On the other hand, the trajectory of the ion is:

\[
z_{\text{ion}} = -v \parallel t
\]  
(2.89)

where \( v \parallel > 0 \),

\[
\mathbf{r}_{\perp \text{ion}} = r_L \left[ \cos(-\Omega_{ci} t), \sin(-\Omega_{ci} t), 0 \right]. 
\]  
(2.90)

with \( \Omega_{ci} > 0 \). Therefore, in the reference frame moving with the ion (primed system):

\[
z' = z + v \parallel t
\]  
(2.91)

\[
\mathbf{r}'_{\perp \text{ion}} = \mathbf{r}_{\perp \text{ion}}
\]  
(2.92)

In the same frame, we also have:

\[
\delta \mathbf{B}_{LH} = [\cos(k (z' - v \parallel t) - w t), \sin(k (z' - v \parallel t) - w t), 0]. 
\]  
(2.93)

If \( z = z_{\text{ion}}(t) \), therefore \( z' = 0 \) (following the particle). In this case the resonance condition results:

\[
-k v \parallel t - w t = -\Omega_{ci} t
\]  
(2.94)

for all times, which can be cast into

\[
w + k v \parallel = \Omega_{ci}
\]  
(2.95)

where all these magnitudes are defined positively. This “normal” resonance interaction between a positively charged ion and a left handed polarized wave propagating against each other (but rotating in the same sense) can be understood in terms of the Doppler effect. Concerning this approach, the frequency of the left handed polarized wave is “seen” by the charged particle with a higher value, so that they are in resonance when it is equal to the ion gyrofrequency.

For the same reason, an electron and a right hand polarized wave propagating towards each other will be in resonance under analogous conditions. Let us then consider in this case the interaction between a negatively charged ion (once again, with negative velocity parallel to \( \mathbf{B}_0 \)) and a right-hand polarized wave whose propagation is parallel to \( \mathbf{B}_0 \). In the reference frame moving with the ion:

\[
\delta \mathbf{B}_{RH} = [\cos(k (z' - v \parallel t) - w t), -\sin(k (z' - v \parallel t) - w t), 0]. 
\]  
(2.96)

with \( v \parallel > 0 \). If \( z = z_{\text{ion}}(t) \), then \( z' = 0 \) (following the particle). In this case the resonance condition is

\[
w + k v \parallel = \Omega_{ce}
\]  
(2.97)

where all the magnitudes are defined positively.
Anomalous resonance

When the same waves and charged particles interact under different conditions, they lead to the so-called “anomalous” cyclotron resonance. In this case the positively charged ions interact with right-hand polarized waves, while negatively charged ions interact with left-hand polarized waves.

One possible case occurs when the positively charged ions overtake the waves ($v_\parallel > V_{ph}$, with particles and waves propagating in the same direction) in such a way that the ions “feel” that the waves are left-hand polarized. Since such ions rotate in the same sense as the left-hand polarized waves, this interaction is denominated “anomalous”.

Let us consider this case in greater detail: positively charged ions and right-hand polarized waves are propagating in the same direction, for example along the $\hat{z}$ axis. Therefore $v_\parallel > 0$, and the wave seen from the ion reference frame is:

$$\delta \mathbf{B}_{RH} = [\cos(k(z' + v_\parallel t) - w t), -\sin(k(z' + v_\parallel t) - w t), 0].$$  \hspace{1cm} (2.98)

The resonance condition is:

$$w - k v_\parallel = -\Omega_{ci}$$  \hspace{1cm} (2.99)

where, once again, all the magnitudes are defined positive. In this case we then find a resonant interaction between a positively charged ion and a right-hand polarized wave. Even though they propagate in the same direction, they can resonate if $v_\parallel > V_{ph} = w/k$. Indeed, the wave frequency seen by the particle is equal to its gyroperiod, while the polarization is reversed: a right-hand polarized wave is seen from the reference frame of the positive ion with a left-hand polarization.

The same kind of anomalous resonance occurs between electrons and left-hand polarized waves. If $v_\parallel > 0$, the wave seen from the electron reference frame results:

$$\delta \mathbf{B}_{LH} = [\cos(k(z' + v_\parallel t) - w t), \sin(k(z' + v_\parallel t) - w t), 0].$$  \hspace{1cm} (2.100)

The resonance condition is then:

$$w - k v_\parallel = -\Omega_{ce}$$  \hspace{1cm} (2.101)

where all the magnitudes are positive. However, in this case it is important to stress that since the left-hand polarized waves have frequencies (in the plasma reference frame) below the proton-cyclotron frequency, resonant electrons are typically relativistic.
2.8. Limits of each model

As we have seen in this chapter, a fundamental assumption of any MHD model is that the plasma acts like a fluid, held together either by frequent collisions or by electromagnetic forces. In these models, it is assumed that kinetic processes arising from the generation of energetic tails in the particle distributions or from temperature anisotropies are not important (at least for the processes being studied), and that the plasma behaviour is well described by a single Maxwellian distribution in ideal MHD or by multiple Maxwellian distributions in a multi-fluid approach. In addition, all fluid models assume quasineutrality, isotropic temperatures, and neglect the displacement current term as well as relativistic effects.

A fundamental area where fluid models differ from one another is in the treatment of Ohm’s law. Indeed, different versions of Ohm’s law have been developed, depending on the relevant spatial and temporal scales being considered. The resulting Ohm’s law in ideal MHD is presented in equation 2.42. If the problem involves structures of the order of the ion gyroradius or the ion skin depth, higher order corrections, specifically Hall and $\nabla p_e$ should be included (see equation 2.41).

In the following, we present estimates of several important parameters associated with some of the underlying assumptions of MHD models in regions near Venus, Mars and Titan. To determine whether an assumption is valid, the typical length $L$ of the structures is compared with the implicit length scale of the assumption. The scales sizes in Table 2.1 are calculated for values applicable to the solar wind (for Mars and Venus) and for O$^+$ ions impinging on Titan. If the model seeks to resolve planetary scales ($R$), then the theoretical assumptions of the model should be valid at scales of the order of $L=0.1$ R or less.

<table>
<thead>
<tr>
<th></th>
<th>Venus</th>
<th>Mars</th>
<th>Titan</th>
<th>Relevance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>6052 km</td>
<td>3395 km</td>
<td>2575 km</td>
<td>$L$</td>
</tr>
<tr>
<td>$\lambda_D/L$</td>
<td>$9.3 \times 10^{-7}$</td>
<td>$6.4 \times 10^{-6}$</td>
<td>$7.4 \times 10^{-5}$</td>
<td>Quasineutrality</td>
</tr>
<tr>
<td>$\lambda_{mfp}/L$</td>
<td>$2.2 \times 10^{4}$</td>
<td>$5.5 \times 10^{5}$</td>
<td>$1.4 \times 10^{8}$</td>
<td>Collision regime</td>
</tr>
<tr>
<td>$(c/w_{pe})/L$</td>
<td>$2.2 \times 10^{-4}$</td>
<td>$1.1 \times 10^{-3}$</td>
<td>$3.8 \times 10^{-3}$</td>
<td>Electron skin depth</td>
</tr>
<tr>
<td>$(c/w_{pi})/L$</td>
<td>$9.7 \times 10^{-3}$</td>
<td>$4.7 \times 10^{-2}$</td>
<td>0.79</td>
<td>Hall effect</td>
</tr>
<tr>
<td>$r_{L}/L$</td>
<td>$6.3 \times 10^{-2}$</td>
<td>0.43</td>
<td>1.6</td>
<td>Particle velocity anisotropies</td>
</tr>
</tbody>
</table>

Table 2.1: Comparison of plasma relevant scale sizes at Venus, Mars and Titan and the planetary radius ($R$), extracted from Kallio et al. [2011].

As shown in Table 2.1, the ratio between the Debye length and the planetary scales,
\( \lambda_D/L \ll 1 \), validates the assumption of quasineutrality in all these cases. The comparison of the mean free paths to the planetary scale size, \( \lambda_{mfp}/L \), shows that except for the dense regions of the atmospheres, the plasma becomes collisionless. In this regard, this fact also demands to be careful concerning the hypotheses that the plasma has a Maxwellian distribution and an isotropic pressure. The next two length scales, \( (c/w_{pi})/L \) and \( (c/w_{pe})/L \), are the characteristic length scales of waves with frequencies near the ion and electron plasma frequencies. While it is a valid assumption to neglect all waves with frequencies of the order of the electron plasma frequency, neglecting waves of the order of the ion plasma frequency, such as ion cyclotron waves, is not valid at scales where Mars and Titan are spatially resolved. The last row of the table, \( r_L/L \), compares the Larmor radius of solar wind/magnetospheric ions the upstream regions of Mars, Venus, and Titan to \( L \). From this comparison it is clear that neglecting gyroradius effects of incident ions is acceptable for Venus, marginally valid at Mars, and completely invalid at Titan. Finally, it is important to note that these values are representative of the \( \text{H}^+ \) ions in the solar wind and not the heavy ions in each body’s upper atmosphere. Even for Venus, the gyroradius of planetary ions can be large relative to scales of the order of \( L \) depending on the strength of the magnetic field and the origin of the planetary ions.
Resumen en castellano

Los plasmas espaciales son generalmente gases extremadamente temprados compuestos de partículas ionizadas que, en promedio, no tienen carga neta. Como resultado de su extremadamente baja densidad (mucho menor que la de un laboratorio de vacío), hay muy pocos encuentros cercanos entre partículas cargadas y por ende, responden principalmente a los campos de fuerza de gran escala en los que se mueven. Determinar estos campos de fuerza puede ser difícil dado que, en la presencia de interacciones Coulombianas, una partícula específica “siente” hasta los efectos de partículas muy lejanas. Sin embargo, las otras partículas requieren ser consideradas solo en promedio y por lo tanto, solo las interacciones colectivas de las partículas son importantes. Más aún, dado que las interacciones colectivas imponen una escala de largo-alcance y dado que, en un plasma magnetizado el campo magnético crea conexiones entre regiones muy distantes, es posible estudiar los plasmas espaciales en términos de una descripción de fluido conductor. En este capítulo presentamos conceptos teóricos fundamentales relacionados con el movimiento de partículas individuales como así también descripciones cinéticas y fluidicas de los plasmas espaciales, entre ellos:

- Movimiento de partículas cargadas en campos magnéticos constantes y uniformes
- Una breve descripción cinética del plasma por medio de la ecuación de Boltzmann y de Vlasov
- La teoría Magnetodinámica (MHD)
- La teoría MHD con los efectos de Hall incluidos
- Resonancias onda-partícula en Hall MHD

Dichos modelos teóricos serán utilizados para interpretar observaciones en los capítulos siguientes.
CHAPTER 3

Instruments and methods of analysis of spacecraft measurements

This chapter describes the technical capabilities of the instruments that provided scientific data for this thesis, which are analyzed throughout the following chapters. We particularly focus on the Mars Global Surveyor and the Cassini missions, since most of the results presented here concern Mars and Titan. In this chapter we also summarize the methods of analysis of spacecraft in situ measurements used in this thesis.

3.1. The Mars Global Surveyor mission

The Mars Global Surveyor was a National Aeronautics and Space Administration (NASA) spacecraft which entered into orbit around Mars on 11 September 1997, after an interplanetary trip of 309 days. The main science objectives of MGS during its primary mission were:

- Characterizing the surface features and geological processes on Mars.

- Determining the composition, distribution and physical properties of surface minerals, rocks and ice.
- Determining the global topography, planet shape, and gravitational field.

- Establishing the nature of the planet’s magnetic field and its interaction with the solar wind.

- Monitoring global weather and the thermal structure of the atmosphere.

- Studying interactions between Mars’ surface and the atmosphere.

MGS was a three-axis stabilized spacecraft equipped with several remote and in-situ instruments: the Mars Orbital Camera, the Thermal Emission Spectrometer, the Mars Orbital Laser Altimeter, the Radio Science Experiment, and the Magnetometer/Electron Reflectometer (MAG/ER) [Albee et al., 2001]. In this thesis we analyzed data provided by the MGS MAG/ER investigation. Below are descriptions of the spacecraft (SC) orbital geometry, the MAG/ER instruments and their capabilities. A more detailed description of the MAG/ER can be found in Acuña et al. [1992].

The Orbital Geometry

The original plan for the MGS mission consisted of 5 phases: the interplanetary trip, the insertion into orbit around Mars (MOI), the aerobraking phase (AB), the mapping phase, and a relay period for future missions. As soon as the second phase was completed, MGS performed elliptical orbits around the planet with periapsis altitudes sufficiently low (80 km) so that the friction caused by the Martian atmosphere could be used to slow it down. By means of this friction, MGS reduced its apoapsis during the aerobraking phase until a final circular orbit was achieved. In the mapping phase, such circular orbits had an altitude of 400 km and an orbital period of 2 hours.

However, at the beginning of the aerobraking phase (in the periapsis of orbit P015), significant dynamic perturbations in the spacecraft were detected. Such perturbations were also observed in the following orbits and attributed to an uncompleted deployment of one of the solar panels used as dragging surface for the aerobraking. A more detailed analysis of available data suggested possible damages in this panel. Because of this, the maximum admissible dynamic pressure was reduced and MGS continued describing an elliptical orbit with a periapsis altitude of 180 km for another year, instead of only 2 months. In other words, in opposition to the original plan, the date for MGS to achieve a circular orbit was postponed for a whole terrestrial year. In order to reach a similar orbit to the initially considered, the AB phase was split in two stages, AB1 and AB2. The latter two stages were separated by a period known as Science Phase Orbits (SPO) in which the aerobraking was suspended. The orbital period of MGS as a function of time is shown in
Figure 3.1: MGS orbital period as a function of time [Albee et al., 1998].

Figure 3.1. Thanks to the varying orbital geometry of the SC, MGS was able to sample most local times and solar zenith angles. This change of plans led to a greater amount of observations of the Martian ionosphere and its induced magnetosphere. Additionally, it allowed to map the Martian crustal magnetic fields closer to the planet and for a period longer that originally planned.

MGS orbited around Mars for more than 9 years, a much longer period than originally planned (2 years) and than that of any other spacecraft sent to that planet. The last communication between MGS and the Earth took place on 2 November, 2006.

The MGS MAG/ER experiment

The Magnetometer and Electron Reflectometer provided a wealth of information on the plasma environment around Mars. Figure 3.2 shows the location of each of the instruments onboard MGS. As it can be seen, all instruments (including the ER) were located in the spacecraft central body, with the exception of the magnetometers whose sensors were located at the outer edges of the solar panels.

The main objectives of the MGS MAG/ER experiment were [Acuña et al., 2001]:

a. To determine the nature of the Martian magnetic field. An important objective in this characterization is to establish whether Mars had a global intrinsic magnetic field.
Figure 3.2: Instruments onboard the MGS spacecraft [Albee et al., 2001].

b. To develop models for such a field.

c. To produce a map of the magnetic field crustal sources.

d. To study the interaction of the solar wind with the upper atmosphere, ionosphere and/or magnetic field of the planet.

The magnetometers (MAG)

The magnetometer experiment consisted of twin triaxial fluxgate magnetometers that provided in situ vectorial measurements of the magnetic field. Each instrument had a full scale dynamic range of ± 4 nT to ± 65536 nT, although the automatic range finder selected the optimum range for the instrument to operate as the ambient magnetic field varies. The MAGs sampled the magnetic field at a rate of 32 samples s\(^{-1}\). However, the resolution of the data that was returned depends on the assigned telemetry rate [Acuña et al., 1998].
Since the magnetometeres were not magnetically isolated from the spacecraft, the spacecraft field had to be determined and subtracted from the data [Acuña et al., 1998]. This is the primary source of uncertainty in the data. In spite of several fits performed to the observations, there is a remaining ~ 1 nT contribution of spacecraft magnetic fields in the MAG data [Acuña et al., 2001]. In many observations during the AB and SPO orbits, this contribution is manifested as an oscillating signal with a period of 100 minutes approximately. This was the period of rotation of MGS during such phases. Finally, note that we have not applied a high-pass filter to the MAG data since there are not enough periods of 100 minutes in each orbit to accurately perform this task.

**The Electron Reflectometer (ER)**

The electron reflectometer was an electron spectrometer which measured the ambient electron flux as a function of energy and time. It consisted of a top hat hemispheric electrostatic analyzer with a field of view (FOV) of $2\pi$ (more specifically, $360^\circ \times 14^\circ$ divided in 16 sectors of $22.5^\circ \times 14^\circ$). The instrument swept through a series of steps in voltage, selecting out 30 logarithmically-spaced energy channels ranging from 10 eV to 20 keV. The electron flux at the selected energy was then recorded as a function of the angular variables. The energy resolution was $\Delta E/E =0.25$ and the maximum integration resolution was 2 s [Mitchell et al., 2001]. Figure 3.3 displays a schematic of this instrument.

The ER owes its name to an innovative method to measure magnetic fields remotely. When the plasma measurements are combined with information about the local magnetic field direction, the pitch angle distribution of the electrons can be determined. Because charged particles can be reflected from regions of higher magnetic field (the magnetic mirror effect), examining the pitch angle distribution of electrons returning from the planet determines the magnetic field strength at the point of reflection. Therefore, the ER did not only provide information on the electron plasma in situ (such as the local distribution function and the energy spectra of the electrons), but also measured the magnetic field remotely.

When analyzing ER data is fundamental to take into account the following effects. The electrons that entered the ER were accelerated by the spacecraft potential (higher than that of the surrounding plasma at the altitudes where it was operational, i.e., above the ionospheric electron peak). Therefore, the observed energies of the electrons are a few eVs higher than the real values. Also, the amount of spacecraft photoelectrons is comparable to that of the ambient electrons for energies below 30 eV. In addition, the FOV covered the entire sky every half spin of MGS ($\sim$ 50 min), a period much longer than the apparent temporal scales associated with the Mars-SW interaction. Also during
this period, MGS typically travels a distance of about $5 \, R_M$. In this thesis we analyze the omnidirectional fluxes (averaged over all directions) with energies higher than $30 \, \text{eV}$.

Finally, it is also important to note that MGS did not have an instrument capable of measuring properties of the ions. Therefore, parameters such as the solar wind plasma velocity, temperature and density were unknown.

### 3.2. The Cassini mission

The Cassini-Huygens mission is a collaboration between NASA, the European Space Agency (ESA) and the Italian Space Agency designed to explore the Saturnian system, including its rings and moons, with a special focus on Titan. After its launch on 15 October 1997 and a seven-year journey that included four gravity-assist maneuvers, Cassini entered into orbit around Saturn on July 1, 2004. It then began an extensive mission that includes more than 100 orbits around the planet and its moons to date.
Some of the main scientific objectives of Cassini include the determination of the three-dimensional structure and dynamic behaviour of the rings and the magnetosphere of Saturn as well as characterization of the surface and atmosphere of Titan. Contained in the latter objective is the study of Titan’s interaction with Saturn’s magnetosphere. Cassini was originally planned to perform 44 Titan flybys but, since the mission has been extended twice until 2017, many more flybys will take place. Up to this day, Cassini has completed more than 110 Titan flybys.

Equipped with twelve instrument packages, Cassini is capable of taking accurate measurements and detailed images in a variety of atmospheric conditions and light spectra. A diagram of the Cassini spacecraft is presented in Figure 3.4. The main body of the orbiter is a stack consisting of a lower equipment module, a propulsion module, an upper equipment module, and a high-gain antenna. Attached to the stack are the Remote Sensing Pallet and the Fields and Particles Pallet with their scientific instruments as well as the Huygens Probe. These two pallets carry most of the scientific instruments. The 11-m magnetometer boom is mounted on the upper equipment module. In this thesis, observations obtained from several instruments onboard Cassini have been analyzed: magnetic field observations are obtained by the Cassini magnetometer (MAG) [Dougherty et al., 2004], the ion composition and plasma flow information are obtained from the Cassini Plasma Spectrometer (CAPS) [Young et al., 2004], and electron number density and spacecraft potential are calculated from the Radio and Plasma Wave Science/Langmuir Probe instrument (RPWS/LP) [Gurnett et al., 2004]. In the following we present a brief description of each of these instruments.

**Magnetometer (MAG)**

The Cassini Magnetic field experiment consists of a Vector Helium Magnetometer (VHM) and a Fluxgate Magnetometer (FGM) capable of measuring the ambient magnetic field over a wide range: ± 256 nT and ± 6565 nT, respectively. The vector measurements provided by the VHM and the FGM have a resolution of 2 Hz and 32 Hz, respectively [Dougherty et al., 2004]. The FGM is located halfway out on the magnetometer boom while the VHM is located at the end of the boom. Due to a malfunction, VHM stopped providing science measurements on 18 November, 2005, almost one month after T8. In this thesis we have used 1 s averaged FGM vector magnetic field measurements.
**Figure 3.4:** Diagram of the Cassini spacecraft, extracted from http://saturn.jpl.nasa.gov/.

Cassini Particle Spectrometer - Electron Spectrometer Sensor (CAPS-ELS)

The Cassini Particle Spectrometer - Electron Spectrometer Sensor (CAPS-ELS) is a hemispheric electrostatic analyzer that detects electrons in the energy range of 0.6 eV - 28 keV with an energy resolution $\Delta E/E = 0.17$ and an angular resolution of $20^\circ$. The FOV of CAPS-ELS is $5.2^\circ \times 160^\circ$ [Young et al., 2004]. In this thesis we have used electron density estimates derived from the electron distribution function moment calculations [Lewis et al., 2008].

Cassini Particle Spectrometer - Ion Mass Spectrometer (CAPS-IMS)

CAPS-IMS samples ions in eight angular sectors (anodes) with each sector having an instantaneous field of view (FOV) of $8^\circ \times 20^\circ$. As seen in Figure 3.5, ions initially enter the top-hat portion of the hemispherical electrostatic analyzer (ESA) through a collimator. A voltage applied to the inner ESA electrodes creates an electric field in the top-hat that deflects ions into the Time-Of-Flight (TOF) analyzer. Same as with the ELS, only
particles within a particular range of energy-per-charge (E/Q) and direction of arrival are transmitted through the ESA to the TOF analyzer. CAPS-IMS takes singles data (SNG) corresponding to energy-per-charge spectra ranging from 1 eV to 50 keV with a spectral resolution of 17%. The 62-step energy sweeps take 4 s to acquire the data and are formatted into 8 energy sweeps per instrument’s internal data acquisition cycle. This cycle, referred to as the A-cycle, lasts 32 s.

**Figure 3.5**: Diagram of the CAPS-IMS and TOF instruments [Young et al., 2004].

Ions that successfully exit the ESA are accelerated by -14.56 kV into one of the eight carbon foils distributed around the entrance of the TOF analyzer. When a molecular ion hits the carbon foil, it is broken up into elementary fragments (e.g. electrons, atomic ions and neutral atoms). From Figure 3.5, it is clear that the presence of a linear electric field (LEF) pushes the electrons outward toward the outer edge of the straight-through (non-mirroring) ST microchannel plate (MCP) where “start” timing events are recorded. Since negative ions have higher momentum, they are pushed slightly outwards. Neutrals follow a rectilinear path through the field and positive ions (whose energies are lower that ~15 keV) are reflected and hit the LEF MCP. Each ion species has its own TOF signature and by testing possible combinations of species one can construct an ion-summed LEF/ST signature, which can be compared to the LEF/ST observations. Then, the TOF analyzer is used to infer detailed compositional analysis in a so called B-cycle. During the B-cycle,
the eight angular sectors are summed together and the 62 energy steps are collapsed to 32 energy steps. The B-cycle lasts 256 s.

The CAPS-IMS sensors are mounted on a rotating platform capable of actuating the CAPS instrument by $\sim 180^\circ$ around an axis parallel to the spacecraft Z-axis in about 3 min. More detailed information of CAPS-IMS are presented in Young et al. [2004], Hartle et al. [2006], Sittler et al. [2010] and Wilson et al. [2012].

These observations are used to characterize the ion population and provide key information on the plasma flow direction, its composition and the relative abundance of its constitutive ion species.

Radio and Plasma Wave Science (RPWS)

The Radio and Plasma Wave Science (RPWS) investigation consists of three orthogonal electric field antennas, three orthogonal search coil magnetic antennas, and a Langmuir probe (LP) [Gurnett et al., 2004]. In the present study, we use the LP and the High and Medium Frequency Receiver observations since they provide two independent estimates for the electron number density [e.g. Edberg et al., 2010]. RPWS observations, in conjunction with the CAPS-ELS measurements, provide estimates for the electron number density at different regions around Titan. Even though the CAPS-ELS moment calculation may underestimate the electron number density at regions where Cassini is negatively charged (since it cannot observe a fraction of low-energy electrons), it is a well designed instrument for regions of hot plasmas such as that of Titan’s surrounding environment [Lewis et al., 2008]. In contrast, the electrostatic wave emissions are observed only at locations where the electron density is $\gtrsim 0.1$ cm$^{-3}$. Therefore, the RPWS instrument provides reliable measurements in regions very close to Titan, which mainly contain cold plasma. The two data sets are complementary and provide a complete characterization of the electron density in Titan’s vicinity.

3.3. Instruments onboard other missions

In the following we provide a brief description of additional instruments considered in the analysis of other IMs. In order to study the magnetic field morphology surrounding comet P/Halley, we have studied magnetic field data provided by the magnetometer onboard the Vega-1 spacecraft. To study properties of plasma waves in the extended exosphere of Venus, we have analyzed data from the magnetometer and the Analyzer of Space Plasma Instrument (ASPERA) onboard Venus Express.
3.3.1. **The Vega-1 magnetometer (MISCHA)**

Vega 1 (along with its twin spacecraft Vega 2) was a Soviet spacecraft whose purpose was to carry probes to Venus and then intercept the comet P/Halley in March 1986. Both spacecrafts were launched in December, 1984, and arrived near Venus in June 1985. After having launched their probes, they took advantage of Venus’ gravity to get the necessary speed boosts to flyby Halley’s comet the following year. Vega 1 and 2 encountered comet’s Halley on 6 and 9 March, 1986, respectively.

The magnetic field experiment MISCHA (Magnetic field in interplanetary space during comet Halley’s approach), consisted of four sensor fluxgate systems, with three sensors mounted on a boom at the end of the solar panels and a gradiometer sensor mounted one meter closer in. These instruments had a dynamic range of ±100 nT and a sensitivity of 0.05 nT [Riedler et al., 1986]. In this thesis we have analyzed high resolution magnetic field measurements (10 Hz).

3.3.2. **The Venus Express magnetometer (MAG) and plasma instrument (ASPERA)**

The Venus Express mission, launched on 9 November, 2005, is the first European space mission to this planet. It reached Venus on 11 April, 2006, when it was put into a highly elliptical polar orbit (periapsis 180-350 km, apoapsis 66000 km) with a period of 24 h. Among other instruments, it carries a magnetometer and an analyzer of space plasmas and energetic atoms which allow to investigate the Venus plasma environment [Zhang et al., 2006].

The Venus Express magnetometer consists of two triaxial fluxgate sensors, one mounted on the tip of a 1 m deployable boom, the other one directly attached to the spacecraft. Based on these dual sensor measurements, it is possible to separate the ambient magnetic field from the one generated by the spacecraft. According to Zhang et al. [2008], the accuracy of the measurements is \( \sim 1 \) nT. The sampling frequency of the measurements analyzed in this thesis is 1 Hz.

The ASPERA instrument comprises four sensors: two energetic neutral atom sensors, the electron spectrometer (ELS) and the ion mass analyzer (IMA). IMA detects ions in the energy range between 10 eV/q and 30 keV/q and the 1-44 amu/charge range, including solar wind as well as planetary ions. It has a time resolution of 192 s and a FOV of \( 90^\circ \times 360^\circ \). In this thesis we have analyzed plasma parameters derived from IMA. More specifically, we have focused exclusively on the solar wind velocity and the proton density, which are currently available on a long-time basis.

Finally, note that when data provided by only one spacecraft are analyzed, it is not
possible to differentiate between temporal variations occurring in the plasma from the
ones that are the result of its proper motion.

3.4. Methods of analysis of the measurements

3.4.1. Minimum Variance Analysis (MVA)

The main objective of the minimum variance analysis (MVA) is to find, from single-
spacecraft magnetometer data, an estimation for the direction normal to a one-dimensional
or approximately one-dimensional current layer, wave front, or other transition layer in
a plasma. Next, the method is developed to determine the unit normal vector, \( \hat{n} \), from
minimum variance analysis of magnetic field vector measurements (MVAB) obtained by
a spacecraft as it moves through the structure in question.

To derive an estimate of \( \hat{n} \), the method identifies the spatial direction along which
the variance of the field-component set \( \{ \mathbf{B}^{(m)} \cdot \hat{n} \} \), \( m = 1, \ldots, M \) is the smallest. In other
words, \( \hat{n} \) is determined by minimization of

\[
\sigma^2 = \frac{1}{M} \sum_{m=1}^{M} |(\mathbf{B}^{(m)} - \langle \mathbf{B} \rangle) \cdot \hat{n}|^2
\]

(3.1)

where the average \( \langle \mathbf{B} \rangle \) is defined by

\[
\langle \mathbf{B} \rangle \equiv \frac{1}{M} \sum_{m=1}^{M} \mathbf{B}^{(m)}
\]

(3.2)

and where the minimization is subject to the normalization constraint \( |\hat{n}|^2 = 1 \).

According to Sonnerup and Scheible [1998], the resulting set of three equations can
be written in matrix form as:

\[
\sum_{\nu=1}^{3} M_{\mu\nu} B_{\nu} = \lambda B_{\mu}
\]

(3.3)

where the subscripts \( \mu, \nu = 1, 2, 3 \) denote the cartesian components along the X, Y, Z
system and

\[
M_{\mu\nu} \equiv \langle B_{\mu} B_{\nu} \rangle - \langle B_{\mu} \rangle \langle B_{\nu} \rangle
\]

(3.4)

is the magnetic variance matrix. From equation 3.3 we see that the allowed \( \lambda \) values
are the eigenvalues \( \lambda_1, \lambda_2, \lambda_3 \) (in decreasing order) of \( M_{\mu\nu} \). Since \( M_{\mu\nu} \) is symmetric, the
eigenvalues are all real and the corresponding eigenvectors, \( \mathbf{x}_1, \mathbf{x}_2 \) and \( \mathbf{x}_3 \), are orthogonal.
The three eigenvectors represent the directions of maximum, intermediate, and minimum
variance of the field component along each vector, respectively. Note that the sense and
magnitude of the eigenvectors remain arbitrary so that, for example, \( x_i, kx_i \) and \(-kx_i\), \((i = 1, 2, 3)\) are all valid eigenvectors. The corresponding \( \lambda \) values represent the actual variances in those field components and are therefore non-negative.

In order to find \( \hat{\mathbf{n}} \), we first determine the matrix \( M_{\mu\nu}^B \), following equation 3.4 (in terms of the measured field data), and then find its three eigenvalues \( \lambda_i \) and the corresponding eigenvectors \( \mathbf{x}_i \). The eigenvector \( \mathbf{x}_3 \) corresponds to the smallest eigenvalue, \( \lambda_3 \), and is used as the estimation for the vector normal to the current sheet or wave front, while \( \lambda_3 \) itself represents the variance of the magnetic field component along the estimated normal. The eigenvectors \( \mathbf{x}_1 \) and \( \mathbf{x}_2 \), correspond to the maximum and intermediate variance, and are therefore contained in the transition layer plane. The set \( \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \), arranged as a right-handed orthonormal triad, provides a suitable basis for further analyses. More generally, for any set of vectors \{\( \mathbf{B}^{(m)} \}\), not necessarily obtained from a spacecraft crossing a transition layer or wave front, the eigenvector set of the variance matrix \( M_{\mu\nu}^B \), derived from the data provides a convenient natural coordinate system in which to display and analyze the data. Note also that the matrix \( M_{\mu\nu}^B \) is independent of the temporal order of the measured vectors.

**Error Estimates**

Following the development by Sonnerup and Scheible [1998], the uncertainties in the directions of the eigenvectors \( (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \) of the variance matrix, \( \mathbf{M}^B \equiv \mathbf{M} \), are estimated by performing a perturbation analysis on the eigenvector equation 3.3 around the unknown noise-free state which is denoted by an asterisk:

\[
(\mathbf{M}^* + \Delta\mathbf{M}) \cdot (\mathbf{x}_i^* + \Delta\mathbf{x}_i) = (\lambda_i^* + \Delta\lambda_i) \cdot (\mathbf{x}_i^* + \Delta\mathbf{x}_i)
\]

(3.5)

Here \( i = 1, 2, 3 \), correspond to maximum, intermediate, and minimum variance associated with \( \mathbf{M}^* \), respectively. The linearized version of this equation becomes

\[
\Delta\mathbf{M} \cdot \mathbf{x}_i^* + \mathbf{M}^* \cdot \Delta\mathbf{x}_i = \Delta\lambda_i \mathbf{x}_i^* + \lambda_i^* \Delta\mathbf{x}_i
\]

(3.6)

Equation 3.6 is written in the unperturbed eigenbasis in which \( \mathbf{M}^* \) is diagonal. Therefore, the jth component of equation 3.6 results,

\[
(\lambda_j^* - \lambda_i^*) \Delta x_{ij} = -\Delta M_{ij} - \Delta \lambda_i \delta_{ij}
\]

(3.7)

where \( \Delta x_{ij} \) is the jth component of the vector \( \Delta \mathbf{x}_i \). Since \( \mathbf{M} \) and \( \mathbf{M}^* \) are both symmetric matrices, we have \( \Delta M_{ij} = \Delta M_{ji} \). As it can be seen from equation 3.7, \( \Delta x_{ij} = -\Delta x_{ji} \).
an equality which expresses the fact that the perturbed eigenvectors must form an orthonormal triad. Equation 3.7 can then be written as:

\[ \Delta x_{31} = -\Delta x_{13} = -\Delta M_{13}/(\lambda_1^* - \lambda_3^*) \]  

(3.8)

\[ \Delta x_{32} = -\Delta x_{23} = -\Delta M_{23}/(\lambda_2^* - \lambda_3^*) \]  

(3.9)

\[ \Delta x_{21} = -\Delta x_{12} = -\Delta M_{12}/(\lambda_1^* - \lambda_2^*) \]  

(3.10)

\[ \Delta x_{ii} = 0 \]  

(3.11)

Note that in the linear approximation, the quantities \( \Delta x_{31} \) and \( \Delta x_{32} \) also represent the angular rotations (in radians) of the eigenvector \( \mathbf{x}_3 \) towards \( \mathbf{x}_1 \) and \( \mathbf{x}_2 \), respectively. Similarly, \( \Delta x_{21} \) represents the angular rotation of \( \mathbf{x}_2 \) towards \( \mathbf{x}_1 \).

Up to this point, we have obtained the errors in the determination of the eigenvectors derived from the MVA in terms of \( \Delta \mathbf{M} \). Nevertheless, we can obtain a formal expression for \( \Delta M_{ij} \) by replacing \( \mathbf{B}^{(m)} \) by \( \mathbf{B}^{(m)*} + \Delta \mathbf{B}^{(m)} \) in the definition 3.4 of \( \mathbf{M} \), where \( \Delta \mathbf{B}^{(m)} \) is the noise component present in the magnetic field measurements. The detailed procedure, which involves taking an average over a large number (ensemble) of realizations of the noise component of the measured field, can be found in Somerup and Scheible [1998]. Neglecting terms of order \( \epsilon^2 \equiv [\lambda_3/(\lambda_2 - \lambda_3)]^2/(M - 1)^2 \) compared to unity, the result is

\[ |\Delta \varphi_{ij}| = |\Delta \varphi_{ji}| = << (\Delta x_{ij})^2 >>^{1/2} = << (\Delta x_{ji})^2 >>^{1/2} = \sqrt{\frac{\lambda_3}{(M - 1)}} \frac{(\lambda_i + \lambda_j - \lambda_3)}{(\lambda_i - \lambda_j)^2}, \quad i \neq j \]  

(3.12)

where \( << \ldots >> \) denotes the ensemble average and \( |\Delta \varphi_{ij}| \) represents the angular uncertainty of eigenvector \( \mathbf{x}_i \) for rotation toward or away from eigenvector \( \mathbf{x}_j \).

The statistical uncertainty in the component of the average magnetic field along the eigenvector \( \mathbf{x}_3 \) is composed of three parts: the uncertainty in the average associated with the corresponding variance \( \lambda_3 \) and the two uncertainties associated with the angular error estimates for \( \mathbf{x}_3 \). Assuming that these errors are independent, we can write the total statistical error estimate for \( \mathbf{< B >} \cdot \mathbf{x}_3 \) as

\[ |\Delta \mathbf{< B >} \cdot \mathbf{x}_3| = \sqrt{\frac{\lambda_3}{(M - 1)}} + (\Delta \varphi_{32} \mathbf{< B >} \cdot \mathbf{x}_2)^2 + (\Delta \varphi_{31} \mathbf{< B >} \cdot \mathbf{x}_1)^2 \]  

(3.13)

Similar expressions can be written for the uncertainties in \( \mathbf{< B >} \cdot \mathbf{x}_1 \) and in \( \mathbf{< B >} \cdot \mathbf{x}_2 \), but these error estimates are usually of less interest.
MVA applied to plane waves

By means of the MVA we can estimate the direction normal to a wavefront (assuming that the wave is a planar fluctuation), this is, the direction of the wavector \( \mathbf{k} \).

The study of the variations of the magnetic field in the plane of maximum and intermediate variance allows to determine the polarization of the plasma waves by means of hodograms. A magnetic hodogram is a curve that is determined as follows: we draw vectors from the origin of the coordinate system, whose longitudes and directions represent the measured data set \( \{ \mathbf{B}^{(m)} \} \) in the principal variance coordinate system and then we connect their ends using segments that follow the temporal sequence in which they were measured. Usually two projections of the hodogram are presented: one corresponding to the plane perpendicular to \( \mathbf{k} \), and another one that contains the latter vector. If \( \lambda_3 \sim \lambda_2, \lambda_1 >> \lambda_3 \), the wave polarization is quasi-linear. In the case of elliptical polarization \( \lambda_3 << \lambda_2 < \lambda_1 \), while in the case of circular polarization \( \lambda_3 << \lambda_2 \sim \lambda_1 \) is fulfilled. The sense in which the magnetic field rotates in the hodogram with respect to the mean magnetic field projection along \( \mathbf{x}_3 \) (magnetic field components that we denominate \( B_1 \) and \( B_2 \)) determines the sense of polarization in the reference frame of the spacecraft.

Moreover, we also determine the angle between the minimum variance eigenvector and the mean magnetic field, i.e., the angle between the wavector \( \mathbf{k} \) (where we assume that it coincides with \( \mathbf{x}_3 \)) and \( \mathbf{B}_0 = \langle \mathbf{B} \rangle \). This angle, denoted as \( \theta_{kB} \), is:

\[
\theta_{kB} = \arccos \left( \frac{\hat{x}_3 \cdot \mathbf{B}_0}{B_0} \right)
\]

where we choose \( 0 \leq \theta_{kB} \leq \pi/2 \). The value of \( \theta_{kB} \) allows to determine if the waves are propagating parallel, perpendicular or oblique to the mean magnetic field in the reference frame of the spacecraft.

### 3.4.2. Discrete Fourier Transform

The Fourier transform of a discrete time series \( x(n) \) is defined as:

\[
X(w) = \sum_{n=-\infty}^{\infty} x(n) \exp(-wni)
\]

where \( X(w) \) is a decomposition of \( x(n) \) in its frequential components. Also note that \( X(w) \) is periodic with a period of \( 2\pi \).

Since we are interested in the frequential components that are present in the expansion of the measured discrete signal (and not in their relative phase shifts), we calculate the
power spectral density $P(w)$ based on the Fourier transform. $P(w)$ is defined as the power density of the signal in the range $(w, w + dw)$, this is:

$$P(w) = |X(w)|^2$$

(3.16)

In other words, $P(w)$ shows the power distribution of the frequencies contained in the original signal. The highest frequency that can be distinguished without ambiguity, the Nyquist frequency, is defined as $f_{Ny} = f_s/2$, where $f_s$ is the sampling frequency of the signal. The frequency resolution of the spectrum is given by the inverse of the period $T$ over which the spectrum has been calculated.

In Chapter 5 we analyze MGS measurements making use of Fourier transforms and the power spectral density. We also determine the time evolution of the power and frequency of these measurements by means of dynamic Fourier spectra known as spectrograms. Such spectrograms consist in analyzing the measurements by grouping them into subintervals (each one defines a window) where the power spectral density is calculated independently. The derived $P(w)$ associated with each window allows to study the evolution of the signal over time. To examine such evolution in more detail, we also overlap those windows.

### 3.4.3. Cross-correlation between discrete signals

The cross correlation $r_{xy}(l)$ between two arbitrary discrete signals $x(n)$ and $y(n)$ with length $N$ is defined as follows:

$$r_{xy}(l) = \frac{\sum_{n=0}^{N-1} (x(n) - m_x) (y(n - l) - m_y)}{\sqrt{\sum_{n=0}^{N-1} (x(n) - m_x)^2 \sqrt{\sum_{n=0}^{N-1} (y(n) - m_y)^2}}}$$

(3.17)

where $i = l, k = 0$ for $l \geq 0$, and $i = 0, k = l$, for $l < 0$. The $l$ index is the parameter of displacement or time delay, and $m_x$ and $m_y$ are the mean values of the corresponding series. The subscripts $xy$ in the sequence $r_{xy}(l)$ indicate what signals have been correlated and their order, for example $x$ preceding $y$ indicates the direction in which one sequence is displaced from the other. In this case, the sequence $x(n)$ has not been displaced while the sequence $y(n)$ was displaced $l$ samples.

Based on the calculation of this parameter for magnetic field and electron flux data sets (provided by MAG/ER), we seek to characterize the oscillation mode present in the plasma surrounding Mars. Indeed, the common application of the cross correlation parameter is to derive the phase shift between two signals. An important point must be stressed: this calculation assumes that the data set $x(n)$ and $y(n)$ are sampled at the same frequency. As we will see in Chapter 5, the sampling frequencies of the MAG magnetometer and the electron reflectometer are not necessarily the same. Therefore, it will be necessary to interpolate one of the two signals to the cadence of the other.
3.4.4. Coherence

The coherence function associated with two given signals $x(t)$ and $y(t)$ is given by:

$$C_{xy}(w) \equiv \frac{|P_{xy}(w)|^2}{P_x(w)P_y(w)}$$

(3.18)

where $P_x(w), P_y(w)$ are the power spectral density of the signal $x(t)$ and $y(t)$, respectively; and $|P_{xy}(w)|$ is the cross spectral density defined as $P_{xy}(w) = X(w)Y^*(w)$.

This real function measures the linear correlation between both time series at each frequency. As it can be seen from Equation 3.18, the coherence function that takes values between 0 and 1 for each value of $w$. By means of this function we will study the coherence of magnetic field observations obtained from the MGS spacecraft.

3.4.5. The deHoffman-Teller analysis

The deHoffmann-Teller frame, hereafter referred to as the HT frame, is a Galilean frame of reference in which the electric field vanishes [deHoffmann and Teller, 1950]. Therefore, in the HT frame, energy is conserved. The existence of a HT frame or, in the experimental context, an approximate HT frame, indicates that a coherent quasi-stationary pattern of magnetic field and plasma velocity such as a wave or current layer, is present. As a consequence, the observed time variation in these events would be, to a first order approximation, due to the steady motion of the pattern relative to the instrument’s frame. In cases where the convection electric field can be used as a proxy for $\mathbf{E}$, the HT analysis then consists in finding the frame velocity $\mathbf{V}_{HT}$ that best agrees with a set of measurements of magnetic field, $\mathbf{B}$, and plasma bulk velocity, $\mathbf{v}$. Such analysis allows to identify the passage of a moving quasi-static structure, whose existence and identification might prove useful for subsequent analyses.

An approximate HT frame can be identified by determining the value of the frame velocity $\mathbf{V}_{HT}$ that minimizes the mean square of the electric field $D(\mathbf{V})$ given by:

$$D(\mathbf{V}) = \frac{1}{M} \sum_{m=1}^{M} |\mathbf{E}^{(m)}|^2 = \frac{1}{M} \sum_{m=1}^{M} |(\mathbf{v}^{(m)} - \mathbf{V}) \times \mathbf{B}^{(m)}|^2$$

(3.19)

where $\mathbf{v}^{(m)}$ and $\mathbf{B}^{(m)}$ are the measurements of plasma bulk velocity and magnetic field, and $m$ goes from 1 to $M$, being $M$ the number of observations obtained during the time interval under study. This critical frame velocity $\mathbf{V}_{HT}$ is known as the deHoffmann-Teller velocity. As shown in Khrabrov and Sonnerup [1998] the minimization condition for $D(\mathbf{V})$ leads to the following equation which we use to derive $\mathbf{V}_{HT}$:

$$\mathbf{V}_{HT} = \mathbf{K}_0^{-1} < \mathbf{K}^{(m)} \mathbf{v}^{(m)} >$$

(3.20)
where
\[ \mathbf{K}^{(m)} = B^{(m)^2} \left( \delta^{(m)}_{\mu\nu} - \frac{B^{(m)}_{\mu} B^{(m)}_{\nu}}{B^{(m)^2}} \right) \]

(3.21)

\( \mathbf{K}^{(m)} \) is a matrix associated with the \((m)\) time interval and its elements \(K^{(m)}_{\mu\nu}\) are related to the \(\mu, \nu\) magnetic field components \(B^{(m)}_{\mu}, B^{(m)}_{\nu}\) and to the magnetic field intensity \(B^{(m)}\) by the expression shown in equation 3.21 (\(\delta^{(m)}_{\mu\nu}\) is the Kronecker delta matrix). The angle brackets \(< \ldots >\) denote the average of the enclosed quantity over the set of measurements and \(K_0 \equiv < \mathbf{K}^{(m)} >\).

A characterization of the quality in the determination of the approximate HT frame can be obtained from the value of the ratio \(D(\mathbf{V}_{HT})/D(0)\) and the correlation coefficient \(R_{HT}\) between the following two electric fields: \(\mathbf{E}^{(m)}_{c} = -\mathbf{v}^{(m)} \times \mathbf{B}^{(m)}\) and \(\mathbf{E}^{(m)}_{HT} = -\mathbf{V}_{HT} \times \mathbf{B}^{(m)}\) [Khrabrov and Sonnerup, 1998].

### 3.4.6. The Walén Test

Magnetic tension forces are expected to be important in regions where Alvénic structures such as rotational discontinuities or current layers are present. In these structures, the components of the plasma velocity tangential to those layers change in response to the \(\mathbf{J} \times \mathbf{B}_n\) force, where \(\mathbf{B}_n = \mathbf{B} \cdot \hat{n}\) and \(\hat{n}\) is normal to the discontinuity plane. Indeed, as stated in Paschmann and Sonnerup [2008], a plasma flowing across such structures satisfies that:

\[ \Delta \mathbf{v} = \pm \Delta \mathbf{v}_A \]

(3.22)

where \(\Delta \mathbf{v}\) and \(\Delta \mathbf{v}_A\) are the variations in the plasma and Alvén velocities, respectively. Equation 3.22 is the so-called Walén relation.

This relation has a much simpler formulation in the HT reference frame [Khrabrov and Sonnerup, 1998]. Indeed, in this reference frame, the Walén relation can be written as (see Appendix A or [Paschmann and Sonnerup, 2008]):

\[ \mathbf{v}' = \pm \mathbf{v}_A \]

(3.23)

where \(\mathbf{v}'\) and \(\mathbf{v}_A\) are the plasma bulk velocity and the Alvén velocity in the HT frame, respectively. As a consequence of this simplification, a combination of the HT analysis together with a Walén test [Khrabrov and Sonnerup, 1998] is usually applied to study plasma acceleration events in the presence of discontinuities or current layers and to establish whether the magnetic tension forces can be accountable for the changes in the kinetic energy of the plasma. As a result, the Walén test consists in finding an approximate HT reference frame, and then plotting the estimated plasma bulk velocity components (after
transformation into the HT frame) against the corresponding components of the calculated local Alfvén velocities. When applied to an ideal Alfvénic structure, the scatter plot should show a correlation coefficient and a slope of ±1. In Chapter 6 we analyze Cassini plasma measurements making use of the HT analysis and the Walén test to determine if magnetic tension is responsible for the changes observed in the kinetic energy of the plasma within the IM of Titan.
Resumen en castellano

Este capítulo describe las capacidades técnicas de los instrumentos que proveyeron los datos para la presente tesis, analizados a lo largo de los siguientes capítulos. Nos centramos principalmente en las misiones Mars Global Surveyor y Cassini dado que la mayoría de los resultados presentados conciernen a Marte y Titán. En este capítulo también resumimos los métodos de análisis de las mediciones in-situ provistas por las sondas espaciales utilizadas en esta tesis.

Respecto a la misión Mars Global Surveyor, se describieron sus objetivos, la geometría orbital de su correspondiente sonda como así también las capacidades de su magnetómetro y reflectómetro de electrones a bordo.

Además describimos brevemente los objetivos de la misión Cassini y el funcionamiento del magnetómetro, el espectrómetro de electrones y protones y el instrumento de ondas de plasma y radio.

En lo que se refiere a estudios realizados con base en las misiones a Venus y al cometa Halley, describimos la capacidades del magnetómetro y el instrumento de plasma a bordo de la sonda Venus Express, y del magnetómetro a bordo de la sonda Vega-1.

Los métodos de análisis que son descriptos son los siguientes:

- Análisis de Varianza Mínima
- Transformada de Fourier Discreta
- Correlación cruzada entre señales discretas
- Coherencia
- El análisis de deHoffman-Teller
- El test de Walén
CHAPTER 4

Magnetic field topology within induced magnetospheres

This chapter is devoted to the study of the magnetic field topology that results from the interaction between a magnetized plasma flow and an atmospheric unmagnetized conductive obstacle. We are particularly interested in the exchange of energy and momentum between the external flow and the conductive body. Indeed, the magnetic field topology surrounding an atmospheric unmagnetized object is mainly the result of gradients in the velocity of the external flow. These gradients are, in turn, the consequence of two processes that couple the two components of this complex system. One of them is the massloading of the plasma flow with ionized particles of atmospheric origin. The second process involves the forces associated with the presence of currents on the obstacle’s effective conducting surface. These currents, as well as the previously mentioned massloading, slows down the magnetized plasma wind in front of the obstacle. In locations where the collisionless regime holds, the streaming magnetic field frozen into the plasma piles up in front of the stagnation region of the flow. At the same time, the magnetic field lines are stretched in the direction of the unperturbed magnetized plasma wind as the stream moves away from the body, giving rise to a magnetic tail.

In a few words, as a consequence of the deceleration of the external magnetized plasma
flow, the streaming magnetic field is draped (wrapped) around the obstacle. This coupling strongly defines the magnetic field topology in the surroundings of atmospheric unmagnetized conductive objects. In this chapter we study their topology from the theoretical and observational points of view. From the theoretical side, we develop an analytical MHD model which allows us to determine the electromagnetic field associated with a perfectly conducting magnetized plasma flow interacting with a spherical conducting body, under steady state conditions. On the observational side, we characterize the magnetic field morphology observed within the induced magnetospheres of the comet P/Halley and Mars, based on measurements obtained by the Vega-I and MGS spacecrafts, respectively.

4.1. The magnetic pile-up and magnetotail formation

In 1957, Alfvén considered a simplified scenario in which a conductive body of radius $R$ and conductivity $\sigma$ interacts with a magnetized plasma flowing with velocity $V_w$. In particular he focused on the case in which these parameters satisfy that $4\pi\sigma RV_w \gg c^2$, a condition equivalent to consider a magnetic Reynolds number $R_m \gg 1$. In this limit, the magnetic diffusion time through the body is much longer than the time for the magnetized plasma wind to sweep past it, and therefore the streaming magnetic field cannot penetrate it. Since the plasma is frozen into the magnetic field, it must slow down ahead of the body and stop just before it. As the plasma slows down, the magnetic field piles up in front of the body. Since the magnetic field continues to be convected sideways by the unperturbed flow, the magnetic field lines bend around the object to form a magnetic tail downstream. These considerations provide the simplest description of the interaction of the SW with a conductive body, and served to explain the formation of cometary plasma tails [Alfvén, 1957]. Figure 4.1 is adapted from the work of Alfvén [1957] on the formation of cometary tails.

As commented earlier, the magnetic field increases as the plasma slows down approaching the object. This effect can be easily derived when the plasma wind speed and the magnetic field are perpendicular to each other and the stationary state has been achieved. In a one-dimesional approximation these fields can be written as $\mathbf{V} = V(x)\mathbf{\hat{x}}$, $\mathbf{B} = B(x)\mathbf{\hat{y}}$, with the $x$ axis being antiparallel to the unperturbed flow velocity, i.e., with $x$ being the distance to the front of the body. In this particular case, the frozen-in induction equation yields:

$$V(x)B(x) = \text{constant} \quad (4.1)$$

Therefore, as the body is approached, the amplitude of the magnetic field increases, an increase that stops close to the body because the frozen-in approximation breaks down.
Magnetic field topology within induced magnetospheres

**Figure 4.1:** Sketch of the interaction of a large conducting body with the solar wind. The solar wind magnetic field cannot penetrate the body, forcing the incoming plasma to slow down ahead and be diverted sideways, whereas the magnetic field lines pile up ahead and a magnetic tail forms downstream. The x-axis referred to in the text is antiparallel to \( \mathbf{V}_w \), the y-axis is parallel to \( \mathbf{B}_w \). Adapted from a sketch of a comet plasma tail by Alfven [1957]. Figure extracted from Meyer-Vernet [2007].

at small scales as it also does the one-dimensional approximation (the flow must divert). Despite the large magnetic field ahead of the object, the magnetic field vanishes inside because a sheet of electric current \( \mathbf{J} \) perpendicular to the plane of Figure 4.1 is formed. This current is driven by the electric field induced in the frame of the body by the motion of the magnetized plasma wind: \( \mathbf{E} = -\frac{\mathbf{V}_w}{\mathbf{r}} \times \mathbf{B}_w \). Regarding the magnetotail structure and orientation, it is important to stress that they depend on the direction of the external magnetic field. The magnetotail consists of two lobes of opposite magnetic polarity generated by the convection of the corresponding field lines, separated by a polarity reversal layer (PRL).

In spite of being able to predict the basic properties observed in these environments, the previous picture misses several important points, such as:

- First, if the magnetized plasma wind is supersonic, it cannot be stopped without forming a bow shock, where the wind slows down to subsonic speeds. Note that the presence of this boundary does not change the maximum magnetic field intensity (in front of the body) which can be derived based on the previous explanation. This is because the total plasma wind pressure is not lost in a shock, but converted into thermal and magnetic pressure once the flow crosses it.

- Second, the observations in our solar system show that the conductivity required to withstand the magnetized solar wind in the case of unmagnetized bodies is provided by their ionospheres rather than by the bodies themselves.
Magnetic field topology within induced magnetospheres

- Third, this picture is also modified when the atmospheric unmagnetized objects possess extended exospheres. In these cases, some of their neutral constituents can be ionized far from their ionospheres. As soon as these newborn ions are created, they are accelerated by the streaming magnetic and convective electric fields in a process known as ion pick-up. When the energy taken from the external flow becomes significant, the magnetized plasma flow is slowed down. This macroscopic phenomenon takes place when the external flow has been significantly massloaded with particles of atmospheric origin. This effect is particularly important in the case of active comets and also in Mars and Venus, albeit more weakly.

- Observations on several objects in the solar system show that the magnetic field draping and pile-up take place abruptly at the MPB. This is due to non-linear effects that involve the ionosphere as well as the exosphere of these obstacles.

Despite all these factors, the basic picture concerning the magnetic field topology around these objects remains unaffected. This is because in the end, the magnetic field topology around these objects relies on the draping of the streaming magnetic field lines, as a consequence of the gradients in the plasma velocity. Additionally, on the downstream side, the draped magnetic field transfers momentum to the atmospheric newborn ions via magnetic curvature forces in the tail, accelerating them. As they catch up with the magnetized external wind, the draping decreases and vanishes, leading to the end of the magnetotail.

An example of the magnetic field morphology in an induced magnetotail can be found in the observations in the environment of comet Giacobini-Zinner (Figure 4.2). The magnetic field measurements were provided by the International Cometary Explorer (ICE) spacecraft. On 11 September, 1985, ICE crossed the tail of the comet at about 8000 km from the nucleus, at a relative speed of 21 km s\(^{-1}\), when it was at 1 AU from the Sun. The figure displays the magnetic field superimposed onto an optical image from ground based observations. As it can be seen, the magnetic field is draped around the comet, producing a magnetic tail 10\(^4\) km wide (dotted line), where the magnetic field peaks at about 60 nT in each lobe, and vanishes in the 1200 km wide plasma sheet.

A second example concerns the magnetic field morphology in the Martian environment from MGS MAG data obtained on 11 October, 1997 (Figure 4.3). This figure shows the magnetic strength near Mars, as MGS travels from the SW down to the ionosphere, and back into the solar wind. The magnetic field starts piling up as soon as the MPB is crossed, an abrupt boundary where the solar wind plasma is replaced by plasma of planetary origin [Nagy et al., 2004]. This feature is out of the scope of a traditional fluid description, for two basic reasons: the solar wind and planetary particles have very dif-
Figure 4.2: Magnetic field measured on the ICE spacecraft when it crossed the tail of comet Giacobini–Zinner at about 1 AU from the Sun, projected onto an image of the comet taken with the Canada–France–Hawaii Telescope. This illustrates the draping of the magnetic field, producing two lobes of opposite polarities. Figure extracted from Meyer-Vernet [2007].

Different properties to be treated as a single fluid; and some of these particles species might not behave as a fluid. In particular, the planetary heavy ions have gyroradii larger that the size of the planet (see section 2.8). Inside the MPB, magnetic field measurements resemble the schematic picture presented in Figure 4.1. In this figure the different regions and discontinuities are also shown. Near CA a drop in the field strength was detected, showing that the SC entered the magnetic cavity, a region where the plasma is strongly shielded from the streaming magnetic field.

In the following we present a description of previous studies concerning the increase of magnetic field draping signatures across the MPB of different objects.
4.2. Magnetic field draping enhancement across the MPB

The measurements made by the magnetometer onboard the Mars Global Surveyor spacecraft [Acuña et al., 1992] showed that Mars does not have a significant intrinsic magnetic field [Acuña et al., 1998]. As a result, there is a direct interaction between the magnetized solar wind and the atmosphere/ionosphere of the planet. This coupling can be described in terms of the interaction between a non-collisional magnetized plasma flow and a neutral cloud being ionized through different mechanisms. Indeed, as part of this interaction, the atmosphere of Mars is subject to several ionizing mechanisms such as photoionization, charge-exchange and electron impact. Also, through several current systems, it generates perturbations in the streaming interplanetary magnetic field, leading to its draping around the Martian effective conducting surface. Regarding the latter point, Bertucci et al. [2003a] identified a sudden and drastic enhancement of the magnetic field line draping taking place at the MPB of Mars. Such identification was performed by means of a correlation technique between two specific magnetic field components (see Israelevich et al. [1994]) applied to MGS MAG measurements\(^1\). This result is in clear contrast with a picture of a progressive draping between the bow shock and the final

\(^1\)The magnetic field draping imposes a strong correlation between the magnetic field components, in clear contrast to regions with high wave activity. As shown in Israelevich et al. [1994], this effect can be clearly appreciated when the magnetic field component parallel to the external flow is correlated with the radial (cylindrical) component to such direction.
planetary obstacle. As soon as the MPB is crossed, the magnetic field becomes suddenly organized, in opposition to what happens in the magnetosheath. In the MPR, the frozen-in magnetic field lines follow a plasma which is denser and cooler, a consequence of the mass loading which contributes to the dominance of heavy ions of exospheric origin [Lundin et al., 1990]; [Crider et al., 2000]. The lower limit of this region is the final planetary obstacle, i.e. its ionopause or the remnant Martian crustal magnetic fields. Interestingly, the enhancement of draping signatures has been observed in numerous MGS orbits, regardless of the presence of crustal magnetic sources and the strength of the gradient in the magnetic field intensity at the MPB.

In the case of Venus, Bertucci et al. [2003b] applied the same technique and identified the MPB around this planet, using Pioneer Venus Orbiter (PVO) magnetometer data. The results showed a drastic change in the magnetic field morphology taking place across a very thin MPB (on the Venusian dayside), the outer edge of the Venusian MPR. As soon as this boundary is crossed, the magnetic field suddenly becomes organized and strongly draped. This sudden enhancement of draping sees independent from the IMF orientation upstream from the bow shock. Just as for Mars, it was not necessarily associated to a strong jump on $|B|$ between the MSH and the MPR since it occurs even if $|B|$ varies smoothly.

With regard to the cometary obstacles, missions such as Vega-1 and Vega-2 to comet P/Halley, Giotto to comet P/Halley and Grigg-Skjellerup [von Rosenvinge et al., 1986] and ICE to comet Giacobini-Zinner [Reinhard, 1986] provided magnetometer measurements that proved the general picture of draping of IMF lines [Alfven, 1957]. The enhancement of draping has also been observed at comet Grigg-Skjellerup MPB, crossed on the dayside by Giotto [Neubauer et al., 1993a], and at comet Giacobini-Zinner (G-Z) magnetotail by ICE [Slavin et al., 1986]. In the latter case, the magnetic field increases in magnitude and adopts a draped configuration once ICE is inside the comet’s tail boundary.

Below, we investigate whether there is evidence of draping enhancement in Vega-1 MISCHA magnetic field data, and, if such enhancement could help in the identification of an MPB.

**Magnetic field draping at Comet Halley: observations from Vega-1**

The term magnetic pile-up boundary was coined by Neubauer [1987] to refer to a sharp boundary found around comet P/Halley and characterized by a sharp increase in the magnetic field strength during the Giotto encounter. Follow-up studies on Giotto magnetometer data contributed to a better understanding of the draping around the
comet’s coma. Israelevich et al. [1994] applied this technique to Giotto measurements and found that the MPB (crossed on the dayside) separates the cometary magnetosheath (where there is no evidence of significant draping) from the MPR, with strongly draped fields.

Two other spacecrafts, Vega-1 and Vega-2, flew by comet Halley on 6 and 9 March 1986, with CA at 8890 and 8030 km from the nucleus, respectively. In contrast to the Giotto data set, the MPB was not observed by the Vega magnetometer (MISCHA) [Schwingenschuh et al., 1986].

In the absence of a sharp signature in $|B|$, we decided to look for an MPB using MISCHA-high resolution (10 Hz) data by finding a spatial place where draping was enhanced. To do so, we employed the technique described in Israelevich et al. [1994] and consisting in the study of the correlation between the flow aligned ($B_x$) and the cross-flow ($B_{rad}$) components of the magnetic field. According to this technique, draping is enhanced if the correlation between $B_x$ and $B_{rad}$ suddenly improves. Figure 4.4 shows the magnetic field strength measured by Vega-1 MISCHA between the BS and CA to comet Halley (top panel) and the SC altitude (bottom panel). CA is indicated with the dashed green line, at 07:20hs. At first sight, a high level of magnetic field strength variability is observed throughout the encounter except between 06:10hs (first dashed red line) and 06:47hs (second dashed red line). Also, we mark two boundaries, ‘C1’ (07:10hs, before CA) and ‘C2’ (07:24hs, after CA), based on a previously identified change in a magnetic field component [Riedler et al., 1986]. These boundaries separate regions with different draping patterns that result from a remnant of a draped IMF.

Our main results are:

- Although magnetic field fluctuations are significantly reduced between 06:10 and 06:47 (this could be associated with Vega-1 being in the MPR, outside the fluctuating fields proper of the MSH), we do not find a significant difference in the correlation coefficients when compared to more variable time intervals before 06:10. Therefore, even though the outer edge of the region with low wave activity (crossed at 06:10) may be attributed to the MPB, significant draping enhancement signatures are not observed there. Also, the magnetic field intensity increases gradually: +35 nT over a large distance (from $3.5 \times 10^5$ km at 06:10 to $9.6 \times 10^4$ km at 07:00), while at the time of the Giotto encounter a very sharp increase of +20 nT over a distance of $1.35 \times 10^5$ km was reported [Neubauer, 1987].

- The data set between 07:00 and 07:35 is divided into intervals that take into account the tangential discontinuities C1 and C2. Figure 4.5 displays the calculated correlation for each sub-interval. The top left panel shows a high correlation co-
Figure 4.4: Top panel: time series of high resolution magnetic field strength \(B_t\) in the magnetosheath around closest approach (CA, green bar at 07:20); fluctuations are significantly lower between 06:10 and 06:47, marked with red bars. Earlier detected features, marked as, C1 and C2, are explained in the text. Lower panel: distance of the SC to the comet [Delva et al., 2014].

...efficient value (-0.91), which increases even more in the next interval (-0.99). We conclude that clear magnetic field draping signatures are first observed at 07:00 hs approximately. This trend also continues after the tangential discontinuity (C1) is crossed, until the end of the interval (07:16). During the time interval from 07:17 to 07:24 the \(B_{z_{CSE}}\) sensor is in saturation, and no correlation analysis can be performed. On the outbound leg, a strong correlation is found again in the time interval 07:24-07:27, that only diminishes as the SC leaves the close neighbourhood of the comet (Fig. 4.5, bottom panels).

This is, in the inbound leg, and inside the bow shock (not shown), the SC observes strong magnetic field fluctuations up to a distance of \(3.3 \times 10^5\) km (06:10). This reduction continues up to a distance of \(1.5 \times 10^5\) km (06:47), where the variability starts raising again. In spite of this, magnetic field draping signatures are weak during the 06:00-07:00 interval. At 07:00, the correlation between the \(B_x\) and \(B_{rad}\) field component is high, indicating a strong enhancement in the IMF draping. Also, the magnetic field strength increases. Therefore, we conclude that Vega-I entered the MPR around that moment.
between $B_x$ and $B_{rad}$, for four intervals around closest approach, the color refers to the respective time interval in Figure 7. For the interval 07:16 to 07:24 no correlation is calculated, because the magnetometer was saturated. [Delva et al., 2014].

Figure 4.5: Correlation between $B_x$ and $B_{rad}$, for four intervals around closest approach. For the interval 07:16 to 07:24 no correlation is calculated, because the magnetometer was saturated. [Delva et al., 2014].

after a gradual transition from the reported start of occurrence of cold cometary ions at 06:55 [Gringauz et al., 1986], down to the fully developed pattern of field draping. Near closest approach (CA), a drop in the field strength was not detected, since the SC did not enter the magnetic cavity (in contrast to the MGS orbit presented in Figure 4.3). On the outbound leg, the SC remained in the MPR up to a distance of about $3.4 \times 10^4$ km (at 07:27). At later times, the strong correlation coefficients are not observed since the SC crossed the MPB. Due to its asymmetric orbit with respect to the comet, the MPB is crossed in a shorter distance on the outbound leg.

In conclusion, the boundaries found at Comet Halley and at Comet Giacobini-Zinner support the idea that the magnetic field topology associated with atmospheric unmagne-
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tized planetary bodies interacting with the SW is also valid for active comets, although the typical scales are much larger. Due to the larger dimensions of the interaction region, the MPB at Halley’s comet as seen by Vega-1 spacecraft appears to be the outer boundary of a wider transition region and magnetic field draping signatures are not observed immediately after it is crossed, as it was shown during the Giotto encounter. Instead, we find a rather gradual transition from the MPB into the MPR [Delva et al., 2014].

Magnetic field morphology of induced magnetotails

Complementary analyses regarding the magnetic field topology within IMs have been focused on their corresponding magnetotails. Various spacecrafts have collected observational results involving their nominal morphology and the dependence of the magnetotail structure on the IMF orientation. In Figure 4.2 we showed measurements provided by ICE across the induced magnetotail of comet Giacobini-Zinner (see also [Slavin et al., 1986]) that revealed a well-defined two-lobe structure with a plasma sheet with a factor of two difference between the magnetic field strength in the outer parts of the lobes and the central tail. In addition, ICE measurements revealed a well-defined outer boundary. In the case of Mars, initial observations [Yeroshenko et al., 1990] by the magnetometer aboard Phobos-2 (acquired during 4 orbits at 2.86 Martian radii) showed that the Martian magnetotail consists of “away” and toward “lobes” containing, respectively, draped IMF lines parallel and antiparallel to the external SW flow. However, the question of the existence of an intrinsic magnetic field was unresolved at that time. With the arrival of MGS at Mars and the confirmation of the absence of an intrinsic magnetic field, MAG measurements were used to characterize the IMF draping [Bertucci et al., 2003a; Crider et al., 2004]. Regarding Venus, its induced magnetotail has been extensively studied by PVO and Venus Express spacecrafts [McComas et al., 1986; Saunders and Russell, 1986; Zhang et al., 2010]. In particular, McComas et al. [1986] determined the average structure of Venus magnetotail in the range between 8 and 12 Venusian radii ($R_V$, with $R_V = 6052$ km) downstream from the planet from PVO magnetometer observations. These authors found that the magnetotail structure consisted of two opposite magnetic lobes separated by a layer whose location was related to the IMF component parallel to the SW, for a nominal Parker spiral pattern. Additionally, Zhang et al. [2010] reported asymmetries in the lobes with respect to the orientation of the interplanetary convective electric field. At Titan, Cassini magnetometer observations confirmed the presence of a bi-polar induced magnetic tail with a well defined PRL [Backes et al., 2005]. The PRL contains the points where the magnetic field component in the direction of the flow changes sign. A similar configuration was found for flybys at even greater distances from the moon [Bertucci
et al., 2007. More recently, Simon et al. [2013] published a comprehensive analysis of the draping of the external streaming (Saturn's) magnetic field around Titan as a function of the direction of the external magnetic field. In their study of the location of the PRL, the authors used and tested an empirical relationship for the tilt of the PRL with respect to the plane containing the center of the body that is parallel to the one generated by the upstream magnetic field and velocity [Simon and Motschmann, 2009].

In addition to these observations, the interaction between magnetized plasma winds and conducting objects have also been studied by means of numerical simulations in a wide range of scenarios. Examples of such studies are Dursi and Pfrommer [2008]; Ma et al. [2002]; Schmidt and Wegmann [2013]; Spreiter and Stahara [1992]; Ledvina and Cravens [1998]. However, there are only a few theoretical works that treat the draping of an external field around a conducting obstacle in detail. To our knowledge, Chacko and Hassam [1997]; Dursi and Pfrommer [2008] are the only analytical examples. In the first paper, the authors derive a asymptotic solutions (very close and very far from the sphere) to the problem of a magnetized plasma flowing around a conducting sphere in steady state conditions. They show that when resistivity is included, the magnetic field drapes around the sphere, forming a layer of very intense magnetic field with a thickness that scales like $O(\eta^{1/2})$, where $\eta$ is the magnetic diffusivity (or electrical resistivity). In the second paper, the authors followed an analytical approach to study the physical processes occurring when an MHD fluid with frozen-in magnetic field conditions flows around a sphere and analyzed the characteristics of the magnetic field near the surface of the body. Both studies are restricted to the particular case where the background magnetic field is perpendicular to the flow velocity.

In an attempt to study the basic properties of the IMs from first principles without this restriction, we perform a characterization of the magnetic field topology that results from the interaction between an ideal magnetized plasma flowing around a spherical conducting body for an arbitrary angle between the upstream flow velocity and the streaming magnetic field. To achieve this goal, we first derive the analytical solution for the electric and magnetic fields associated with this problem under a steady state MHD description.

### 4.3. An analytical MHD description of the magnetic field draping

Let us consider an MHD plasma flow with frozen-in magnetic field conditions whose velocity field $\mathbf{V}(r)$ corresponds to that of a flow around a sphere of radius $R$ under steady-
state conditions. First of all, it is important to notice that this problem is inherently three-dimensional: if the external plasma is not magnetized, the problem still has a symmetry about the flow axis; however, when we consider an arbitrary magnetic field direction frozen into the flow, this symmetry is lost. The frozen-in magnetic field condition is in general justified in the environments we are interested in, because of their very high magnetic Reynolds numbers. Therefore, to a first approximation, we compute the electric and magnetic fields neglecting the effects of resistivity. To see the range of validity of this assumption, we heuristically consider the main changes introduced by resistivity in section 4.3.3.

4.3.1. Determination of the electric field

As shown in Chapter 2, the following equations belong to the ideal MHD description (under steady-state conditions):

\[ \nabla \times (\mathbf{V} \times \mathbf{B}) = 0 \quad (4.2) \]
\[ \nabla \cdot \mathbf{B} = 0 \quad (4.3) \]

where

\[ \mathbf{E} = -\frac{\mathbf{V}}{c} \times \mathbf{B} \quad (4.4) \]

We solve this set of equations outside the sphere for a given stationary velocity field which is associated to an ideal, irrotational \((\nabla \times \mathbf{V} = 0)\) and incompressible \((\nabla \cdot \mathbf{V} = 0)\) flow around the sphere. For simplicity, we neglect any change in the flow pattern caused by the back-reaction of the magnetic field. We choose the origin of our coordinate system at the center of the sphere and the x-axis being anti-parallel to the fluid velocity at infinity. We assume a homogeneous magnetic field \(\mathbf{B}_0\) at infinity whose component perpendicular to the fluid velocity \(\mathbf{V}_0\) points towards the positive z-coordinate axis. Figure 4.6 shows the adopted coordinate system.

Thus, the velocity field in spherical coordinates is given by the expression:

\[ \mathbf{V} = -(1 - R^3/r^3)\mathbf{V}_0 \cos(\theta) \hat{r} + (1 + R^3/2r^3)\mathbf{V}_0 \sin(\theta) \hat{\theta} \quad (4.5) \]

where \(r, \theta\) and \(\mathbf{V}_0\) are the radial distance, the polar angle and the intensity of the velocity field at infinity, respectively.

We define the impact parameter \(p\) of a given line of flow, that is, the distance \(p\) of a streamline from the x-axis for \(r \to \infty\), as

\[ p = \sqrt{1 - R^3/r^3} \ r \sin(\theta) \quad (4.6) \]
Equations 4.2 and 4.4 allow us to express the electric field in terms of the electrostatic potential $\Phi$ ($E = -\nabla \Phi$), and therefore

$$\mathbf{V} \cdot \nabla \Phi = 0$$

Equation 4.7 implies that the electric potential does not vary along the streamlines of the flow. The boundary conditions associated to equation 4.7 are that the surface of the sphere is an equipotential surface and that at large distances from the object, the electric field goes to an asymptotic uniform value which in our case points along the negative $y$-axis (see equation 4.4). Therefore the solution has to fulfill the following boundary condition:

$$\Phi(r \to \infty) = E_0 \ r \sin(\theta) \cos(\phi)$$

where $\phi$ is the azimuthal angle related to the spherical coordinate system considered in this study. The solution to equation 4.7 consistent with the stated boundary condition (equation 4.8) is

$$\Phi = E_0 \ r \sqrt{1 - R^2/r^2} \sin(\theta) \cos(\phi) = E_0 \ p \cos(\phi)$$

Finally, using equation 4.9, the convective electric field in spherical coordinates results:
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\[
E_r = -E_0 \frac{[1 + R^3/2r^3]}{\sqrt{1 - R^3/r^3}} cos(\phi) sin(\theta) \tag{4.10}
\]

\[
E_\theta = -E_0 \sqrt{1 - R^3/r^3} cos(\phi) cos(\theta) \tag{4.11}
\]

\[
E_\phi = E_0 \sqrt{1 - R^3/r^3} sin(\phi) \tag{4.12}
\]

Equation 4.4 together with the flow velocity and the electric field, allow us to determine the magnetic field components perpendicular to \( V \) at each point. However, in order to obtain the total magnetic field we need to take into account equations 4.2 and 4.3 simultaneously.

### 4.3.2. Determination of the magnetic field

To determine the magnetic field topology for any possible background magnetic field orientation, we decomposed the problem into two separate contributions: (1) the problem in which the angle between the background magnetic field \( B_0 \) and the velocity flow at infinity \( V_0 \) is 90°; (2) the problem in which the angle between \( B_0 \) and \( V_0 \) at infinity is 0°. Because of the linearity of the idealized problem with respect to \( B \) (having fixed the velocity field \( V \)), the general solution for any possible angle between both fields is simply a linear combination between these two solutions. Hence, we decompose the magnetic field at infinity as:

\[
B_0 = B_{0\parallel} + B_{0\perp} \tag{4.13}
\]

We first consider the case of a uniform background magnetic field pointing towards the positive \( z \)-coordinate system axis \( B_{0\perp} \). In spherical coordinates, this field can be written as:

\[
(B_{0\perp})_r = B_{0\perp} sin(\theta) sin(\phi) \tag{4.14}
\]

\[
(B_{0\perp})_\theta = B_{0\perp} cos(\theta) sin(\phi) \tag{4.15}
\]

\[
(B_{0\perp})_\phi = B_{0\perp} cos(\phi) \tag{4.16}
\]

The spherical components of the differential equations 4.2 and 4.3 are:
\[
[\nabla \times (V \times B)]_r = \frac{\partial}{\partial \theta} [\sin(\theta) (V_r B_\theta - V_\theta B_r)] + \frac{\partial}{\partial \phi} (V_r B_\phi) = 0 \tag{4.17}
\]

\[
[\nabla \times (V \times B)]_\theta = \frac{\partial}{\partial r} [r (V_r B_\theta - V_\theta B_r)] - \frac{1}{\sin(\theta)} \frac{\partial}{\partial \phi} (V_\theta B_\phi) = 0 \tag{4.18}
\]

\[
[\nabla \times (V \times B)]_\phi = \frac{\partial}{\partial r} (r V_r B_\phi) + \frac{\partial}{\partial \theta} (V_\theta B_\phi) = 0 \tag{4.19}
\]

\[
\nabla \cdot B = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 B_r) + \frac{1}{r \sin(\theta)} \frac{\partial}{\partial \theta} [\sin(\theta) B_\theta] + \frac{1}{r \sin(\theta)} \frac{\partial B_\phi}{\partial \phi} = 0 \tag{4.20}
\]

For the flow velocity field \( V \) given in Equation 4.5 and from Equation 4.19 we can derive the differential equation for \( B_\phi \):

\[
\frac{\partial B_\phi}{\partial r} + \frac{V_\theta}{r V_r} \frac{\partial B_\phi}{\partial \theta} = -\frac{3 B_\phi R^3}{2 r (r^3 - R^3)} \tag{4.21}
\]

On the other hand, to determine \( B_r \) and \( B_\theta \) we multiply equation 4.17 by \( r \) and add Equation 4.18 multiplied by \( \sin(\theta) \), to obtain

\[
\frac{\partial K}{\partial r} + \frac{V_\theta}{r V_r} \frac{\partial K}{\partial \theta} = 0 \tag{4.22}
\]

where \( K = r \sin(\theta) (V_r B_\theta - V_\theta B_r) \).

The resulting linear inhomogeneous first-order partial differential equations 4.21 and 4.22 can be solved by the method of characteristics. To this end, we consider \( r \) as a parameter in the characteristic equations and express the variables \( \theta \) and \( \phi \) in terms of \( r \). Therefore, along a streamline:

\[
\frac{d}{dr} = \frac{\partial}{\partial r} + \frac{V_\theta}{r V_r} \frac{\partial}{\partial \theta} \tag{4.23}
\]

since \( V_\phi = 0 \) (see Eqn (4.5)).

Along a streamline with impact parameter \( p \) at infinity, Equation 4.21 can be rewritten as:

\[
\frac{dB_\phi}{dr} = -\frac{3 B_\phi R^3}{2 r (r^3 - R^3)} \tag{4.24}
\]

while Equation 4.22 becomes:

\[
\frac{dK}{dr} = 0 \tag{4.25}
\]
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We integrate Equation 4.24 using the boundary condition for the magnetic field at infinity (equation 4.16). Therefore:

\[ B_{\phi \perp} = \frac{B_{0 \perp \cos(\phi)}}{\sqrt{1 - R^3/r^3}} \] (4.26)

while from Equation 4.25 we obtain that

\[ K = -p V_0 B_{0 \perp \sin(\phi)} \] (4.27)

Note that Equations 4.26 and 4.27 are intimately related to Equations 4.10, 4.11 and 4.12. We can check the results regarding the electric field (obtained in section 4.3.1) in the following way: having determined \( B_{\phi} \) and \( K \), we calculate:

\[ E_r = -\frac{V_0}{c} B_{\phi} = -(1 + R^3/2r^3) \frac{V_0}{c} \cos(\theta) \frac{B_{0 \perp \cos(\phi)}}{\sqrt{1 - R^3/r^3}} = -E_0 \frac{[1 + R^3/2r^3]}{\sqrt{1 - R^3/r^3}} \cos(\phi) \sin(\theta) \] (4.28)

\[ E_\theta = \frac{V_r}{c} B_{\phi} = -(1 - R^3/r^3) \frac{V_0}{c} \sin(\theta) \frac{B_{0 \perp \cos(\phi)}}{\sqrt{1 - R^3/r^3}} = -E_0 \sqrt{1 - R^3/r^3} \cos(\phi) \cos(\theta) \] (4.29)

\[ E_\phi = -(\frac{V_r B_\theta - V_\theta B_r}{c}) = -\frac{K}{r \sin(\theta)} = \frac{p V_0 B_{0 \perp \sin(\phi)}}{c r \sin(\theta)} = E_0 \sqrt{1 - R^3/r^3} \sin(\phi) \] (4.30)

where \( E_0 = B_{0 \perp} V_0/c \).

Equation 4.27 couples the magnetic field components \( B_r \) and \( B_\theta \). By making use of equations 4.26, 4.27 and 4.20 and applying the method of characteristics along streamlines, we obtain the following differential equations for \( B_r \) and \( B_\theta \) (see also Dursi and Pfrommer [2008] for a similar derivation):

\[ \frac{dB_r}{dr} + \left[ \frac{2}{r} - \frac{2r^3 + R^3}{2 r (r^3 - R^3)} \left( 1 + \frac{1}{\cos^2(\theta)} \right) \right] B_r = -\frac{B_{0 \perp \sin(\phi) \sin(\theta)}}{r \sqrt{1 - \frac{R^3}{r^3} \cos^2(\theta)}} \] (4.31)

\[ \frac{dB_\theta}{dr} + \left[ \frac{2}{r} - \frac{2r^3 + R^3}{2 r (r^3 - R^3)} + \frac{9 r^2 R^3}{(2r^3 + R^3)(r^3 - R^3)} \right] B_\theta = \frac{2 B_{0 \perp \sin(\phi)(r^3 + 2R^3)}}{r \sqrt{1 - \frac{R^3}{r^3} \cos(\theta)(2r^3 + R^3)}} \] (4.32)
The resulting linear inhomogeneous first-order ordinary differential equations are solved by an integrating factor which is obtained from the homogeneous equations. Finally, the magnetic field components $B_r$ and $B_\theta$ are:

$$B_{r\perp} = \frac{(r^3 - R^3)}{r^3} \cos(\theta) \left\{C_1 + B_{0\perp} \sin(\phi) \int_\infty^r \frac{p r^4 \, dr}{(r^3 - R^3 - p^2 r)^{3/2} \sqrt{r^3 - R^3}} \right\} \quad (4.33)$$

$$B_{\theta\perp} = \frac{(2r^3 + R^3)}{r^{5/2} \sqrt{r^3 - R^3}} \left\{C_2 \pm 2 B_{0\perp} \sin(\phi) \int_\infty^r \frac{r^3 (r^3 + 2 R^3) \sqrt{r^3 - R^3} \, dr}{(r^3 - R^3 - p^2 r)^{1/2} (2r^3 + R^3)^2} \right\} \quad (4.34)$$

where the upper signs refer to the region $0 \leq \theta \leq \pi/2$ and the lower sign to $\pi/2 \leq \theta \leq \pi$ and $C_1$ and $C_2$ are integration constants determined to satisfy the boundary conditions upstream from the object.

To obtain the magnetic field configuration associated with the more general case of an oblique (uniform) background magnetic field at infinity, we now consider the case of $B_0 = B_{0\parallel}$ (therefore $B_{0\perp} = 0$). Note that: $\nabla \cdot B = \nabla \cdot V = 0$, $\nabla \times (V \times B) = 0$ and the boundary conditions (fields at infinity) $B_0 = B_{0\parallel}$, $V = V_{0\parallel}$ are fields which point in the direction of the positive/negative x-axis, respectively. Because of this, as long as we are restricted to a uniform magnetic field at infinity in the x-direction, the $-V$ and $B$ fields are characterized by the same topology at any point in space. Therefore the resulting magnetic field components in this case are:

$$B_{r\parallel} = (1 - R^3/r^3)B_{0\parallel} \cos(\theta) \quad (4.35)$$

$$B_{\theta\parallel} = -(1 + R^3/2r^3)B_{0\parallel} \sin(\theta) \quad (4.36)$$

$$B_{\phi\parallel} = 0 \quad (4.37)$$

As we previously pointed out, the general solution for any possible orientation of the magnetic field at infinity is simply a linear combination between the two solutions proposed in equation 4.13. If $\theta_0$ is the angle between the background magnetic field and the x-axis (see figure 4.6), then $|B_{0\parallel}| = B_0 \cos(\theta_0)$ and $|B_{0\perp}| = B_0 \sin(\theta_0)$.

Figure 4.7 shows the magnetic field lines around the spherical obstacle on the x-z plane, for the case where $B_0 \perp V_0$ (hence $B_0$ parallel to the z-axis). We can observe the following properties:

- There is an increase of the magnetic field intensity at the surface of the sphere and also in the downstream region, where the largest values are reached in the surroundings of the $Z = 0$ plane.
The magnetic field structure consists of two mirror symmetric magnetic hemispheres separated by a flat polarity reversal layer located on the plane \( Z = 0 \) (the thick black bar in figure 4.7). This change of the polarity of the flow-aligned component of the magnetic field as well as the increase of the magnetic field strength close to the surface of the sphere are main features associated to a draping configuration.

Figure 4.8 shows the magnetic field lines around the spherical obstacle (on the y-z plane) for the case in which \( \theta_0 = 60^\circ \). For non-perpendicular cases between magnetic field and the flow direction, the above mentioned symmetry is broken, giving rise to an inverse polarity reversal layer (IPRL). The IPRL, observed in figure 4.8 corresponds to the set of points where \( B_x = 0 \) right above the PRL. Therefore, in these cases there are two layers where the flow aligned-component of the magnetic field reverses its polarity. One of these layers, the PRL is located in the \( Z = 0 \) plane, just as it is observed in the strictly perpendicular \( B_{0z} \) case. In contrast to this, the location and shape of the IPRL varies with \( \theta_0 \). In the next section, we perform a more detailed analysis of the properties of each of these layers.
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**Figure** 4.8: Predicted magnetic field lines around the sphere for $\theta_0 = 60^\circ$. Based on [Romanelli et al., 2014a].

### 4.3.3. Influence of resistivity effects on the PRL and the IPRL

All previous results were obtained under the assumption that resistivity effects were negligible. While the resistivity effects are really important in the surroundings of the PRL, they are not expected to affect the location and shape of the IPRL significantly. This difference is due to the fact that while the intensity of the magnetic field is infinite at both sides of the PRL, in the proximity of the IPRL it remains finite and varies rather smoothly.

**Polarity reversal layer**

Chacko and Hassam [1997] showed that the divergent behavior of the magnetic field in certain locations is due to the absence of resistivity effects, which therefore need to be taken into account. They also found that the electrostatic potential remains unaffected at length scales such that $p/R > O(R_m^{-1/4})$, where $R_m$ is the magnetic Reynolds number. Moreover, they identified a boundary layer totally contained in the tail region inside of which the resistivity effects are relevant. As shown in the previous section, this is the case for the PRL since it is always located in the $Z = 0$ plane. The reader interested
in the analytical computation of the magnetic field in this region (with the inclusion of diffusivity effects) is referred to Chacko and Hassam [1997].

**Inverse polarity reversal layer**

Since the resistivity effects are not essential in the region near the IPRL, we determine its position and shape for different values of \( \theta_0 \), using the theoretical approach presented in the previous section. Figure 4.9 shows the location and geometry of the IPRL in the X-Z plane for \( \theta_0 = 45^\circ, 65^\circ \) and \( 85^\circ \). For small values of the x-coordinate (i.e. x such that \(-1R < X_{IPRL} < 2R\)) the IPRL is well draped around the obstacle. For positions further downstream (x \(<\) \(-3R\)) the IPRL becomes aligned with the flow. Additionally, in the downstream part with flow-aligned IPRL, this layer is clearly shifted to larger distances from the Z = 0 plane for larger values of the angle \( \theta_0 \). Figure 4.10 shows the z-coordinate corresponding to the location of the IPRL in the downstream region (in particular, for \( X_{IPRL} = -3R\)) for \( \theta_0 \) in [\( 0^\circ, 90^\circ \)]. As it can be seen, the tendency mentioned above (larger shift for larger \( \theta_0 \)) is observed for all \( \theta_0 \) angles.

![Figure 4.9: Location of the inverse polarity reversal layer (IPRL) in the x-z plane for different angles between the background magnetic field and the flow direction. The dashed lines indicate the location for \( \theta_0 = 45^\circ, 65^\circ \) and \( 85^\circ \) [Romanelli et al., 2014a].](image)

**4.3.4. Overall morphology of the magnetic field. Lessons for future studies**

In summary, our results show that when the external magnetic field is strictly perpendicular to the direction of the flow, the induced magnetic tail formed downstream from the obstacle consists of two mirror-symmetric magnetic hemispheres separated by
a flat PRL, which is normal to the direction of the cross-flow component $B_{0\perp}$ (see also Dursi and Pfrommer [2008] for this particular case). However, if flow-aligned component $B_{0\parallel}$ is nonzero, the mirror symmetry breaks down giving rise to the IPRL whose location depends on the orientation of the background magnetic field. Furthermore, we show that the IPRL displacement will be along $B_{0\perp}$ for negative $B_{0\parallel}$, whereas it will be antiparallel to $B_{0\perp}$ for positive $B_{0\parallel}$. In this regard, even though we do not compute the shift associated with the PRL (because of the absence of resistivity), the fact that the solution for the magnetic field is the result of the solution of the purely perpendicular and the purely parallel cases shows that the sign of the latter controls the direction of the shift of the magnetic lobes. Moreover, since the inclusion of resistivity effects do not break the mirror symmetry observed under a strictly perpendicular case, the shift of the PRL for other magnetic field orientations (for a fixed velocity field) must take place in the same direction than that of the IPRL [Romanelli et al., 2014a].

Figure 4.11 presents a scheme of these results applied for an induced magnetosphere interacting with the SW for different IMF orientations showing the three possible scenarios mentioned above (in the upper, middle and lower panel, respectively) as seen in a ’draping’ (DRAP) coordinate system, where the X-axis is antiparallel to the external flow and the Z-axis is parallel to $B_{\perp}^{\text{MF}}$. If we focus on the tail ($X_{\text{DRAP}} < 0$), the following conclusions can be drawn:
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1. If $B_{XDRAP}^{IMF} = -B_{||}^{IMF} = 0$, negative $B_{XDRAP}$ values are restricted to the $Z_{DRAP} > 0$ sector. Conversely, positive $B_{XDRAP}$ values will be confined to locations where $Z_{DRAP} < 0$.

2. If $B_{XDRAP}^{IMF} = -B_{||}^{IMF} > 0$, every location with $Z_{DRAP} < 0$ will display positive $B_{XDRAP}$. However, regions with $Z_{DRAP} > 0$ will display both positive and negative $B_{XDRAP}$ values.

3. If $B_{XDRAP}^{IMF} = -B_{||}^{IMF} < 0$, every location with $Z_{DRAP} > 0$ will display negative $B_{XDRAP}$. However, regions with $Z_{DRAP} < 0$ will display both positive and negative $B_{XDRAP}$ values.

To investigate whether this correlation between the spatial distribution of the magnetic lobes and the IMF orientation is observed in the induced magnetosphere of Mars, we analyze MGS MAG data obtained during the mission’s pre-mapping orbits (AB1 and SPO). In these periods, the spacecraft performed 570 elliptical orbits during which it sampled both the pre-shock and the magnetotail region. These analyses are presented and developed in the next section.

4.4. Dependence of the location of the Martian magnetic lobes on the IMF direction: observations from MGS

4.4.1. MAG data analysis: selection criteria

We considered MGS MAG measurements projected in the aberrated Mars Solar Orbital (MSO) coordinate system with a sampling frequency of 0.33 Hz and 1.33 Hz. This coordinate system is centered on Mars with the $X_{MSO}$ axis pointing opposite to the mean solar wind flow direction in the planet frame of reference and assuming an aberration of $4^\circ$, $Z_{MSO}$ being perpendicular to Mars’s orbital plane and positive to the ecliptic north, and $Y_{MSO}$ completing the right-hand system.

For every orbit in this study, we extracted those MAG data obtained both inside the MPB and upstream from the Martian bow shock. To that purpose, we used the MPB and BS fits by Vignes et al. [2000]. In addition, and to filter out the potential effects of crustal magnetic fields (see Chapter 1), we discarded all MAG measurements occurring inside the MPB for which the $X_{MSO}$ coordinate is higher than $-1.5 R_{M}$. The justification for this criterion is based on the work by Brain et al. [2003]. These authors have shown that the influence of the crustal magnetic fields at Mars can reach altitudes of up to 1400 km (i.e. $\sim 0.41 R_{M}$).
**Figure 4.11:** Scheme of an induced magnetosphere for different IMF orientations seen from the DRAP coordinate system. In all panels the solar wind flows antiparallel to the X-axis and the Z-component of the IMF is positive. The PRL is located between both magnetic lobes (where $B_{X\text{DRAP}} = 0$). The upper, middle and lower panels correspond to $B_{X\text{DRAP}}^\text{IMF} = 0$, $B_{X\text{DRAP}}^\text{IMF} > 0$ and $B_{X\text{DRAP}}^\text{IMF} < 0$, respectively. The PRL is located at $Z = 0$, in the northern and the southern hemisphere, respectively. [Romanelli et al., 2015].

Magnetic field topology within induced magnetospheres
To determine the upstream IMF we compute the average from the first 1000 magnetic field measurements outside the BS fit [Vignes et al., 2000]. Depending on the data gaps, there are orbits with inbound and outbound IMF estimates and orbits with only one IMF estimate (obtained from MAG observations in only one of the two legs). This procedure ensures that the IMF is determined from an average over planetary length scales (MGS typically travels a distance of the order of $\sim 1 \text{ } R_M$ perpendicular to the $X_{MSO}$ axis during the averaging period) and allows us to derive the mean magnetic field in approximately the same spatial regions for all orbits. It also provides an acceptable determination of the IMF since it reduces the time interval between the MAG observations obtained upstream from the Martian bow shock and the ones observed in the Martian magnetotail. Furthermore, in all the following analysis it is essential to ensure that the IMF does not vary significantly while MGS crosses the magnetic tail structure. In this regard, all orbits that have two IMF estimates are better suited to test the stationarity of the IMF than those with only one estimate. Based on this point, all orbits with two clearly different IMF estimates or with one poor IMF estimate are discarded from our analysis (we get back to this point at the end of this section). As a result, we find that 71 orbits satisfy all these criteria, from which 21 have two IMF estimates. Our main hypothesis is that the IMF derived for each of these orbits does not change significantly between the times where MGS is about to cross the BS and the ones where it is inside the MPB.

Figure 4.12 shows the trajectory of the selected orbits in the aberrated MSO cylindrical coordinate system. In all of them, MGS approaches Mars in the downstream region from very high altitudes. Then the spacecraft encounters the Martian BS (outer black line, fit from Vignes et al. [2000]) at locations with $X_{MSO}$ coordinates ranging between $\sim -4.5 R_M$ and $\sim -3.7 R_M$. After the shock has been crossed, MGS enters the magnetosheath and reaches the MPB (inner black line, fit from Vignes et al. [2000]). Inside this boundary, MGS MAG provides magnetic field measurements of the Martian magnetic lobes while approaching the planet. After closest approach has been reached, MGS crosses both boundaries again and returns to the upstream region.

**Different states of the magnetized SW: the DRAP coordinate system**

To analyze MAG observations in the magnetotail region while considering different states of the solar wind, we have performed a rotation of the coordinate system. This coordinate system, already referred to as the DRAP coordinate system [Neubauer et al., 2006; Bertucci et al., 2007], is defined taking into account the IMF direction as well as the SW velocity. Since MGS lacks measurements of ion properties, we assume that the solar wind flows in the aberrated Sun-Mars direction. The DRAP coordinate system is
Figure 4.12: MGS trajectory during the selected orbits shown in the aberrated MSO cylindrical coordinate system. This coordinate system, centered at Mars, has its $X_{MSO}$ axis pointing opposite to the mean solar wind flow direction in the planet frame of reference assuming a 4° aberration. $Z_{MSO}$ is perpendicular to Mars's orbital plane and positive to the ecliptic north and $Y_{MSO}$ completes the right-hand system. [Romanelli et al., 2015].

Therefore defined as follows:

\[
X_{\text{DRAP}} = X_{\text{MSO}}
\]

\[
Z_{\text{DRAP}} = \left[ 0, B_{Y_{\text{MSO}}}, B_{Z_{\text{MSO}}} \right] / \sqrt{B_{Y_{\text{MSO}}}^2 + B_{Z_{\text{MSO}}}^2}
\]
\[ Y_{\text{DRAP}} = Z_{\text{DRAP}} \times X_{\text{DRAP}} \] (4.40)

where \( B_{Y_{\text{MSO}}, B_{Z_{\text{MSO}}}} \) are the \( Y \)– and \( Z \)– MSO components of the background magnetic field calculated in the upstream region of Mars. In the case of orbits with two IMF estimates we have taken the average between both vectors. By changing from the MSO to the DRAP coordinate system we ensure that the background magnetic field is in the \((X - Z)_{\text{DRAP}}\) plane: in the DRAP coordinate system \( B_{Z_{\text{DRAP}}}^{\text{IMF}} \) is always positive, \( B_{X_{\text{DRAP}}}^{\text{IMF}} \) is equal to the value of the \( X \)– MSO component of the IMF and \( B_{Y_{\text{DRAP}}}^{\text{IMF}} \) is zero.

Note that changing from the MSO to the DRAP coordinate reference frame requires to know the IMF direction, but it does not depend on the magnetic field intensity. We then characterize the Martian magnetotail structure using measurements obtained for different orbits in terms of the IMF orientation, i.e., in terms of the angle between it and the \( X_{\text{DRAP}} \) direction. Hereafter we denote the angle between any magnetic field measurement and the \( X_{\text{DRAP}} \) axis as \( \theta \). Since the spacecraft magnetic field intensity has an upper limit of 1 nT at the location of the MAG sensors [Acuña et al., 2001], the maximum deviation of \( \theta \) due to this artifact is \( \Delta \theta = \pm \cos^{-1} \left( \frac{B_{Y_{\text{MSO}}}}{\sqrt{B_{X_{\text{MSO}}}^2 + B_{Z_{\text{MSO}}}^2}} \right) \). In this expression and the following all magnetic field measurements are normalized to 1 nT. This leads to an uncertainty in the cone angle of the interplanetary magnetic field which is then \( \theta_{\text{IMF}} \pm \Delta \theta_{\text{IMF}} \). Additionally, the maximum deviation of the direction of the IMF-component perpendicular to the \( X_{\text{DRAP}} \) axis \( (B_{\perp_{\text{IMF}}} = B_{Z_{\text{DRAP}}}^{\text{IMF}}) \) is \( \Delta \alpha = \tan^{-1}(B_{\perp_{\text{IMF}}})^{-1} \) (with \( B_{\perp_{\text{IMF}}} \) expressed in nT).

Figure 4.13 displays the decomposition of the averaged IMF according to the DRAP coordinate system. In particular, the upper panel presents the projection of the IMF onto the \((X - Z)_{\text{DRAP}}\) plane while the lower one shows the projection onto the \((Y - Z)_{\text{DRAP}}\) plane. Since by definition, \( B_{\perp_{\text{IMF}}} \) is always parallel to the \(+Z_{\text{DRAP}}\) direction, the angular uncertainty in the determination of this direction, \( \Delta \alpha \), generated by the spacecraft fields defines a region where the sign of the \( Z_{\text{DRAP}} \) component (of a vector position) is undefined. This region is shown in figure 4.13 in grey and is bound by the straight lines \( Z_{\text{DRAP}} = Y_{\text{DRAP}}/B_{\perp_{\text{IMF}}} \) and \( Z_{\text{DRAP}} = -Y_{\text{DRAP}}/B_{\perp_{\text{IMF}}} \). This issue is taken into account in the analysis regarding the location of the magnetic lobes presented in the next section.

All selected orbits have a well defined IMF direction in the following sense: all orbits with an IMF estimate such that \( \theta_{\text{IMF}} \pm \Delta \theta_{\text{IMF}} \) was not able to define the sign of the \( B_{X_{\text{DRAP}}}^{\text{IMF}} \) component were discarded. In addition, we also discarded orbits with different inbound and outbound \( B_{X_{\text{DRAP}}}^{\text{IMF}} \) signs.

Also with regards to our criteria, we note that some poor IMF estimates, leading to
Figure 4.13: IMF decomposition in the DRAP system. Upper panel: projection onto the \((X - Z)_{DRAP}\) plane. The angle between the IMF and the \(X_{DRAP}\) axis is denoted by \(\theta_{IMF}\). Lower panel: projection onto the \((Y - Z)_{DRAP}\) plane. By definition, in the DRAP coordinate system \(B_{\perp}^{IMF}\) is pointing in the \(+Z_{DRAP}\) direction. \(\Delta\alpha = \arctan \left( B_{\perp}^{IMF} \right)^{-1}\) (with \(B_{\perp}^{IMF}\) in nT) is the uncertainty in the determination of this direction. [Romanelli et al., 2015].
exclude orbits from the analysis, might be the sole result of taking a MAG data average over magnetosheath fields. Indeed, the Martian bow shock location is highly variable [Vignes et al., 2000]. In spite of this fact, we decided to stick to this procedure to make sure that the selected orbits are a reliable and statistically significant set of data to study the dependence of the location of the Martian magnetic lobes on the IMF orientation.

To evaluate the impact of potential IMF variabilities on shorter time scales, we determine MAG averages over data-windows with 300 and 500 data points outside of the BS fit. We do not find any significant difference with the statistical results presented in the next section.

4.4.2. Results: Magnetotail structure and dependence on $B_{XDRAP}^{IMF}$

IMF configuration

The criteria presented in the previous section allow the characterization of each of the 71 selected orbits according to the IMF cone angle $\theta_{IMF}$, and the orientation of the DRAP coordinate system in terms of the aberrated MSO one. Due to the experimental uncertainty $\Delta \theta_{IMF}$, we first classify these orbits into two groups depending on the value of $\theta_{IMF} \pm \Delta \theta_{IMF}$. One group contains the cases with $0^\circ < \theta_{IMF} \pm \Delta \theta_{IMF} < 90^\circ$ and the other one contains the cases with $90^\circ < \theta_{IMF} \pm \Delta \theta_{IMF} < 180^\circ$. The first group is composed of 32 orbits (with $B_{XDRAP}^{IMF} > 0$), 12 of which have IMF averages from inbound and outbound measurements. The second group contains 39 orbits (with $B_{XDRAP}^{IMF} < 0$), with 9 of them having IMF averages obtained from inbound and outbound measurements.

As we have already stated in the previous section, the angular uncertainty $\Delta \alpha$ might affect the predicted location of the Martian magnetic lobes. Thus, when analyzing MAG data obtained inside the MPB, we do not consider data located inside the region limited by the straight lines $Z_{DRAP} = Y_{DRAP}/B_{YDRAP}^{IMF}$ and $Z_{DRAP} = -Y_{DRAP}/B_{YDRAP}^{IMF}$ (i.e. the grey area shown in the lower panel of figure 4.13).

Magnetic field topology inside the magnetic pile-up boundary

Figure 4.14 displays the trajectory of MGS inside the MPB projected onto the $(Y - Z)_{DRAP}$ (left column) and $(X - Z)_{DRAP}$ (right column). The upper and lower rows describe the location of MAG measurements under positive and negative $B_{XDRAP}^{IMF}$ conditions respectively. Each point represents a single MAG measurement and is color coded according to the local values of $\theta$ ($0^\circ = $ black, $180^\circ = $ copper). These points have an associated angular uncertainty of $\Delta \theta < 20^\circ$, ensuring that their color will not change significantly because of experimental errors.
Figure 4.14: MGS trajectory inside the MPB projected onto the DRAP coordinate system. The upper (lower) panel corresponds to orbits with $B_{X_{DRAP}}^{IMF} > 0$ ($B_{X_{DRAP}}^{IMF} < 0$). The left (right) panel displays the projection of the trajectory of MGS in the $(Y-Z)_{DRAP}$ plane ($X - Z)_{DRAP}$. The dashed lines mark the limits between the different regions inside the MPB. From the upper to the lower one, they are the OP, IP, IN and ON regions. The percentage bars shown at the right correspond to bins where the local magnetic field is almost antiparallel or parallel to the nominal SW flow. The T-lobe and A-lobe bins correspond to MAG measurements where $0^\circ < \theta \pm \Delta \theta < 25^\circ$ and $155^\circ < \theta \pm \Delta \theta < 180^\circ$, respectively. [Romanelli et al., 2015].
To investigate whether there is a relationship between the spatial distribution of the martian magnetic lobes and the IMF orientation, MGS trajectory information is additionally divided in four regions according to $Z_{DRAp}$. The first two regions are located inside the nominal wake. These are the inner-positive (IP) and inner-negative (IN) regions, with $0 < Z_{DRAp} < 1$ and $-1 < Z_{DRAp} < 0$, respectively. The remaining two regions are located outside the wake, but still within the MPB. These are the outer-positive (OP) and outer-negative (ON) regions, with $Z_{DRAp} > 1$ and $Z_{DRAp} < -1$, respectively.

A few features are clearly noticeable from these plots. First of all, we find that most magnetic field measurements in the lower hemisphere ($Z_{DRAp} < 0$) are oriented toward the planet. Analogously, MAG measurements oriented away from Mars dominate in the upper hemisphere ($Z_{DRAp} > 0$). This shows that MAG measurements are in qualitative agreement with all the schemes presented in Figure 4.11.

Beyond this zeroth-order behavior, a careful consideration of the orientation of the magnetic field in the IP, IN, OP and ON regions under $B^1_{xIMF} > 0$ and $B^1_{xIMF} < 0$ reveals interesting features. In particular, if we consider the occurrence of fields both nearly parallel (the away-lobe bin or A-lobe bin) $\theta \pm \Delta \theta = [155^\circ - 180^\circ]$ and antiparallel (the toward-lobe bin or T-lobe bin) $\theta \pm \Delta \theta = [0^\circ - 25^\circ]$ to the SW direction, we find that:

1) In the ON region 100% of the magnetic field measurements belong to the toward bin (T-bin) for positive $B^1_{xDRAp}$ conditions.

2) In the ON region, $\sim$ 10% of the measurements belong to the away bin (A-bin) for negative $B^1_{xDRAp}$ conditions.

3) In the OP region 100% of the MAG measurements belong to the A-bin for negative $B^1_{xDRAp}$ conditions.

4) In the OP region, $\sim$ 50% of the magnetic field measurements belong to the toward bin (T-bin) for positive $B^1_{xDRAp}$ conditions.

These statistical results are displayed through the percentage bars located on the right of Figure 4.14. Interestingly, all these results consistently show the existence of a shift of the Martian magnetic lobes towards higher and lower values of $Z_{DRAp}$ (with respect to the canonical position shown in the upper panel of figure 4.11) when the $B^1_{xDRAp}$ component is, respectively, positive or negative.

The analysis of MAG data occurring within the IP and IN regions confirms the same behavior as above, even though the trends are not as sharp as those observed in the outer regions. In the IP region both toward and away bin measurements are observed under positive $B^1_{xDRAp}$ conditions (T-bin: $\sim$ 19%, A-bin: $\sim$ 81%). In this case, the mix is less evident than in the OP region, but still clearly showing a significant fraction of MAG...
measurements belonging to the T-bin in the northern hemisphere. There is also a mix of T-bin and A-bin measurements in the IN region when $B_x^{IMF} < 0$ (T-bin: 93.5%, A-bin: 6.5%) and, once again, the IN region shows a lower degree of mixing when compared to the ON region. However, the observed increment in the percentage associated with the A-bin going from the IN to the ON region (with $B_x^{IMF} < 0$) supports all previous results. Similarly, we do not observe 100% of the points belonging to the A-bin and the T-bin in the IP region (case $B_x^{IMF} < 0$) and in the IN region (case $B_x^{IMF} > 0$), respectively. However, we do observe high percentages for both cases (90.2% for the first case and 94.41% for the latter one) and we also observe 100% of the cases in the expected bins when MGS is further away from the $Z_{DRAP} = 0$ plane ($Z_{DRAP} > 1$ in the case of $B_x^{IMF} < 0$ and $Z_{DRAP} < -1$ in the case of $B_x^{IMF} > 0$).

4.5. Discussion and Conclusions

The results derived in this chapter agree with the idea that the IMF perturbations that result from the interaction between atmospheric unmagnetized planets or active comets and the SW are approximately the same. Note however that the spatial scales associated with the latter objects are much larger. Related with this point, our analysis of Vega-1 MISCHA observations around comet P/Halley also suggest that the MPB is the external boundary of a wide region in which magnetic field draping is indeed observed, although it does not take place as abruptly as in the cases of Mars or Venus.

Also, we have found a remarkable agreement between the magnetic field morphology observed in the outer regions inside the Martian MPB (section 4.4) and the proposed theoretical model, developed in section 4.3. Even though the expected tendencies in the inner regions are also clearly observable, we suggest that the slight deviations might be the result of the blurring produced by some of the causes discussed next. The first cause concerns the properties of the incoming solar wind. Because MGS lacks an instrument capable of measuring the SW direction, we considered that the SW flows along the Sun-Mars line, taking also into account the motion of Mars around the Sun. However, since the DRAP coordinate system orientation with respect to the aberrated MSO coordinate system depends on the direction of this flow, some of the cases that deviate from the expected trend might be (at least partially) the result of the SW not being totally aligned with the nominal Mars-Sun line. A second possible cause is related to temporal variations of the IMF. Since we analyze data provided only by one spacecraft, it is not possible to measure the IMF while MGS is inside the MPB. In this regard, even though we have performed a careful study of the MGS MAG data, analyzing orbits with two IMF
derivations that do not show differences in the sign of the $B_x^{\text{IMF}}$ component and orbits with a well defined IMF value (inbound or outbound leg), temporal variations cannot be completely ruled out. As a result, a possible cause for the observed blurring is the flapping of the PRL (and therefore, the motion of the magnetotail lobes) in response to these temporal variations. In this regard, numerical simulations shown in Modolo et al. [2012] suggest that the magnetic lobes adopt a new quasi-equilibrium configuration associated with a new IMF orientation in approximately 2 minutes. Interestingly, the blurring is only present in two of the inner bins, but not in the outer ones. Finally, when deriving the IMF from the average of MAG MGS data during a particular data-window, we are implicitly assuming that this field is uniform. Departures from this hypothesis are also able to generate deviations from the expected trend.

In favor of the causality of the IMF orientation on the spatial location of the Martian magnetic lobes, we also note that there is a higher percentage of T-lobe cases in the IP region if $B_x^{\text{IMF}} > 0$ than in the $B_x^{\text{IMF}} < 0$ case. In the same direction but less pronounced, there is slightly higher percentage of A-lobe cases belonging to the IN region if $B_x^{\text{IMF}} < 0$ than in the $B_x^{\text{IMF}} > 0$ case.

Moreover, the same correlation is observed even for larger bins. Indeed, we perform an analogous statistical study (to the one presented in the previous section) for the angular ranges $\theta \pm \Delta \theta = [0^\circ - 45^\circ]$ and $\theta \pm \Delta \theta = [135^\circ - 180^\circ]$, and we do not find any significant difference in the derived conclusions. Taking into account these considerations, and the result that only 5.59 % of the points in the IN bin ($B_x^{\text{IMF}} > 0$) and 9.80 % in the IP bin ($B_x^{\text{IMF}} < 0$) present deviations, we conclude that there is a good agreement between our results and the proposed theoretical scenario. This interpretation is in agreement with previously published studies reporting the influence of the orientation of the IMF on the location of the PRL and the IPRL [Simon et al., 2013; Romanelli et al., 2014a] and the magnetotail lobes [McComas et al., 1986]. Finally, it is interesting to note that the displacement of the magnetic lobes following the orientation of the IMF has also consequences on the location of the region where planetary particle acceleration is expected. Therefore, this effect should be taken into account when it comes to derive estimates for the planetary plasma escape rate, at least inside the magnetic pile-up boundary.
Resumen en castellano

El presente capítulo está dedicado al estudio de la topología del campo magnético que resulta de la interacción entre un flujo de plasma magnetizado y un obstáculo atmosférico no magnetizado. Estamos particularmente interesados en el intercambio de energía y cantidad de movimiento entre el flujo externo y el objeto conductor. En efecto, la topología del campo magnético que rodea a un objeto atmosférico no magnetizado es, principalmente, el resultado de la presencia de gradientes en la velocidad del flujo externo. Dichos gradientes son, a su vez, la consecuencia de dos procesos que acoplan las dos componentes de este complejo sistema. Uno de ellos es la inyección de partículas ionizadas de origen atmosférico al flujo de plasma (proceso conocido como carga de masa). El segundo proceso involucra fuerzas derivadas de corrientes presentes en la superficie conductora efectiva del obstáculo. Estas corrientes, como así también la inyección de masa, reducen la velocidad del viento de plasma magnetizado en frente del obstáculo. En las regiones donde el régimen no colisional es válido, el campo magnético “congelado” al plasma se apila frente a la región de estancamiento del flujo. Al mismo tiempo, las líneas de campo magnético son estiradas en la dirección del viento externo mientras el flujo se aleja del objeto, dando lugar a una magnetocola. En este capítulo estudiamos la topología del campo magnético en los alrededores de objetos atmosféricos conductores no magnetizados desde puntos de vista teóricos y observacionales. Desde el punto de vista teórico, desarrollamos un modelo analítico MHD que permite determinar el campo electromagnético asociado con un flujo de plasma magnetizado perfectamente conductor interactuando con una esfera conductorra, bajo condiciones estacionarias. Desde el punto de vista observacional, caracterizamos la morfología de campo magnético observada dentro de las magnetosferas inducidas del cometa Halley y Marte, basándonos en observaciones obtenidas por medio de las sondas espaciales Vega-1 y MGS, respectivamente.

Los trabajos realizados durante esta tesis dieron lugar a las siguientes publicaciones:


This chapter is focused on the study of low frequency electromagnetic plasma waves observed in the extended exospheres of Mars and Venus. We characterize their properties (frequency, polarization, coherence, etc.), and discuss their connection with that of the planetary exospheres (density, temperature) and the different microscopic interaction processes leading to their generation. In this context, the word “microscopic” is used to emphasize that the value of several parameters characterizing these waves (wave growth rate, saturation amplitude) depend on the type of the velocity distribution function associated with one or several of the interacting charged particle species [Gary, 1993]. A particular example of such processes takes place in the upstream region of both planets (i.e., upstream from their respective bow shocks) when exospheric neutral particles are initially ionized and subsequently picked-up by the solar wind, causing several wave plasma modes to become unstable.

The presence of PCWs is of fundamental importance when it comes to the interaction of the SW and Mars or Venus since, at those altitudes\(^1\), they are the main channel

\(^1\)The exobases of Mars and Venus are located at 220 km and between 200 and 290 km from the planetary surface, respectively [Delva et al., 2011a].
through which there is transfer of energy and momentum between the incoming SW and the planetary particles. At those exospheric altitudes, collisions between particles are not frequent and therefore they do not play a significant role in the dynamics of the system.

In the following section we develop an introduction to this topic with emphasis on the Martian case, since a significant part of this chapter is devoted to that planet. However, the plasma instabilities leading to the observed plasma waves depend on the presence of newborn ions immersed in a background magnetized plasma flow. The associated electromagnetic plasma waves are then expected and observed in the upstream regions of Mars, Venus and also active comets.

5.1. Introduction: Ion pick-up and plasma wave generation processes

For planets without intrinsic global magnetic fields such as Venus or Mars, the existence of neutral exospheres that extend beyond their respective bow shocks\(^2\), has major implications for the interaction processes taking place between them and the SW [Delva et al., 2011a; Mazelle et al., 2004]. Indeed, the interaction between each atmosphere and the SW starts far beyond the BS where particles from the extended exosphere, mainly hydrogen (the lighter element present in both planetary atmospheres), are ionized several planetary radii away from the object [Chaufray et al., 2008].

In general, planetary neutrals are ionized by photoionization, charge exchange and electron impact [Zhang et al., 1993]. In the case of Mars, these particles are ionized mostly through the first two processes [Modolo et al., 2005]. These ionization mechanisms add a small amount of energy to the ions with respect to their parent neutrals. As the latter are approximately at rest with respect to the planet (velocities lower than the escape velocity, which is 5 km s\(^{-1}\) for Mars), the planetocentric velocities of the ions are also negligible. As a result, when the parent neutral becomes ionized, the newborn charged particle is injected to the SW frame with a velocity close to \(-V_{SW}\). Hence, this ion describes a helical trajectory around the interplanetary magnetic field with a perpendicular velocity \(|V_{SW} \sin(\alpha_{V,B})|\) and a parallel drift \(|V_{SW} \cos(\alpha_{V,B})|\), where the angle between the SW velocity and the local IMF \(\alpha_{V,B}\) at the time of pick-up determines the initial pitch angle of the newborn ion. A scheme of this decomposition is shown in Figure 5.1. Based on these considerations, the velocity distribution function of the newborn planetary ions are often assumed to be drifting rings or ring-beam distributions. Asymptotic cases are

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\(^2\)The subsolar point of the Martian and Venusian bow shocks are located at 1.65 and 1.32 planetary radii from the center of each planet, respectively [Delva et al., 2011a].
“pure-ring” distributions, for which $\cos(\alpha_{V,B}) = 0$, and “pure-beam” distributions, which correspond to $\sin(\alpha_{V,B}) = 0$.

\[|V_{\perp 0}| = V_{SW} |\sin(\alpha_{V,B})|\]

\[-V_{SW} \quad \alpha_{V,B} \quad B\]

\[|V_{\parallel 0}| = V_{SW} |\cos(\alpha_{V,B})|\]

**Figure 5.1:** Parallel and perpendicular velocity of a newborn ion.

The physics of planetary ion pick-up, i.e. the dynamics derived from this phenomenon, is exactly the same as the one previously studied at comets [see for instance, Mazelle and Neubauer [1993]; Tsurutani et al. [1989]; Tsurutani [1991]; Gary [1991], and references therein]. Linear theories of cometary plasma waves [Brinca, 1991; Wu and Davidson, 1972; Wu and Hartle, 1974] show that a large variety of growing modes can be fed by the highly unstable distributions of newborn planetary ions. In appendix B we derive all the relevant parameters associated with a linear electromagnetic plasma wave instability that results from the interaction between a background magnetized plasma flow and a cold, field-aligned beam distribution. This case of study has two clear advantages: it has an analytical solution, and it has a significant relevance since this type of configuration is highly unstable when compared to other newborn planetary ion distribution functions.

The so-called electromagnetic “ion-ion” instabilities [Gary, 1993] obtain free energy from the relative streaming of the two ion populations along the magnetic field. In this regard, Tsurutani and Smith [1986] and Tsurutani et al. [1987] have also shown that the angle $\alpha_{V,B}$ is a fundamental variable, since it determines the wave-generation efficiency by governing both the resonant frequency and the growth rate of any unstable mode. The dependence of the dominant instability and wave polarization on the IMF cone angle $\alpha_{V,B}$ at the time of pick up can be summarized as follows. If $V_{SW}$ is parallel to the IMF and the newborn ions form a cold beam in the solar wind frame, the electromagnetic ion-
ion right-hand (RH) resonant instability will be predominant [Gary, 1993]. On the other hand, if $\mathbf{V}_{SW}$ is perpendicular to the IMF, the newborn ion ring distribution function will drive the electromagnetic ion-ion left-hand (LH) mode unstable. At moderate angles of $\alpha_{V,B}$ the RH instability is still predominant. Brinca and Tsurutani [1989] found that the maximum growth rate of the LH instability is larger than that of the RH instability for $\alpha_{V,B} > 75^\circ$, whereas according to Convery and Gary [1997] (see also Gary and Madland [1988]), this cone angle cutoff is $\alpha_{V,B} = 90^\circ$. We emphasize the following two aspects concerning these instabilities:

1)- Both the RH and LH instabilities belong, under certain conditions, to the group of cyclotron resonant instabilities. As explained in Chapter (2), a cyclotron resonant instability occurs when the conditions are such that the charged particles (species i, in this case the newborn ion species) experience an electromagnetic wave of frequency $\omega$ and wave vector $\mathbf{k}$ with a Doppler-shifted frequency close to their own cyclotron frequency $\Omega_{ci}$ or a multiple of this frequency. The general cyclotron resonance condition can be written:

$$\omega - k \cdot \mathbf{v}_{\|}^{\text{ion}} \pm n \Omega_{ci} = 0$$

(5.1)

where $\mathbf{v}_{\|}^{\text{ion}}$ is the newborn ion velocity component parallel to $\mathbf{B}$, $\Omega_{ci}$ is the ion gyrofrequency and the signs $+$ and $-$ depend on the polarization of the waves ($+$ for RH and $-$ for LH waves). Fundamental cyclotron, and higher order cyclotron resonances are associated with $n = 1$ and $n > 1$, respectively. Moreover, when the resonance occurs, the ions sense the waves in their own reference frame as left-hand polarized, with the oscillating electric field rotating in the same sense as their gyromotion about the background magnetic field. This establishes a strong wave-particle interaction with a net energy exchange between them.

2)- Both resonant instabilities have maximum growth rates at $\mathbf{k} \times \mathbf{B} = 0$, where $\mathbf{k}$ is the propagation wave-number [Brinca and Tsurutani, 1989]. In this case, only the fundamental cyclotron resonance is possible, i.e., only $n = 1$ has to be considered in Equation (5.1).

In relation to point 1), the ion-ion RH instability (for propagation parallel to $\mathbf{B}$) satisfies the expression $\omega - k \cdot \mathbf{v}_{\|}^{\text{ion}} + \Omega_{ci} \sim 0$ for moderate $\alpha_{V,B}$ for values of $\omega$ and $\mathbf{k}$ close to the ones with maximum linear growth rates [Gary et al., 1989; Brinca, 1991]. Note that the term $\omega_{\text{ion}} = \omega - k \cdot \mathbf{v}_{\|}^{\text{ion}}$ is the frequency of the electromagnetic plasma wave (of frequency $\omega$ and wave vector $\mathbf{k}$ in the SW frame) seen from the newborn ion reference frame. In addition, since all spacecrafts capable of measuring plasma wave properties around Mars or Venus have negligible planetocentric velocities when compared to that
of the SW, the following expression also holds:

\[
\omega_{sc} = \omega - \mathbf{k} \cdot \mathbf{v}_{\parallel}^{sc}; \quad \text{where} \quad \mathbf{v}_{\parallel}^{sc} = -[\mathbf{V}_{sw} \cdot \hat{k}] \hat{k}
\]  

(5.2)

where \(\omega_{sc}\) is the frequency of the plasma wave in the SC frame and \(\hat{k} = \mathbf{k}/|\mathbf{k}|\). Because of these factors and since the initial velocity of the newborn ion (in the planetary frame) is also negligible with respect to \(\mathbf{V}_{sw}\), in this case is:

\[
\omega_{sc} \sim -\Omega_{ci}
\]

(5.3)

Therefore, waves generated by the RH instability (triggered, in turn, by the exospheric ions interacting with the SW ions) are characterized by a frequency which, in the SC frame, must be close to the local newborn ion cyclotron frequency with a left-handed polarization. In those cases where the parallel propagating ion-ion LH instability prevails, the Doppler correction is very small and therefore the waves will be observed with a left-hand polarization in both the plasma and SC frames at the local newborn ion cyclotron frequency. This then suggests that the occurrence of waves at the local ion cyclotron frequency of a particular ion species in the SC frame can be associated with the occurrence of the pick up of such ions. In this sense, the presence of plasma waves is, a priori, evidence of the existence of ionized exospheric particles and an important diagnostic tool of their physical properties.

Local ion cyclotron waves have been reported at Mars [Russell et al., 1990; Brain et al., 2002; Mazelle et al., 2004; Bertucci et al., 2005], at Venus [Delva et al., 2008a,b] and in the environment of active comets [Tsurutani and Smith, 1986; Johnstone et al., 1987; Glassmeier et al., 1989; Tsurutani, 1991; Mazelle and Neubauer, 1993]. In the case of Mars, the first observation of waves at the local proton cyclotron frequency upstream of the BS was made by the Phobos-2 spacecraft [Russell et al., 1990]. These waves had small amplitudes (∼ 0.15 nT), they were left-hand, elliptically polarized in the SC frame, and propagated at a small angle to the mean magnetic field. PCWs have also been observed by MGS [Brain et al., 2002; Mazelle et al., 2004]. Brain et al. [2002] performed a statistical analysis of the properties of these waves during the pre-mapping phases. In that case, the frequency, polarization and propagation angle of the waves detected were similar to those determined from Phobos-2 observations. Their amplitudes, however, were 2 to 3 times greater in the latter detections. A few years later, Wei and Russell [2006] analyzed 85 events during the AB1 phase and discussed the generation mechanisms at large distances, as well as the possible distribution of waves depending on the direction of the convective electric field. Analogous and complementary analyses to the ones by Wei and Russell [2006] were applied by Delva et al. [2009] to PCWs observed upstream from the Venus
bow shock. In this study the magnetic field measurements (corresponding to 450 orbits) were obtained by the VEX MAG instrument.

In this chapter we carry out a study of the PCWs detected by MGS MAG/ER in the region upstream from the Martian bow shock during the pre-mapping phases. We analyze the frequency, propagation and polarization properties of these waves, the associated occurrence rate, and discuss the generation mechanisms and their relationship to the neutral densities at the exosphere of Mars. We also analyze the spatial distribution of these waves in a magneto-electric coordinate system centered on Mars (MBE). Additionally, we also study the properties of electromagnetic plasma waves observed by VEX MAG/ASPERA in the upstream region Venus. We discuss the implications of our results and compare them with recent studies.

5.2. Waves Upstream from Mars’ Bow Shock

We begin this study by analyzing MGS MAG measurements with a time resolution of 0.75 s. To minimize the influence of spacecraft fields, we focus on periods where the strength of the IMF is equal to or larger than 4 nT. Also, we consider the averaged fluxes over all directions with energies higher than 30 eV whose maximum time resolution is 2 s. Below this energy value, the electron distribution function is affected by spacecraft photoelectrons.

The analyses performed on MAG/ER data consists of a characterization of their spectral properties, their polarization, their degree of coherence, their magnetic connection to Mars’ bow shock, and their spatial distribution in an electromagnetic coordinate system. A more detailed description is given below.

5.2.1. Methods of Analysis

We used the following techniques from Chapter 3:

- **Dynamic spectra and correlation**: we generate dynamic Fourier spectra of the magnetic field components in the MSO coordinate system. To gain greater insight into the properties of the observed PCWs, we also calculate the cross-correlation between the fluctuations present in the electron flux measurements for specific energy channels, and in the component of the magnetic field along its mean value.

- **Polarization and coherence (MVA and the Coherence Matrix)**: the wave-vector and polarization of PCWs are obtained by applying the MVA explained in Chapter (3) [Smith and Tsurutani, 1976; Khrabrov and Sonnerup, 1998; Sonnerup
and Scheible, 1998] to MAG data. An additional tool of analysis is to calculate the coherence matrix of the magnetic field for a 0.015 Hz interval around the local proton cyclotron frequency (remaining contribution of spacecraft fields) [Fowler et al., 1967; McPherron et al., 1972]. The elements of this matrix provide information about the coherence [Tsurutani et al., 2009], polarization and ellipticity within the selected frequency range. These parameters are explicitly indicated in Equations (21), (20) and (23) in Rankin and Kurtz [1970], respectively.

Additionally, we used the following methods of analysis:

- **Magnetic connectivity with the Martian bow shock**: we determine the spatial location of MGS wave observations with respect to the Martian foreshock since this study, as we will see, can provide additional evidence in favor of these waves as being generated from newborn exospheric ions [Mazelle et al., 2004]. For this purpose, we use a static model of the bow shock [Vignes et al., 2000] and look for an intersection point with the field line sampled by the spacecraft, assuming a uniform IMF.

- **Wave Spatial distribution**: since several theoretical studies [Modolo et al., 2005] suggest a dependence of the spatial distribution of pick-up ions upon the convective electric field \( \mathbf{E} \), we investigate if a similar pattern is found on the distribution of waves. By assuming \( \mathbf{V}_{SW} = -400 \, \text{km} \, \text{s}^{-1} \, \hat{x}_{MSO} \), we study the spatial distribution of waves (including their amplitude), with respect to the direction of the convective electric field \( \mathbf{E} = -\frac{\mathbf{V}_{SW}}{c} \times \mathbf{B} \), by introducing an “electromagnetic” coordinate system (MBE) which is centered at Mars and with the \( \mathbf{Z}_{MBE} \) axis is parallel to \( \mathbf{E} \), \( \mathbf{X}_{MBE} \) is antiparallel to \( \mathbf{V}_{SW} \) (\( \mathbf{Y}_{MBE} \) completes the right hand triad). The IMF cone angle is also included in this analysis.

All these methods have been applied on selected time intervals to illustrate the main properties of these waves and then on 10-minute overlapping intervals to study changes in the properties of these waves along the MGS trajectory. The duration of these segments was chosen in such a way to contain a significant number of proton cyclotron periods.

5.2.2. Examples of Observations

Figures 5.2 and 5.3 show typical examples of wave activity present in MAG/ER measurements. Figure 5.2 illustrates an example of PCWs seen by MAG upstream from the Martian bow shock. The top three panels show the magnetic field components in the MSO reference frame. At first glance, the oscillations show a well-defined frequency,
large amplitudes (up to 4 nT) and a high degree of coherence. In this interval, the average magnetic field ($\mathbf{B}_o$) is quite steady: its magnitude is slightly above 8 nT and its orientation makes an angle of $34^\circ$ with respect to the Mars-Sun direction. Figure 5.3 shows the omnidirectional electron flux measurements by ER for energies 116, 191 and 314 eV (with time resolution of 2s). Oscillations similar to those observed by MAG can be found. Because they are observed in all energy levels, we assume that they are also present in the total electron density.

![Figure 5.2: Magnetic field measurements during part of the orbit P216 in MSO coordinates, (April 3, 1998) [Romanelli et al., 2013].](image)

5.2.3. Dynamic spectra and correlation

When simultaneous MAG and ER measurements are compared, we find that the component of the magnetic field parallel to $\mathbf{B}_o$ and the omnidirectional electron fluxes for different energies are in phase. Figure 5.4 shows the fluctuations in the parallel component of the field (upper panel), and in the electron flux at 116 eV (lower panel) during part of the orbit P232. The cross-correlation of the two time series shown in the enclosed panel displays a peak at zero displacement with a correlation value of 0.7, falling down to 0.44 at $\pm2$ s. We repeat this analysis for other events and for different energy channels with similar results, which is consistent with both time series being in phase, considering an
error of \( \pm 2 \) s associated with the cadence of the ER instrument.

We also find that most of the waves observed by MGS MAG/ER have frequencies in the SC frame which coincide with the local proton cyclotron frequency \( \Omega_{cp} \) within a \( \pm 1 \) nT uncertainty. Figure 5.5 shows a dynamic Fourier spectrum of the \( Y_{MSO} \) component of the magnetic field for a portion of the orbit P216. The peak in the frequency of the oscillations is systematically close to the calculated proton cyclotron frequency for several hours.

### 5.2.4. Polarization (MVA)

MVA shows that the waves are planar, propagate almost parallel to the background magnetic field and have a left-hand circular polarization (in the SC frame) with respect to \( \mathbf{B}_o \). Figure 5.6 shows an example of the results of MVA applied on 10 cyclotron periods of data. In this case, MVA yields a high \( \lambda_2/\lambda_3 \) ratio (50.3) indicating that the waves are planar. Also the analysis yields a small angle between \( \mathbf{k} \) and \( \mathbf{B}_o \) \( (\theta_{kB} = 8^\circ) \) showing that their propagation is quasi-parallel with respect to the background field. The hodogram describing the maximum and intermediate variance components clearly shows a left hand polarization, as the mean magnetic field for this interval points out of the page. In the
Figure 5.4: Fluctuations in the parallel component of the magnetic field and in the electron flux measurements (E=116 eV) during part of the orbit P232, (April 11, 1998) [Romanelli et al., 2013].

As a result, an estimation of $\theta_{k_B}$ from 10-proton-cyclotron period windows for which $\lambda_2/\lambda_3 > 10$ [Knetter et al., 2004] yields a value of $\theta_{k_B} \sim 9.9^\circ$. Consistently with such MVA results, we find that these waves typically have more power in the direction perpendicular to the mean-field direction than along it, once again indicating that the wavevector $k$ makes a small to moderate angle with the mean-field direction.

5.2.5. Polarization and coherence (Coherence Matrix)

The apparent coherence of the waves shown in Figure 5.2 is confirmed with an estimate of the coherence coefficient for a 0.015 Hz interval around the local proton cyclotron frequency. Figure 5.7 shows an example of observations of waves which maintain high degrees of coherence ($\geq 0.7$) and polarization ($\geq 70\%$) for about an hour. Their ellipticity coefficient indicates a left-hand elliptical polarization ($<-0.6$). Similar analyses applied to other orbits yield similar properties which remain unchanged for long periods of time (typically 1 hour).

We also see that the wave amplitudes decrease with radial distance from the planet,
Figure 5.5: Fourier dynamic spectrum of $B_{Y_{MSO}}$, Orbit P216, (April 3, 1998). The calculated local proton cyclotron frequency from MAG data is plotted in black for reference (line in the middle). The black lines at the sides correspond to the error bars associated with the spacecraft fields [Romanelli et al., 2013]. $P$ is the power spectral density defined in Chapter (3).

supporting the idea that Mars is the source of these waves. Figure 5.8 illustrates what is observed in orbit P216. A similar behavior is observed for other orbits.

5.2.6. IMF dependence: Connection to the Martian bow shock

The properties of PCWs are the same inside and outside the nominal Martian fore-shock. The results are summarized in Table 5.1, which shows the average properties of PCWs for selected orbits that were chosen based on their particularly high values of polarization and coherence. For example, orbits P204 and P207 have similar wave properties: they both are characterized by a high degree of polarization ($\geq 90\%$), a large coherence ($0.9$) and a left-hand ellipticity ($-0.72$). $\theta_{kB}$ is less than $15^\circ$ and their IMF cone angles differ in approximately $8^\circ$. Furthermore, in both cases the waves are observed in the same region (approximately between $(2.09, -1.27, -2.22) R_M$ and $(1.7, -1.38, -4.21) R_M$ in MSO coordinates). However, whereas in P204 the magnetic field lines are connected to the bow shock, this does not occur in P207. The difference arises because in the first case, $B_o$ is approximately $(3, -1.5, 3)$ nT and in the second one $(3, -5, 0.7)$ nT, strongly suggesting
Figure 5.6: MVA hodogram corresponding to a 10 cyclotron period interval during orbit P216. The mean magnetic field points out of the page \( \mathbf{B}_o = [-1.02, -0.18, 8.27] \) nT. The cross indicates the start of the time series analyzed [Romanelli et al., 2013].

that these are not associated with SW backstreaming ions only present within the ion Martian foreshock [Mazelle et al., 2004].

5.3. Overall Wave Occurrence

So far, the analysis of the wave properties was limited to particular events. In order to study the occurrence rate of these events on a long term basis (i.e. several months of data), strict criteria to define a PCW are required. In the following, we analyze all the SPO period (with IMF strength \( \geq 4 \) nT) considering that an oscillation is a PCWs if satisfies that:

- Frequency close to the calculated proton cyclotron frequency \( \pm 1 \) nT.
- MVA: events with \( \lambda_2/\lambda_3 \geq 10 \).
- Events with coherence \( \geq 0.7 \), ellipticity \( \leq -0.5 \), degree of polarization \( \geq 70 \% \).
Figure 5.7: Properties of the ultra low frequency waves observed by the MGS spacecraft during a part of the orbit P204 (March 28, 1998) [Romanelli et al., 2013].

As we explained in Section 5.2, we study the temporal variation of the different wave properties breaking time series into 10-minute long overlapping orbital segments.

Figure 5.9 shows the wave amplitude versus planetocentric distance upstream from the BS for all SPO orbits. The wave amplitude is estimated from the power spectral density in an interval centered at the proton cyclotron frequency with a width of 0.015 Hz. Cyan crosses correspond to the SPO1 subphase, while brown open circles indicate SPO2 events. The black curve displays the average amplitudes for 0.5 \( R_M \) bins, showing a general decrease of amplitudes with increasing distance. Assuming that the source for these waves are exospheric pick-up protons, the obtained result is to be expected for at least two reasons: the ion density decreases with distance, and for smaller MGS altitudes, waves have more time to grow as they are convected downstream by the SW.

Figure 5.10 shows the amplitude of the waves (estimated in the same way as in figure 5.9) as a function of the observed IMF cone angle for all SPO orbits. In the absence of velocity vector measurements, \( \alpha_{VB} \) is obtained assuming that \( V_{SW} \) is parallel to the Sun-Mars line. In particular, in Figures 5.10 and 5.11 we are considering the angle between the directions of the SW velocity and the IMF since this angle determines the initial newborn ion velocity distribution function. The cyan crosses correspond to SPO1 and the brown
Figure 5.8: The relative wave amplitudes decrease with radial distance from the planet. Orbit P216, (April 3, 1998). An exponential fit $y = A \exp \left(-\frac{r}{a}\right)$ yields a scale height $a = (4.15 \pm 1.5) \ R_M$ [Romanelli et al., 2013].

open circles to SPO2, while the black curve displays the average amplitudes for $10^6$ bins. This figure does not show any statistical shift between these two populations. The $10^6$ bin curve shows a slight shift toward small amplitudes as the IMF cone angle increases. This trend is similar to the behavior of the saturation amplitude of the instability shown in Cowee et al. [2012]. However, Cowee et al. [2012] suggested that the saturation level is not likely to be reached in the upstream region of Mars, which is not necessarily the case for the waves observed by MGS. Differences might be caused by the non homocedasticity of the $\alpha_{V,B}$ distribution and the $\alpha_{V,B}$ estimation errors. For these reasons, direct comparison of our observational results with their numerical results is not straightforward.

We also analyze the spatial distribution of the orbit segments where waves were found, making use of the MBE coordinate system. Figure 5.11 (a) shows the wave amplitudes (color coded) for the projection of the trajectory on the $(Y_{MBE}, Z_{MBE})$ plane for all SPO orbits. Figure 5.11 (b) shows the $\alpha_{V,B}$ value instead. In both figures, closed and open circles correspond to SPO1 and SPO2 respectively. The analyses do not show any clear difference in the spatial distribution of the waves between these two sub-phases: 60% of
Table 5.1: List of selected orbits. Orbit number and wave properties. From left to right: degree of polarization, ellipticity, coherence, $\lambda_2/\lambda_3$, $\theta_{kB}$, $|B|$, Cone Angle $\alpha_{V,B}$, Connection to the Martian Bow Shock (Yes, No, Both cases) [Romanelli et al., 2013].

| Orbit | D. Pol | Ell | Coh | $\lambda_2/\lambda_3$ | $\theta_{kB}$ (°) | $|B|$ | $\alpha_{V,B}$ (°) | Connection |
|-------|--------|-----|-----|----------------------|------------------|--------|------------------|------------|
| 204   | -0.72  | 0.90| 31.4| 8.9                  | 4.86             | 51.9   | Yes              |
| 207   | -0.72  | 0.90| 32.8| 14.2                 | 5.94             | 59.6   | No               |
| 208   | -0.67  | 0.80| 17.4| 10.0                 | 5.04             | 54.1   | Yes              |
| 209   | -0.67  | 0.78| 13.9| 19.3                 | 9.04             | 49.8   | Yes              |
| 211   | -0.68  | 0.83| 15.3| 15.6                 | 4.65             | 56.6   | Both             |
| 215   | -0.65  | 0.73| 18.1| 38.4                 | 13.5             | 40.2   | No               |
| 216   | -0.69  | 0.89| 20.2| 17.6                 | 7.31             | 33.6   | Both             |
| 217   | -0.70  | 0.84| 19.6| 24.3                 | 4.62             | 31.5   | No               |
| 239   | -0.70  | 0.87| 37.6| 6.14                 | 4.10             | 65.2   | No               |
| 241   | -0.68  | 0.83| 32.1| 17.8                 | 7.20             | 76.9   | No               |
| 242   | -0.69  | 0.81| 19.7| 20.6                 | 15.0             | 71.7   | No               |
| 249   | -0.63  | 0.76| 10.0| 29.6                 | 4.87             | 32.6   | Both             |
| 257   | -0.64  | 0.75| 16.1| 15.2                 | 4.44             | 61.7   | Yes              |
| 258   | -0.68  | 0.82| 30.4| 21.9                 | 4.63             | 54.1   | No               |
| 260   | -0.67  | 0.82| 33.3| 14.1                 | 5.27             | 42.8   | Yes              |
| 267   | -0.70  | 0.83| 42.6| 19.6                 | 5.02             | 68.4   | No               |
| 268   | -0.67  | 0.80| 23.1| 12.7                 | 5.06             | 24.4   | Yes              |


the waves occur above the $Z_{MBE} = 0$ plane and 48% are below it, while the spacecraft is 48% and 52% of the time in each hemisphere. As a result, the spatial distribution of PCWs, in contrast with the spatial distribution of pick-up ions, does not seem to be affected by the SW’s convective electric field. In fact, the spatial occurrence of the wave amplitudes and the cone angle $\alpha_{V,B}$ are not affected by the electric field either. Moreover, these waves are observed even when $E$ is relatively weak.

Another interesting aspect of this study is the clear difference in the occurrence rate of the waves between SPO1 and SPO2: whereas PCWs are present in 62% of the SPO1 upstream observations, they only appear in 8% of the time spent by MGS in the upstream region during SPO2. Also, the waves during SPO1 last longer and have a higher degree of polarization and coherence than those observed in SPO2. This difference in the occurrence of PCWs is in principle not due to changes in the $\alpha_{V,B}$ as the distributions for both subphases peak at around 60° (Parker’s spiral angle at Mars 55°) with a
standard deviation of 19°, suggesting that the pick-up geometry might have not changed significantly from SPO1 to SPO2. Hence, the difference in the occurrence of the waves between SPO1 and SPO2 might be related to temporal changes in the properties of the Martian hydrogen exosphere and/or ionization rates that lead to variations in the density of pick-up protons. This particular hypothesis is investigated in the next section.

5.4. Temporal variability of PCWs upstream from Mars: Implications for Mars distant hydrogen exosphere

PCWs occurrence rate

In order to improve the temporal coverage of our previous study we analyze (in addition to SPO orbits) all MGS MAG measurements obtained during the AB1 period. In our search for waves at $\Omega_p$, we set an upper limit for the studied upstream segments of every AB1 and SPO orbit at an altitude of 20,400 km, i.e., $6 \, R_M$. This limit is chosen in
**Figure** 5.10: Amplitude of the PCWs as a function of the IMF cone angle for all SPO orbits. (Criteria stated at the beginning of section 5.3) [Romanelli et al., 2013]. Cyan crosses correspond to SPO1 observations, brown open circles correspond to SPO2 events.

such a way to ensure a homogeneous altitude coverage throughout both orbital periods. Upstream portions are identified using the BS fit derived in Vignes et al. [2000].

Also, since we have established that the waves propagate almost parallel to the IMF, the criterion used to identify them in this enlarged data set is based on the Fourier power spectral density (PSD) of the transverse wave component with respect to the IMF ($\delta B_\perp$). The PSD of $\delta B_\perp$ is the norm of a vector whose components are the power spectral densities of the components of $\delta B_\perp$. Just as before, the PSD is estimated over sliding 10-minute intervals to ensure a large number of wave periods while allowing changes in the IMF. Consecutive intervals have an overlap of 1 minute. We define that waves at $\Omega_{cp}$ are detected when the average PSD within the frequency band $f_{cp} \pm 0.015$ Hz (where $\Omega_{cp} = 2\pi f_{cp}$ and 0.015 Hz is the error in $f_{cp}$ associated with remaining spacecraft fields) is larger than the average PSD for frequencies $f > f_{cp} + 0.015$Hz, plus its standard deviation (STD), multiplied by a constant $k>1$. Being $f_N$ the Nyquist frequency (0.167...
Figure 5.11: Trajectory of the spacecraft SPO orbits with PCWs (MBE coordinate frame). Upper panel (a): Color coded by the amplitude of the waves. Lower panel (b): Coded by color by the cone angle associated with the MAG measurements where it is observed PCWs. Closed and open circles correspond to SPO1 and SPO2, respectively [Romanelli et al., 2013].
Proton Cyclotron Waves upstream from Mars and Venus

and 0.667 Hz for the two low frequency waves sampling rates available), this condition is

\[
< \text{PSD}(f) >_{f_{cp}+0.015\text{Hz}}^{f_{cp}-0.015\text{Hz}} k \{< \text{PSD}(f) >_{f_{cp}+0.015\text{Hz}}^{f_{cp}-0.015\text{Hz}} + < \text{STD}(f) >_{f_{cp}+0.015\text{Hz}}^{f_{cp}-0.015\text{Hz}} } \]

(5.4)

The value of \( k \) is chosen by empirically inspecting several orbits with and without a clear spectral line at \( \Omega_{cp} \). We find that a value \( k=2.5 \) is acceptable, as all selected orbits show signatures at \( \Omega_{cp} \), and their number is adequate for statistical purposes. Similar criteria have been applied on VEX MAG measurements [Delva et al., 2011a].

Figure 5.12 shows an example of a positive detection of waves at the local proton cyclotron frequency. The plot shows the PSD of \( \Delta B_{\parallel} \) as a function of the frequency \( f \) for the interval 10:45:23 - 10:55:23 on 3 April, 1998. The spectrum shows a peak at \( f_{cp} = 0.125 \text{ Hz} \) in the SC frame. The power contained in the 0.105 - 0.135 Hz band (black lines) is higher than the averaged power contained at higher frequencies (gray dashed line), plus its STD (gray solid line).

To be able to quantify the temporal variability of the PCWs, we first define an upstream occurrence wave rate \( R \) associated with each orbit. \( R \) is defined as the number of intervals meeting the criterion above, divided by the total number of intervals (MGS orbits have orbital periods that vary between 12 and 40 hours depending on the orbital phase). In order to investigate the long-term temporal variability of this parameter, we take the average of \( R \) every 15 orbits with a 14-orbit overlap. The averaged occurrence rate is denoted as \( <R> \). The latter value allows to determine how the PCWs occurrence rate varies on Martian seasonal and annual scales.

The derived values of \( R \) and \( <R> \) between 14 September 1997 and 17 September 1998 (approximately half a Martian year) are shown in Figure 5.13a. Data gaps correspond to the AB1-SPO1 transition which took place between 20 February and 26 March 1998, and the solar conjunction hiatus between 30 April and 26 May 1998. In what follows, we describe the behavior of \( <R> \).

Soon after MOI, around the southern spring equinox (12 September 1997), \( <R> \) increases until a local maximum of 32\% in late October 1997. After a brief decline in the occurrence to 21\% in early November 1997, \( <R> \) undergoes a strong increase, which ends around Mars perihelion (7 January 1998), where waves are observed in 92\% of the time. After 12 January, the wave abundance drops until it hits the first data gap. During the entire SPO1, high values of \( <R> \) (around 80\%) are observed. After solar conjunction, a more gradual decrease is observed in SPO2, with values around 20\% during early southern autumn. Therefore, in spite of the altitude restriction and the applied criteria, occurrence rates are consistent with estimates obtained in the previous section and also by Brain et al. [2002]. Moreover, given the strong difference in wave
Figure 5.12: PSD of $\delta B_\perp$ for 10:45:23-10:55:23 on 3 April 1998 for a case of positive detection of waves at $\Omega_{cp}$. Black lines indicate the frequency range corresponding to the error in the estimate of $f_{cp}$ due to spacecraft fields Acuña et al. [2001]. The average power density for frequencies higher than the upper limit of the interval is indicated in gray dashed lines. The gray solid line shows the average plus the STD. The latter value is 3.92 times smaller than the power in the interval around $\Omega_{cp}$ [Bertucci et al., 2013].

abundance for orbits with similar geometry, and the absence of a manifest influence upon the convective electric field or the IMF direction, Figure 5.13a supports the hypothesis that any variability in the wave occurrence at $\Omega_{cp}$ at a given location might be attributed to temporal effects.

Implications for the Martian hydrogen exosphere

Ideally, to determine whether there is a correlation between the observed changes in $< R >$ and the state of the distant Martian exosphere would require to empirically determine key properties such as its numerical density and temperature as well as the ionization rates and cross sections. Unfortunately, the former parameters can only be measured at altitudes of the order of two atmospheric scale heights above the exobase\(^3\). In

\(^3\)The atmospheric scale height at Mars is 12 km for CO$_2$ and 750 km for H and O.
Figure 5.13: (a) Occurrence of upstream waves at $\Omega_{cp}$ below 20,400 km altitude from September 1997 through September 1998. Rates per orbit (R, gray dots) and 15-orbit averaged ($<$R$>$, black dots) are displayed. (b) Modeled H density above Mars South Pole and dayside-averaged H density at 15,400 km altitude for minimum and mean solar activity from August 1997 to October 1998. Mars perihelion (PH, 7 January 1998), southern hemisphere spring equinox (SSE, 12 September 1997), southern hemisphere summer solstice (SSS, 6 February 1998), and southern hemisphere autumn equinox (SAE, 14 July 1998) are indicated [Bertucci et al., 2013].

Other words, the structure of the upper atmosphere can only be estimated from indirect observations or numerical simulations. Most of the numerical models use a Chamberlain [1963] approach based on isothermal equilibrium from which the vertical profile of the exospheric number density is a decreasing exponential whose scale height is a function of the exobase temperature and density, provided by models or observations. In such modeled exospheric densities, the long-term (i.e. timescales larger than diurnal) variations
Pr♦t♦♥ ❈②❝❧♦tr♦♥ ❲ ❛ ✈ ❡s ✉♣str❡❛♠ ❢r♦♠ ▼❛rs ❛♥❞ ❱ ❡♥ ✉s ❝♦♠❡ ♠❛✐♥❧② ❢r♦♠ ✉❧tr❛ ✈✐♦❧❡t ✭❯❱✮ ❤❡❛t✐♥❣ ♦❢ t❤❡ ▼❛rt✐❛♥ t❤❡r♠♦s♣❤❡r❡✳ ❆s ❛ r❡s✉❧t✱ s♦❧❛r ❝②❝❧❡✱ ❛♥♥ ✉❛❧✱ ❛♥❞ s❡❛s♦♥❛❧ ✐♥✢✉❡♥❝❡s ❡①♣ ❡❝t❡❞✳

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In view of the above considerations, a fundamental point is to determine how representati√ve the presence of waves at Ω cp is for the local neutral density. Concerning this point, Cowee et al. [2012] suggests that an increase in the amplitude of waves at Ω cp is the result of an increase in the local pickup ion density and in the cumulative ion production upstream of the observation point. Since increments in the exospheric neutral densities lead to increments in the local newborn ion density and in the cumulative ion productions, higher wave occurrence rates at Ω cp are an additional expected outcome. This is to be expected, of course, as long as nonlinear effects do not degrade the wave spectra below the detection threshold. In summary, when it comes to compare changes in the PCWs rate with changes in the hydrogen exospheric densities the underlying assumption is that wave generation mainly depends on the H density and that the ionization rates remain constant during this process.

By taking into account this hypothesis, we determine the Martian hydrogen densities for minimum and mean solar activity conditions by means of a 3D exospheric model [Chaufray et al., 2012]. This model incorporates nonuniform densities and temperatures at the exobase from a 3D Laboratoire de Météorologie Dynamique Global Climate Model (LMD-GCM) model [González-Galindo et al., 2009]. The values, initially given as a function of solar longitude, are then converted to time using MGS ephemeris. Figure 5.13b displays the derived H exospheric density at 15.400 km altitude above the Martian South Pole and the density averaged over the entire dayside hemisphere at the same altitude, between August 1997 and October 1998. This altitude level was chosen based on the average altitude of MGS for the chosen data set. From this panel, three basic properties
become immediately apparent:

- H densities increase with solar activity, which reveals the UV control of the thermosphere.

- The influence of solar declination and heliocentric distance is manifested in density curves displaying annual maxima around Mars perihelion and southern summer solstice (6 February 1998) and lower values during spring and autumn.

- H densities are not symmetric around their maxima. This is probably due to heliocentric distance effects (perihelion and aphelia occurred on 29 January 1997 and 16 December 1998, respectively). In addition to this trend, the model predicts oscillations in density in timescales of a few months. The amplitude of these oscillations is smaller for the dayside profiles, probably indicating a seasonal effect.

As it can be seen, even though the wave growth rate depends on several factors (e.g., SW velocity, neutral exospheric density, ionization rates), the observed variability in < R > could be related with the annual and seasonal changes in the H profile predicted by our exospheric model. First, the average wave occurrence and the H density curves show maxima around perihelion/southern summer solstice and display lower values around the equinoxes, suggesting that the occurrence of waves could be influenced by the expansion or contraction of the planet’s exosphere in response to annual/seasonal changes in solar UV heating. Second, wave occurrence and density curves display oscillations in timescales on the order of a few months. However in this case, the timescales of the density oscillations predicted by the exospheric model seem to be longer, degrading any possible correlation. Third, densities for the South Pole and dayside show similar behaviors suggesting that at those altitudes, H density and ultimately wave occurrence could be more sensitive to annual rather than seasonal changes. Finally, the influence of a possible change in solar activity remains elusive since, as previously stated, the low variability in the F10.7 flux suggests that MGS measurements might have taken place during solar minimum conditions.

In order to gain more insight about what conditions play a fundamental role in the generation, evolution and stability of the observed PCWs, in the next section we perform additional analyses of data provided by VEX in the upstream region of Venus. In particular, VEX MAG and ASPERA instruments are capable of measuring the magnetic field and solar wind properties in this environment, upstream from the Venussian bow shock. Both instruments, then provide more information about the external conditions favorable for the presence of these ultra-low frequency plasma waves.
5.5. Waves Upstream from Venus’ Bow Shock

When it comes to studying the PCWs occurrence rate upstream from the Venusian BS based on VEX data, there is an important difference compared to that of MGS data. As shown in the previous section, the PCWs occurrence rate at Mars appears to be correlated with the Martian heliocentric distance. Moreover, even though solar activity might have an effect in its value, it is not possible to quantify its influence since all available MGS measurements were taken under solar minimum conditions. In contrast with this situation, heliocentric distance effects on Venus are not expected to be as significant as in the Martian case. While the eccentricity of the Martian orbit is 0.0935, the eccentricity of the Venusian orbit is 0.0067\(^4\). However, VEX data are available for different solar activity regimes, therefore allowing to assess the study of other related effects.

In this section we analyze MAG [Zhang et al., 2008] and ASPERA plasma data [Barabash et al., 2007a; [Lundin, 2011] obtained from the VEX mission for two Venusian years (10 May 2006 and 10 August 2007) under solar minimum conditions and two Venus years in the rising phase of the Solar Cycle 24 (1 March 2011 and 31 May 2012) under conditions close to solar maximum. This selection then allows to determine whether there are variabilities in the efficiency of the PCWs generation mechanisms related to changes in the magnetized SW plasma properties that are due, in turn, to differences between both solar activity regimes.

Magnetic field data are obtained in the Venus Solar Orbital (VSO) reference frame with a sampling frequency of 1 Hz. The VSO reference frame is centered at Venus, with the \(X_{VSO}\) axis pointing towards the Sun (and opposite to the SW velocity \(V_{SW}\) which is taken to be radially outward from the Sun), the \(Z_{VSO}\) axis perpendicular to Venus’ orbital plane and positive to ecliptic north, and \(Y_{VSO}\) axis completing the right-handed system. As in the case of Mars, we use a static model of the bow shock [Zhang et al., 2008] to determine the times when VEX was located in the upstream region of Venus. We select MAG data from 20 min up to 4 h before and after the BS crossing, and calculate the power spectra using 10 minute data windows. To compare these results with previous studies, we identify PCWs in an automated way based on the power in a frequency interval near the local proton cyclotron frequency (see Delva et al. [2008b], Delva et al. [2015]).

Based on these first MAG data analyses, we find that PCWs at Venus are less conspicuous than PCWs at Mars. However they are observed up to a distance of 9 Venus

\(^4\)The difference between the aphelion and perihelion is \(42.61 \times 10^6\) km and \(1.46 \times 10^6\) km, for Mars and Venus, respectively.
Proton Cyclotron Waves upstream from Mars and Venus

radii \((1R_V = 6052 \text{ km})\) from the Venus-Sun line at both solar activity regimes. We also observe a significant difference between them; while there is a total number of 439 PCWs during near solar maximum conditions, we only find 153 PCWs events when the Sun is close to solar minimum conditions [Delva et al., 2008b]; [Delva et al., 2015]. This clear difference in the wave occurrence raises the question: what are the factors that make PCWs at Venus to be more frequently observed at solar maximum (by a factor of approximately 3) than at solar minimum? In the following we perform a detailed analysis of the solar wind magnetic field and plasma conditions at Venus, trying to understand what are the physical causes of these observed differences.

Solar Wind Magnetic Field Conditions at Venus

First, we check the stability of the IMF cone angle in the undisturbed solar wind in bins of 30°, since a lower amount of waves are expected when the external conditions are more variable [Delva et al., 2011b]. We find that in both solar activity regimes, the IMF cone angle is rather stable (within a 30° interval) for a duration of 20 min, although stable conditions for 40 min or longer are sometimes possible. In general, PCWs are observed under quasi-(anti) parallel conditions of the IMF and the solar wind velocity, as expected from theoretical models (see Figures 2, 4, 5 in Delva et al. [2015]).

Given these similarities concerning the IMF stability and orientation under which PCWs are observed, we now focus our attention towards possible changes in the SW plasma properties at Venus. Two fundamental variables are the solar wind velocity and the solar wind proton density, both measured by VEX ASPERA.

Solar Wind Velocity Conditions at Venus

As for the Martian case, we assume that the SW velocity flows radially outward from the Sun. However, based on the ASPERA instrument [Barabash et al., 2007a]; [Lundin, 2011] we are able to derive the solar wind speed data upstream of the bow shock averaged to one value per day. The histograms of \(V_{SW}\) presented in Figure 5.14 (blue lines; left: solar minimum, right: near solar maximum) show that \(V_{SW}\) is about 100 km s\(^{-1}\) higher near solar maximum. In addition, we determine the solar wind speed for days with PCW occurrence. The red lines in Figure 5.14 display histograms of the relative occurrence of PCWs as function of \(V_{SW}\). As it can be seen, in solar minimum PCWs occur more frequently for lower to normal \(V_{SW}\) (200–400 km s\(^{-1}\)). Near solar maximum, waves occur for somewhat higher values (300–400 km s\(^{-1}\)). Even though this follows the previous trend for all cases, there are, however, only a few wave occurrences for \(V_{SW}\) higher than 500 km s\(^{-1}\).
Figure 5.14: Histogram (in %) of observed solar wind speed (ASPERA, blue) and of 10 min PCW intervals (MAG, red) as a function of the observed solar wind speed, for (left) solar minimum and (right) near solar maximum [Delva et al., 2015].

As a result, we conclude that regardless of the level of solar activity, PCWs are more frequently observed for slow to mean solar wind speeds of 200-400 km s$^{-1}$. Based on theoretical approaches and numerical simulations, it has been shown that the growth rate of the waves decreases with a decreasing relative velocity of the beam with respect to the background plasma [Gary et al., 1986]. The PCW observations do not seem to be in agreement with this prediction. However this may be the result of having a limited volume with available newborn ions. In other words, the supply of newborn ions to the instability is limited because of the relatively small volume of the Venus upper atmosphere. For low $V_{SW}$ a longer time is needed to cross that region, and more newborn ions can contribute with their free energy to the growing instability. Therefore, it may be possible that also for low solar wind speed the wave growth rate is sufficiently large to produce the observed PCWs.

**Solar Wind Density Conditions at Venus**

Concerning the SW proton density derived from ASPERA measurements [Lundin, 2011], daily averages might be misleading due to the fact that the instrument might in the shadow of some spacecraft parts at specific times during the orbit, leading to a lower density estimation. Because of this we use the full measured unaveraged data for the
times of interest. Even if densities are underestimated, we expect this underestimation to occur both for solar minimum and maximum conditions. Figure 5.15 shows the number of wave occurrences (in 10 min intervals) per day, as function of the ASPERA solar wind proton density ($n_p$) for these times. At solar minimum (Figure 5.15, left), days without PCWs display $n_p$ values ranging between 0.5 and 20 cm$^{-3}$. A higher number of PCWs occur only for densities lower than $\sim 5$ cm$^{-3}$. Near solar maximum (Figure 5.15, right), the overall solar wind proton number density for times without PCWs ranges from 0.5 to 8 cm$^{-3}$. The trend for more PCWs for lower densities ($n_p < 3$ cm$^{-3}$) is also clearly visible. This leads to the conclusion that PCWs occur preferably for lower background solar wind density. This also supports the fact that the relative density of the newborn planetary protons with respect to the background SW protons play a significant role when it comes to the PCWs generation mechanisms. Since the measured SW proton density here is generally lower for solar maximum (change by a factor 0.5), the higher number of PCW occurrences is in agreement with this result.

![Diagram showing PCW occurrences vs. SW H+ number density](image)

**Figure** 5.15: Observed number of PCW (10 min) intervals per day, as function of the SW proton number density (ASPERA), for (left) solar minimum and (right) near solar maximum. A general tendency of more PCWs for lower SW density is seen in both cases [Delva et al., 2015].
Effect of EUV on the Venus Hydrogen Exosphere

Hybrid numerical simulations show that planetary ion densities as low as ~0.01% of the solar wind proton density are sufficient to produce observable ion cyclotron waves [Cowee et al., 2012]. Moreover, continual supply of newborn ions under constant field conditions leads to faster wave growth and longer survival of the waves [Cowee et al., 2008]. For solar minimum, the observation of the upstream PCWs has proven that the ratio of the planetary proton density to the background solar wind proton density at Venus is high enough to support wave growth to observable amplitudes, preferably under stable (anti)parallel conditions [Delva et al., 2011b]. This poses the following question: what is that ratio near solar maximum? For the time interval under study, the EUV photon flux is a factor 1.5-2 higher than for solar minimum (data courtesy of SOHO-SEM) and therefore the ionization rate at the Venus exosphere will be increased as well. This is expected to have an important effect on the number density of newborn protons in the exosphere. Models for the thermosphere of Venus calculate the number density of cold hydrogen up to an altitude of 400 km for different levels of solar activity, and predict a lower value for solar maximum (only 20% of that at solar minimum) [Fox and Sung, 2001]. On the basis of these thermospheric results, Lichtenegger et al. [2013] modeled the hot hydrogen corona and the planetary $H^+$ density up to an altitude of 60000 km ($\sim 10 R_V$). Although starting from a lower hydrogen thermospheric density during solar maximum, the higher solar EUV flux leads to an increased ionization rate of the hot hydrogen. This results in a somewhat higher number density for $H^+$, while at higher altitudes the increment is about 40% compared to solar minimum. The observed lower density (change by a factor 0.5) of the solar wind protons, and the higher density of exospheric protons (increase by a factor 1.4 according to numerical models) for high levels of solar activity, lead to an expected enhancement of the ratio between the density of planetary protons to the solar wind background by a factor 2-3. A higher number of newborn planetary ions under constant pick up geometry supply more energy to the growing waves, leading to a higher occurrence rate of PCWs near solar maximum.

In short, we find more upstream PCWs at Venus near solar maximum compared to solar minimum. From the previous analysis, we then conclude that in addition to quasi-(anti) parallel conditions of the IMF and the SW velocity direction, the higher relative density of Venus exospheric protons with respect to the background SW proton density is the key parameter for the higher number of observable proton cyclotron waves near solar maximum.
5.6. Discussion and Conclusions

The first fact to point out is that all previous analyses show that the waves in the region upstream of the Martian and Venusian bow shocks are observed, in the SC frame, at the local proton cyclotron frequency and are left-hand elliptically polarized. There is a general consensus to interpret these PCWs as being the result from the pick-up process of protons from the exospheric hydrogen of Mars and Venus which extend beyond their bow shocks [Russell et al., 1990]. Indeed, several studies (e.g., [Brinca, 1991; Tsurutani, 1991]) have shown that ring-beam distributions, formed by pick-up ions, can drive several SW wave plasma modes unstable, mainly low frequency waves around the gyrofrequency of the pick-up ions (in the planetary or SC frame). In particular, numerical studies applied to cases with super-Alfvenic ion drift velocities $v_{\|}^{\text{ion}}$ [Gary, 1993] show that two low-frequency MHD modes can resonantly interact with the pick-up ion population to produce amplification at $\Omega_{ci}$. These are the RH polarized co-propagating (in the direction of $v_{\|}^{\text{ion}}$) fast mode and a counter-propagating LH polarized Alfven wave. In the case of a sub-Alfvenic drift velocity (pick-up ion distribution closer to a ring) the LH polarized Alfven/proton cyclotron mode turns out to be the most unstable. In other words, which resonant mode dominates depends on the initial distribution function of the pick-up protons, which in turn, depends on the IMF cone angle. In the previous cases, the pick-up ions sense the unstable wave electric field in their own reference frame as nearly coincident with their gyration motion, ensuring an intense wave-particle interaction. Due to an anomalous Doppler shift, the polarization of the RH fast mode is reversed to left-handed in the SC which is also the newborn ion frame.

When it comes to the analyses performed on the MGS MAG data, the dispersion obtained in the values of the IMF cone angle (where $V_{SW}$ was assumed to be along the Sun-Mars direction) indicates that in principle both modes, RH and LH, could be present in MAG observations. In the absence of ion measurements it is impossible to obtain a value for the phase velocity of these waves. In particular, the angle between $k$ and $V_{SW}$ (that is necessary to compute the Doppler correction) at which the polarization changes remains unknown. However, following theoretical results [Cowee et al., 2012] one could assume that the phase speed is of the order of the 10\% of the $V_{SW}$, in which case the cutoff angle would be around 95°. In that case, most of the waves observed would then correspond to the RH mode. LH waves have been observed at comet Grigg-Skjellerup [Neubauer et al., 1993b] for IMF cone angles near 90° [Cao et al. [1998] and references therein].

Additionally, MVA analysis shows that the PCWs observed at Mars are planar and propagate almost parallel to $B$. This nonzero $\theta_{kB}$ could be the result of nonlinear effects
not addressed in this study [Mazelle et al., 2004], and is responsible for a small level of compressibility (typically, $\delta B_{\text{par}}/B \approx 0.25$). The compressibility of these waves was studied from the cross-correlation between the fluctuating electron flux (with the electron fluxes taken as a proxy of the electron density) and the parallel magnetic field component. The maximum values for the cross-correlation coefficient between both time series are associated with phase shifts between 0 and 2s. The quality of this result is limited by the following two factors: the influence of the magnetic fields generated by the MGS spacecraft and the time resolution of the ER instrument (2s). These two sources of error impose a tradeoff in our aim to obtain the best possible cross-correlation between the two time series. With this in mind we calculated the cross-correlation for several intervals with mean magnetic fields between 4 and 8 nT. For all the cases considered, the results indicate a maximum in the correlation coefficient located within 2s from zero time lag. This means that there is a systematic correlation between the two data sets within the errors caused by the spacecraft fields and the ER time resolution. But also, these results indicate that anticorrelation is not possible (errors are not as high as half a proton cyclotron period).

The analysis of MGS MAG measurements also shows that the frequency, polarization and coherence of these waves do not seem to depend on the direction of the IMF and in particular on the connection to the shock (see Table 1). It is then not surprising that the observed waves are not associated with the foreshock. Moreover, even though waves generated by backstreaming ions might be quasi-monochromatic, their frequency, in the SC frame, is below and far enough from the local proton cyclotron frequency [Mazelle et al., 2004; Shan, L. et al., 2014]. In addition, the amplitude of the observed waves shows a decrease as both the IMF cone angle and the radial distance from the planet increase. The latter fact supports the idea that Mars is indeed the source of these waves.

An interesting feature of the observed PCWs at Mars is their high coherence. Even though there is not a definitive answer for this phenomenon, there are several studies that suggest that this could be the result of a nonlinear wave-particle interaction capable of producing gyrating ion distributions upstream from the Martian bow shock. This has been observed at the Earth’s foreshock [Mazelle et al., 2003]; [Mazelle et al., 2004]. Indeed, the generation of highly coherent waves has also been studied for the case of comet Grigg-Skjellerup during the visit of the Giotto probe. The observations collected by this probe have shown that highly coherent waves with a frequency close to the ion gyrofrequency of water group ions were identified upstream of the shock [Mazelle et al., 1997] coinciding with non gyrotropic distributions of implanted ions [Neubauer et al., 1993b]; [Coates et al., 1993].
**Proton Cyclotron Waves upstream from Mars and Venus**

The lack of influence of the convective electric field \( E \) on the spatial distribution as well as the amplitude of the PCWs observed in the upstream region of Mars suggests that the link between the spatial distribution of the pick up ions and the waves is not obvious [Cowee et al., 2012]. Related to this point, Wei and Russell [2006] studied the wave occurrence in the MGS AB1 period, as seen from a reference frame that takes into account the convective electric field. They observed that the PCWs at large distances (85 events present in 9 orbits with \( B > 5.6 \text{ nT} \)) occur intermittently and predominantly in the hemisphere where \( E \) points away from the planet (+E hemisphere). In order to explain this behavior the authors proposed a mechanism where an exospheric hydrogen atom is ionized and then accelerated by the SW convective electric field. A charge exchange collision would then transform it into a fast neutral able to reach distant regions. At that location the neutral is once again ionized and generates cyclotron waves downstream of the planet and on the +E hemisphere. In contrast with this result, we find that (during SPO1 and SPO2) the wave spatial distribution for distances smaller than 6 Martian radii does not seem to depend on the orientation of the SW convective electric field (even after more restrictive criteria have been applied). Furthermore, waves are found even when \( |E| \) is relatively weak. We also observe PCWs far away into the -E hemisphere, although there is no known mechanism to move positive ions against the electric field. One explanation to this is that the wave distribution does not necessarily follow that of pick-up ions. It is important to note that pick-up ions generate waves whose wavelengths are of planetary scale [Bertucci, 2003]. Ion density is not likely to be homogeneous over such large spatial distances. The difference between the results by Wei and Russell [2006] and the ones presented here could be due to the fact that MGS sampled different regions during AB1 and SPO. On the other hand, it is also important to point out that the lack of correlation between the spatial distribution of the PCWs and the convective electric field has also been observed at Venus [Delva et al., 2011a].

Finally, the analyses performed on both MGS and VEX data show interesting results regarding the occurrence rate variability of the PCWs observed in the upstream regions of Mars and Venus, respectively. In the case of Mars, we found a very clear difference in this rate as well as in the properties of PCWs when comparing MGS observations obtained during the SPO1 and SPO2 subphases. Moreover, a statistical study of the IMF cone angle did not show any significant difference between both periods, thus suggesting that the pick-up geometry might have not changed significantly from SPO1 to SPO2. Motivated by these observations, we studied the temporal variability of these waves in an enlarged data set containing the AB1 period. By virtue of a particular property of waves propagating along the IMF and originating from exospheric ion pickup, the analysis of the
occurrence of upstream transverse fluctuations at $\Omega_{cp}$ at high southern latitudes shows that the abundance of these waves varies with time. Indeed, MAG observations show that this rate is significantly higher around the Martian perihelion or southern summer than around the spring and autumn equinoxes. Moreover, this variability is found to display a similar long-term trend as those of the densities of exospheric H obtained from models, which take the effect of thermospheric heating by solar UV radiation into account. Therefore, in spite of the complexity in the ion pickup and plasma wave generation and evolution processes, these results suggest a correlation between the occurrence of these waves and the temporal evolution of the distant Martian H corona, assuming that wave generation mainly depends on the H density and that ionization rates remain constant during the period studied. Concerning these results it is also important to take into account that all MGS measurements (pre-mapping period) were obtained under solar minimum conditions, which does not allow to assess possible solar activity influences on the Martian exosphere, and therefore on the PCWs. Fortunately, this is not the case for VEX measurements. Indeed, VEX data are available close to both solar maximum and minimum. In this case, however, heliocentric distance effects on the PCWs occurrence rate are not expected to be as significant as in the Martian case. Indeed, the difference between the aphelion and perihelion is approximately 30 times higher in the orbit of Mars than in that of Venus. Regarding the analyses performed on VEX MAG and ASPERA data, we find a higher occurrence PCWs rate when the the measurements were obtained at solar maximum compared to solar minimum. These results also suggest that, in addition to quasi-(anti)parallel conditions of the IMF and the SW velocity direction, the higher relative density of Venus exospheric protons with respect to the background SW proton density is the main responsible for the higher (factor 3) number of observable proton cyclotron waves near solar maximum.
Resumen en castellano

Este capítulo está centrado en el estudio de ondas electromagnéticas de plasma de baja frecuencia observadas en las exósferas extendidas de Marte y Venus. Específicamente, caracterizamos sus propiedades (frecuencia, polarización, coherencia, etc.) y discutimos su conexión con aquellas de las exósferas planetarias (densidad, temperatura) y los diferentes procesos de interacción microscópicos que dan lugar a su generación. En este contexto, la palabra “microscópico” es utilizada para enfatizar que el valor de varios parámetros caracterizando estas ondas (tasa de crecimiento, amplitud de saturación) dependen del tipo de función distribución de velocidades asociadas a una o varias especies de partículas cargadas interactuantes [Gary, 1993]. Un ejemplo particular de estos procesos tiene lugar en la región aguas arriba de ambos planetas (es decir, aguas arriba de sus respectivas ondas de choque) cuando partículas exosféricas neutras son inicialmente ionizadas y subsecuentemente capturadas por el viento solar, haciendo que varios modos normales del plasma resulten inestables.

La presencia de ondas a la frecuencia del ciclotrón de protones es fundamentalmente importante cuando se trata de la interacción entre el viento solar y Marte o Venus ya que, a aquellas altitudes\(^5\), son el principal mecanismo a través del cual hay procesos de transferencia de energía y cantidad de movimiento entre el viento solar y las partículas planetarias. A estas altitudes exosféricas, las colisiones entre partículas no son frecuentes\(^6\) y por lo tanto no tienen un rol significativo en la dinámica del sistema.

Los trabajos realizados durante esta tesis dieron lugar a las siguientes publicaciones:


\(^5\)Las exóesferas de Marte y Venus están localizadas a 220 km y entre 200 y 220 km de altitud, respectivamente [Delva et al., 2011a].

\(^6\)El camino libre medio de un protón en el entorno de Marte y Venus es del orden de 1 UA [Meyer-Vernet, 2007].
This chapter is devoted to the study of the plasma properties surrounding Saturn’s largest satellite Titan and the ionospheric plasma acceleration processes taking place in its induced magnetotail. Such acceleration processes are of fundamental importance when estimating the magnitude of the atmospheric escape due to its interaction with its plasma environment. Using Cassini data, we have selected 3 flybys that crossed Titan’s magnetotail and analyzed if tension forces are responsible for observed changes in the kinetic energy of Titan’s plasma. Additionally, we derive average values for the ionospheric fluxes flowing away from Titan for each of these flybys.

6.1. Titan and the Kronian magnetized plasma

Titan—the largest moon of Saturn— is perhaps the most complex example of the interaction between an atmospheric unmagnetized object and its plasma environment in our solar system. Usually located within the magnetosphere of Saturn, Titan orbits at an average distance of 20.3 Saturn radii (1 $R_s$ = Saturnian radius= 60268 km) from the ringed planet and it is subjected to highly variable external conditions. [Bertucci
et al., 2011]. This is partially the result of the proper variability of the outer Kronian plasma flow, a multispecies flux of charged particles (H\(^{+}\) and water group ions W\(^{+}\) defined as a combination of O\(^{+}\), OH\(^{+}\), H\(_{2}\)O\(^{+}\), and H\(_{3}\)O\(^{+}\)) proper of the Saturnian magnetosphere, which (to a first order approximation) corotates with the planet. The plasma flow convects the magnetic field lines of Saturn\(^{1}\) and impinges on Titan, the only natural satellite known to have a dense atmosphere. Indeed, while Titan rotates around Saturn\(^{2}\) in the corotating direction with a mean orbital velocity of 5.57 km s\(^{-1}\), the Kronian plasma component in this direction has an average velocity of 120 km s\(^{-1}\).

Similarly to the cases of Mars, Venus and also active comets, currents set in Titan’s highly conductive ionosphere acts as an obstacle to the flowing plasma which, in addition to the exospheric mass loading, tries to keep away Saturn’s magnetic field from the moon. In contrast to the SW impinging the other atmospheric unmagnetized objects, the Kronian plasma is subsonic, and hence no bow shock is formed upstream from Titan. In spite of this, the plasma flowing past Titan drapes the Saturnian magnetic field around the moon, creating an induced magnetosphere.

Interestingly, due to the difference between the Kronian plasma flow and Titan orbital velocities, the induced magnetotail is leading Titan in its orbit, as shown in Figure 6.1. As a result of the orbital motion of the moon around Saturn, changes in the relative orientation between the solar and co-rotation wakes (and also solar and magnetospheric inputs over Titan) also take place. In the ideal scenario of a Kronian plasma flowing in the corotating direction, the moon’s local time with respect to Saturn (Saturn Local Time or SLT) is a reliable parameter to measure the departure of the flow direction from the solar radiation flux. In such cases, these two directions will only be parallel around 18:00 SLT (a configuration similar to Mars, Venus and comets), and anti-parallel around 06:00 SLT. As it can also be seen in this figure, the orbital radius of Titan is a bit smaller than the average location of the sub-solar point of Saturn’s magnetopause. Results derived from Cassini data by Achilleos et al. [2008] show that such standoff distance for Saturn is well described by a bimodal distribution with mean values at 22 and 27 \(R_{\oplus}\). Therefore, as previously mentioned, Titan is almost all the time located within the magnetosphere of Saturn. Bertucci et al. [2009] estimated that this is the case for about 95\% of the time. However, when the solar wind dynamic pressure is significantly higher than usual, Titan may also be found in the Saturnian magnetosheath [Bertucci et al., 2008] or even in the solar wind plasma [Bertucci et al., 2015]. Interestingly, only in the latter case, a

\(^{1}\)Saturn’s magnetic moment is \(\sim 4.6 \times 10^{28}\) G cm\(^{3}\) \(\sim 580\) times the dipole magnetic moment of the Earth.

\(^{2}\)Titan’s orbital eccentricity is 0.03, and the orbital plane is inclined 0.35 degrees relative to the Saturnian equator.
bow shock ahead of Titan has been observed.

\[ E = -(v \times B) \]

\textbf{Figure 6.1:} Scheme of the environment surrounding Titan. Wake orientations at different local times around the moon’s orbit. The green wake is associated with the presence of the Kronian plasma, the yellow wake is associated with the solar radiation direction. Figure extracted from Coates [2009].

When it comes to the external (Saturnian) magnetic field frozen-in to the plasma, additional variabilities have also been observed. First of all, in the outer parts of Saturn’s magnetosphere, the magnetic field near the equatorial plane is highly stretched and forms a disk-like structure [Arridge et al., 2007]. Usually, the Kronian magnetodisk continues up to the magnetopause on the dayside and transitions into the magnetotail on the nightside. However, it can be absent near the dayside when the magnetosphere is compressed by the solar wind. This mostly happens when the magnetopause distance is smaller than 23 \( R_s \) [Gombosi, et al., 2009]. On the nightside and flanks of the magnetosphere, the magnetodisk is always present. Second, the plasma sheet associated with the Saturnian magnetodisk presents a bowl-like shape not found in any other known magnetosphere [Arridge et al., 2008]. Moreover, a seasonal effect of this structure has also been observed [Gombosi, et al., 2009]. Figure 6.2 displays a scheme illustrating the distortion of Sat-
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Titan’s magnetosphere together with Titan’s orbit below Saturn’s magnetodisk\(^3\). The left panel shows the magnetic field lines and the current sheet in the noon-midnight plane. The right panel presents the three-dimensional structure of the current sheet together with Titan’s orbit below it\(^4\). This was the case, for instance, when Cassini arrived in 2004 during Saturn’s northern hemisphere winter. The measurements of the magnetic field and plasma density then revealed that the plasma sheet, looking like a giant bowl, was warped and laid north of the equatorial plane. Regarding the Saturnian magnetodisk seasonal effect, Figure 6.3 shows modeled magnetic field lines of Saturn’s magnetospheric field based on Voyager and Cassini data sets for three different planetary orientations resulting from Saturn’s motion around the Sun\(^5\). Also, Saturn is a rapidly rotating planet whose planetary rotation influences the magnetospheric structure even in the outer magnetosphere where Titan orbits. Indeed, André et al. [2008] has observed that the magnetic field in the whole magnetosphere of Saturn is modulated at periods close to the planetary radio rotation period. Thus, these variations are perceived as spatial and temporal changes in the upstream conditions at Titan’s location.

Later, Bertucci et al. [2009] analyzed Cassini data and demonstrated that the direction and magnitude of the magnetic field upstream from Titan vary depending mainly on whether Titan is located above or below Saturn’s magnetospheric current sheet. The variabilities in the distance between Titan and the current sheet are then the result of global magnetospheric oscillations at Saturn, which change the elevation of the current sheet with respect to the rotational equatorial plane. When Titan is above or below the current sheet, the background magnetic field orientation points away from or towards Saturn, respectively. When Titan is in the current sheet, the background magnetic field is significantly weaker than before but the external plasma is hotter, leading to higher beta values compared to the previous case. An study that allows to quantify the importance of the hot and cold particle pressure in shaping the magnetic and plasma conditions near Titan can be found in Achilleos et al. [2014].

In addition to the external variabilities, Titan possesses a complex atmospheric and ionospheric chemistry [Ågren, 2012]. Titan’s atmosphere is very dense and the surface pressure is 50\% higher than the surface pressure on Earth. Since the gravity on Titan (1.35 m s\(^{-2}\)) is much weaker than on Earth, there is approximately ten times more gas per unit area above Titan’s surface than above the Earth’s surface. More than 90\% of the atmosphere consists of molecular nitrogen (N\(_2\)). There are also a few percent of methane

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\(^3\)The angle between the Saturnian dipole magnetic moment and its rotational axis is less than 1 degree.

\(^4\)The field orientation in the subsolar region is in fact more dipolar than displayed.

\(^5\)Saturn’s axial tilt is \(\sim 26.73^\circ\).
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Figure 6.2: Schematics illustrating the distortion of Saturn’s magnetosphere. (left) The distorted plasma/current sheet and magnetic field lines in the noon-midnight meridian. (right) A three-dimensional view of this distortion and the resulting bowl-shaped current sheet corresponding to winter in the Saturn’s northern hemisphere. The orbits of Titan and Hyperion are included showing that they are underneath the sheet. Figure extracted from Arridge et al. [2008].

(CH₄) and several tenths of percent of molecular hydrogen (H₂). In spite of its heliocentric distance, Titan’s obstacle to the incoming plasma strongly depends on photoionization [Ågren et al., 2007]. Indeed, solar radiation and energetic electrons in Saturn’s magnetosphere (the latter more important for the nightside of Titan’s atmosphere) ionize the upper parts of Titan’s atmosphere, creating its ionosphere. As a consequence, the angle between the incoming Kronian plasma flow and the EUV flux from the Sun affects the plasma interaction taking place in the moon’s environment (see Figure 6.1).

Interestingly, the interaction of CH₄ in the upper atmosphere with the Sun’s ultraviolet light from the Sun also produces hydrocarbons and other complex molecules. The most dominant species in the lower atmosphere are thought to be ethane (C₂H₆/C₂H₇⁺), ethylene (C₂H₄/C₂H₅⁺) and acetylene (C₂H₂/C₂H₃⁺), which can condense and precipitate to Titan’s surface, forming liquid hydrocarbon lakes or rivers [Lebreton et al., 2005]. Additionally, airborne hydrocarbons combine with nitrogen and produce other compounds, such as hydrogen cyanide (HCN/H₂CN⁺). Hydrogen cyanide can join together with other molecules forming polymers. Even though proteins (polymers made out of amino acids) are not likely to be found in Titan’s ionosphere due to the very cold environment far away from the Sun and the lack of oxygen, it is worth mentioning that some of the compounds
Figure 6.3: Representative field lines from the latest field model of Saturn’s magnetospheric field constructed from Voyager and Cassini data sets. Field lines are shown in the noon-midnight meridian for three situations of the dipole tilt -26°, 0° and 26°. Figure extracted from Gombosi, et al. [2009].

of hydrogen, nitrogen and carbon present in Titan’s atmosphere are the building blocks of the organic molecules on which life on Earth is based.

In agreement with the description given above, CAPS-Time of flight analyses on the pickup ion composition for the first Titan flyby (TA) suggested the presence of H⁺, H₂⁺, N⁺/CH₄⁺, CH₄⁺ and N₂⁺ with possible contribution of CH₃⁺, CH₅⁺, HCNH⁺ and C₃H₅⁺ [Hartle et al., 2006]. Similarly to the other atmospheric unmagnetized objects, currents set in Titan’s ionosphere attempt to prevent the external (Saturn’s) magnetic field from penetrating the moon⁶. However, in the particular case of this moon, the external plasma is severely slowed down as a result of being mass loaded by these highly massive ions, proper of Titan upper atmosphere. This effect is so intense that the convection time of magnetic field lines to past Titan ionosphere varies between 20 minutes and 3 hours [Bertucci et al., 2008]. Once again, as a consequence of the desacceleration of the Kronian magnetized plasma, the ionospheric particles gain energy, leading place to the erosion of moon’s atmosphere.

⁶In most of the cases, currents set in Titan’s ionosphere are not sufficiently strong to standoff the incoming magnetized plasma. As a result, a magnetized ionosphere is usually observed.
In the following section we focus our analysis on the presence of ionospheric particles downstream from Titan. More specifically, we study the changes in the kinetic energy of these atmospheric ions and determine what is the main acceleration process taking place in this region. In order to do this, we perform analyses of particles, waves and magnetometer measurements provided by Cassini during 3 selected Titan flybys across its magnetotail.

### 6.2. Ionospheric plasma in Titan’s tail: the presence of acceleration processes

Several observations in Titan’s tail have shown that ionospheric plasma populations are being transported downstream from the moon as a result of the interaction with the Kronian plasma. Gurnett et al. [1982] studied the structure of Titan’s wake from Voyager 1 measurements and suggested that the plasma observed in the neutral sheet was originated at the ionosphere of the moon. Gurnett et al. [1982] also estimated that the total loss rate of ionospheric ions is about $1.2 \times 10^{24}$ ions s$^{-1}$. More recently, Wahlund et al. [2005] analyzed the cold plasma environment around Titan and provided the first ionospheric outflow determination derived from Cassini observations: this value was estimated to be $10^{25}$ ions s$^{-1}$. Coates et al. [2007] and Szego et al. [2007] reported on the presence of ionospheric plasma at several Titan radii in the tail region during Cassini flyby T9 and Modolo et al. [2007a,b] estimated an associated outflow ranging between 2 and $7 \times 10^{25}$ ions s$^{-1}$. Sittler et al. [2010] studied Cassini data during the T9 and T18 flybys and estimated that the ionospheric flux flowing away from Titan (for the so-called Event 1 during the T9 flyby) is $\sim 7 \times 10^6$ ion cm$^{-2}$s$^{-1}$. Edberg et al. [2011] analyzed Radio and Plasma Wave Science - Langmuir Probe observations during consecutive and similar Cassini Titan flybys T55-T59. They found a region with high (1 - 8 cm$^{-3}$) plasma densities in the tail/nightside of the moon at locations progressively farther downtail from pass to pass. They described their results as a steady structure of ionospheric plasma escaping from Titan. Even though Edberg et al. [2011] suggested three possible acceleration mechanisms which could have contributed to this ionospheric outflow, they could not conclude which mechanism is responsible for the observed acceleration. Cassini Plasma Spectrometer electron and ion observations at Titan’s distant tail during flybys T9, T63 and T75 have also been investigated by Coates et al. [2012]. They identified the presence of ionospheric plasma in that region. Some of the electron spectra indicate a direct magnetic connection to Titan’s dayside ionosphere, while ion observations reveal heavy (m/q~16 and 28) as well as light (m/q = 1-2) ion populations streaming down the...
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tail. They suggested that the ambipolar electric field may be the driver of the ion escape. They estimated total plasma loss rates from Titan of the order of \( \sim 10^{24} \) ions s\(^{-1}\) for the three flybys. In addition to the previous studies, Cui et al. [2010] derived a total ion loss rate from Titan of \( 1.7 \times 10^{25} \) ions s\(^{-1}\) based on measurements made during nine close encounters of Cassini with the moon. Also, an analysis of the ionospheric composition for T40 flyby close to Titan can be found in Westlake et al. [2012]. Based on Cassini Ion and Neutral Mass Spectrometer observations obtained at altitudes ranging between 2225 km and 3034 km, the authors found significant densities of \( \text{CH}_3^+ \), \( \text{HCNH}^+ \) and \( \text{C}_2\text{H}_5^+ \). Taking into account this composition as well as the ion velocities, they suggested that these ions must have been created below the exobase and transported to the detection altitude by a combination of thermal pressure and magnetic forces. They also pointed out that this outward flow might link the gravitationally bound ionosphere with the more distant wake.

In the following we investigate the role of the magnetic tension forces in the acceleration of ionospheric plasma in the wake of Titan. As shown in Chapter 3, these forces are expected to be important near Titan’s plasmasheet as well as in regions where Alfvénic structures are present. Moreover, in these structures, the components of the plasma velocity tangential to those layers change in response to the magnetic tension forces, satisfying the Walén relation. To do this study we analyze Cassini plasma observations obtained during flybys T17, T19 and T40 since in these cases Cassini’s trajectory explored Titan’s induced magnetotail and was allowed to characterize Titan’s plasma outflow. Figure 6.4 displays the trajectory of Cassini during the three flybys, in Titan Ionospheric Interaction coordinates (TIIS). In this Titan-centered coordinate system the \( \mathbf{X}_{\text{TIIS}} \) axis points in the direction of ideal corotation, the \( \mathbf{Y}_{\text{TIIS}} \) axis points towards Saturn and the \( \mathbf{Z}_{\text{TIIS}} \) axis completes the right-handed system. As it can be seen, Cassini visited locations where the nominal wake and the induced magnetic lobes are expected\(^7\). Analysis of ion fluxes, angular distribution, mass composition and energy spectra allows the estimation of the plasma velocity in Titan’s wake. This information, combined with electron number density and magnetic field observations yields the local Alfvén speed and the ionospheric fluxes flowing away from Titan. After verifying that the MHD formalism can be applied for the considered data sets, we perform a Walén test. Additionally, we perform a MVA of the magnetic field measurements to determine if they display signatures of the existence of current layers compatible with tension forces [Sonnerup and Scheible, 1998]. The next section presents Cassini observations followed by a step-by-step description of the

\(^7\)A detailed description of the expected magnetic field topology surrounding a conductive object was provided in Chapter 4.
analysis undertaken for this investigation. The detailed analyses are presented for the T40 flyby, but similar studies have been performed for T17 and T19 flybys.

Figure 6.4: Trajectory of Cassini in TII$S$ coordinates for flybys T17, T19 and T40.

6.2.1. Observations

Cassini’s Titan flyby T40 took place on 5 January, 2008 with the closest approach at an altitude of 949 km (at 21:26:24 UTC). Saturn local time was 11.3 h, as a result the angle between Titan’s nightside and Titan’s nominal corotation wake is close to 90°. Figure 6.5 shows CAPS-IMS, CAPS-ELS, RPWS and MAG observations for this flyby in the time interval 20:30 - 22:30 UTC. The top panel shows the 8-anode average of the CAPS-IMS singles observations. The middle panel displays the electron number density derived by CAPS-ELS moment calculation (blue marks), LP analysis (red marks) and deduced from the upper hybrid frequency (green marks). Electron number density deduced from ELS in the time interval 21:08 to 21:45 are not shown. The bottom panel presents the magnetic field components measured by MAG in TII$S$ coordinates. The blue, green and red curves show the $B_{x\text{TII}}$, $B_{y\text{TII}}$, $B_{z\text{TII}}$ magnetic field components,
respectively, while the black curve displays the magnetic field intensity $B_{TII}$. 

![Graph showing magnetic field components](image)

**Figure 6.5:** CAPS-IMS, RPWS and MAG observations for T40 flyby (2008, DOY 5). Top panel shows the 8 anodes average of the CAPS-IMS observations. Middle panel displays the electron number density derived by CAPS-ELS moment calculation (blue marks), LP analysis (red marks) and deduced from wave observations (green marks). Bottom panel presents the magnetic field components measured by MAG: $B_{x\,TII}$ (blue curve), $B_{y\,TII}$ (green curve), $B_{z\,TII}$ (red curve) and the magnetic field intensity $B_{TII}$ (black curve). [Romanelli et al., 2014b].

Cassini is located in Saturn’s magnetosphere between 20:30 and 20:50 UTC. This plasma region is characterized by an electron number density ranging between 0.01 and 0.1 cm$^{-3}$. At about 20:50 UTC, Cassini enters Titan’s induced magnetosphere and the electron density increases while the spacecraft approaches Titan’s ionosphere. Between 20:53 and 21:15 UTC, the observed energy of the ion plasma smoothly decreases from $\sim$ 50 eV to $\sim$ 5 eV. At the same time, MAG measurements show that Cassini is in the away ($B_{x\,TII} > 0$) lobe of Titan’s magnetotail. The entry into the induced magnetosphere can be noticed from the increment in $B_{x\,TII}$ while the other two components decrease.
Note that in the case of ideal magnetospheric plasma corotation, the direction of the $\mathbf{X}_{\text{TITIS}}$ magnetic field component is the same as the direction of the ideal external plasma flow. Cassini remains in the away lobe of Titan’s magnetotail until the reversal of the $\mathbf{B}_{\text{TITIS}}$ magnetic field component at about 21:21 UTC. Finally, Cassini leaves Titan’s induced magnetosphere in the time frame 21:47 - 21:50 UTC. In this study we test if the observed changes in the kinetic energy of the plasma (Figure 6.5, top panel) can be associated with magnetic tension forces present during the same time intervals.

Next sections describe the analyses performed on CAPS/RPWS/MAG measurements in order to estimate the bulk velocity of the plasma and the local Alfvén velocity in Titan’s induced tail. For practical purposes, the analyses are carried out in the Kronocentric Solar Orbital (KSO) coordinate system. This coordinate system is centered at Saturn with the $\mathbf{X}_{\text{KSO}}$ axis pointing towards the Sun, the $\mathbf{Z}_{\text{KSO}}$ axis being perpendicular to the plane of Saturn’s orbital motion and pointing north of the ecliptic and $\mathbf{Y}_{\text{KSO}}$ completing the right-handed system.

### 6.2.2. Plasma Flow Direction

The determination of the flow direction is computed from the CAPS singles (SNG) data, and makes use of the instantaneous FOV of CAPS-IMS, the actuator position and the spacecraft orientation. There is no mass discrimination in the SNG data but elevation and azimuthal information allows us to reconstruct the angular distribution. During one actuator scan the instrument is able to cover a wider region which, due to its geometrical capabilities, is limited to about $2\pi$ steradians (for a fixed spacecraft orientation). The sky map seen by the instrument is parametrized by two angular spherical coordinates, $\delta$ and $\theta$. The latitude $\delta$ is the angle between the $\mathbf{Z}_{\text{KSO}} = 0$ plane and the position $\mathbf{r}$, it ranges from $-90^\circ$ to $+90^\circ$. The longitudinal angle $\theta$ is contained in the $\mathbf{X}_{\text{KSO}} - \mathbf{Y}_{\text{KSO}}$ plane and takes values between $-180^\circ$ and $+180^\circ$, where $\theta = 0^\circ$ corresponds to the direction of the $\mathbf{X}_{\text{KSO}}$ axis. Taking into account the intrinsic angular acceptance of the analyzer, the actuator motion and the spacecraft orientation, we derive the spatial coverage of each anode at each acquisition time by means of SNG data. By integrating over all energy bins, we can estimate the total number of counts received for each anode at each accumulation time. Plotting this information in a $\delta-\theta$ map for a short time interval (basically one actuator scan) allows the reconstruction of the angular distribution of the plasma in the spacecraft frame, in the KSO coordinate system. The flow direction is determined when the core (peak) of the distribution function is observed and identified with specific values of $\delta$ and $\theta$. The uncertainties in the spherical coordinates are $\Delta\delta = 20^\circ$ and $\Delta\theta = 20^\circ$.

Figure 6.6a displays the single-ion fluxes measured by CAPS from 21:09 - 21:12 UTC.
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(during flyby T40) as a function of $\delta$ and $\theta$ and also points out the position associated with the flow direction. In this case, the angles are $\delta = 35^\circ$ and $\theta = -130^\circ$. The plasma flow direction has been determined for different time intervals when the incoming plasma was in the FOV of the instrument. These results are summarized in Table 6.1, columns 3 and 4.

**Figure** 6.6: a) Normalized single ion fluxes (colorcoded) measured by CAPS from 21:09 - 21:12 hs as a function of $\delta$ and $\theta$. The mark also indicates the position associated with the flow direction: $\delta = 35^\circ$ and $\theta = -130^\circ$. b) Energy spectra of the single ion fluxes observed by each of the 8 anodes of CAPS at 21:10:30 hs. The red curve identifies the anode number 5 where the ion flux takes its highest value [Romanelli et al., 2014b].

### 6.2.3. Plasma Flow Speed

Determining the plasma speed requires information such as the plasma composition, the observed energy and the spacecraft potential. Unfortunately, TOF measurements do not always allow a mass determination since the ion flux might not be intense enough. The knowledge of the ion composition is then limited to only a few time intervals. However, after assuming that the different ion species are travelling at the same velocity $v$, it is possible to use SNG data in a complementary way. In this case, different peaks in the energy spectrum reflect the different ion composition of the plasma under study. Most of
the time, two energy peaks are observed and correspond to two ion mass groups. TOF data analyses, when sufficient counts are available, are used to identify ion masses more accurately. In these cases, ratios of the main ion mass groups deduced from the TOF analysis are consistent with the energy ratio of the SNG spectra. This result suggests that the different ion populations are travelling with the same speed. As a consequence, the MHD formalism can be applied in this environment. The plasma speed is \( v = \sqrt{2 E_s/m_s} \), where \( E_s \) is the energy of the peak corresponding to the population of mass \( m_s \) (after corrections involving the spacecraft potential \( U_{SC} \)). As a result, the ratio between the energy of these two peaks is equal to the ratio between the masses of the corresponding populations (\( E_{1,2} = 1/2 m_{1,2} v^2, E_1/E_2 = m_1/m_2 \)). This ratio remains relatively constant for the different ion spectral signatures of the flyby, suggesting that the plasma composition does not vary significantly during the time intervals being studied.

The velocity vector \( \mathbf{V}_{KSO} \) is determined as

\[
\mathbf{V}_{KSO} = -v (\cos(\theta) \cos(\delta) \mathbf{\hat{X}}_{KSO} + \sin(\theta) \cos(\delta) \mathbf{\hat{Y}}_{KSO} + \sin(\delta) \mathbf{\hat{Z}}_{KSO})
\]  

(6.1)

In this study we consider an uncertainty in the energy values of \( \Delta E = 0.2 E \). The minus sign in equation 6.1 reflects that the plasma flow direction is antiparallel to the normal of the spherical map seen by CAPS.

The speed of the plasma flow has been calculated for the same time intervals where the plasma flow direction has been derived (previous subsection). Information on the derived energy and mass of each population is summarized in Table 6.1.

Next, we show an example based on TOF and SNG data for one event during flyby T40. The left panel in Figure 6.7 displays the best TOF measurements, in terms of counts, in the tail region under consideration. The plot shows the number of counts measured by the Straight Through detector at about 21:16 UTC as a function of energy per charge (E/q [eV]) and time of flight channels. The ion energy ranges from 1 to approximately 10 eV, and several TOF channels exhibit significant counts. The determination of the corresponding mass has been examined with a simulation model developed by Nelson and Berthelier. This simulation model mimics the instrumental response to various ion (molecular and atomic) masses and energies and has been validated by comparing simulation results with CAPS spare model calibration tests. The expected signature on the ST for a 10-50 eV CH\(_5^+\) ion beam is presented in green in the top right panel of Figure 6.7. The largest peak at TOF bin 150 corresponds to a start time generated by an electron impact on the ST micro-channel plate and a stop time due to a neutral carbon impact on the ST MCP. Additionally, for CH\(_5^+\) the peak at TOF bin 100 corresponds to start and stop times induced by an electron and a negatively charged carbon, while the broader contribution of the signature, from TOF bin 30 to 60, is due to a start time of an
electron impact and a stop time of a secondary electron created after a neutral hydrogen or carbon impact on the high voltage rings. The simulation analysis suggests that these peaks are due to the existence of \( \text{CH}_3^+ \), \( \text{CH}_4^+ \), \( \text{CH}_5^+ \), \( \text{HCNH}^+ \) and \( \text{C}_2\text{H}_5^+ \). Water group ions show \( 0^- \) signature on TOF ST at TOF bins 115-125 (for an incident ion energy below 100 eV). The absence of significant counts at these TOF bins allows excluding water group ions from the plasma composition. The lower figure in this panel shows the number of counts derived from TOF data (red curve) and the TOF simulated result (black curve) when these ion species are considered. We find a good agreement between these two curves; the peak marked as (1) is mainly related to the presence of \( \text{C}_2\text{H}_5^+ \) while the peak marked as (3) is associated with the existence of \( \text{CH}_3^+ \), \( \text{HCNH}^+ \) and \( \text{C}_2\text{H}_5^+ \). The peaks corresponding to the heavier ions, marked as (2) and (4) are related to the \( \text{CH}_3^+ \), \( \text{CH}_4^+ \), \( \text{CH}_5^+ \) (electron for the start signal and neutral carbon for the stop signal) and the \( \text{HCNH}^+ \) and \( \text{C}_2\text{H}_5^+ \) species, respectively. Therefore, the TOF analysis of this event indicates the presence of particles that can be classified in two groups: a first group with masses ranging between \( 15 - 17 \) amu and a second with masses between \( 28 - 31 \) amu. As shown next, these two mass groups are fully consistent with the energy spectra observed at 21:10 UTC and also farther in the tail region (\( m_1/m_2 \) determined from TOF is close to \( E_1/E_2 \) determined from SNG for different time intervals). Figure 6.6b shows the energy spectra of the single ion fluxes observed by each of the 8 anodes of CAPS at 21:10 UTC. This is the time when the core of the distribution was identified (see the example given in the end of section 6.2.2). The energy values \( E' \) (associated with the ion fluxes) derived from CAPS-IMS have been corrected by the spacecraft potential \( U_{SC} \) measured by the LP (shown in Table 6.1, column 11). The energies of the ion fluxes seen from Cassini’s reference frame \( E \) are related to \( E' \) and \( U_{SC} \) by the following equation:

\[
E = E' + q U_{SC}
\]

(6.2)

The red curve identifies the anode where the ion flux takes its highest value (number 5 in this case). Two peaks in the ion fluxes can be seen in this curve; the first one with energy \( E_1 = 15.6 \text{ eV} \) has an associated flux \( F_1 = 2.1 \times 10^7 \text{ cm}^{-2}\text{sr}^{-1}\text{s}^{-1} \) and the second one with energy \( E_2 = 9.1 \text{ eV} \) has an associated flux \( F_2 = 7.6 \times 10^6 \text{ cm}^{-2}\text{sr}^{-1}\text{s}^{-1} \). If both peaks are related to two different particle species travelling at the same plasma flow speed \( v \), the ratio between the energy of the peaks \( E_1/E_2 \) is equal to the ratio between the particle masses of each species \( m_1/m_2 \) and therefore the speed of the flow is given by \( v = \sqrt{2 E_1/m_1} = \sqrt{2 E_2/m_2} \). In this case, \( E_1/E_2 = 1.7 \) and therefore \( m_1/m_2 = 1.7 \). As a consequence of this, and making use of the TOF results, we estimate that \( m_1 = 28 \) amu and \( m_2 = 17 \) amu. Thus in this case \( m_1/m_2 \sim 1.65 \), a value consistent with the proportion derived from the energy ratio \( E_1/E_2 \). This composition indicates that the
plasma has an ionospheric origin, even at few Titan radii away in the wake. The ion composition deduced from the TOF are fully compliant with the INMS ion composition in Titan’s topside ionosphere [Westlake et al., 2012]. This result is also consistent with the higher electron densities found in the wake (top panel, Figure 6.5) with respect to that of Saturn’s background plasma [Wahlund et al., 2005].

**Figure** 6.7: Number of counts on the ST detector, color coded with a logarithmic scale, plotted as a function of the energy per charge (E/q [eV]) and the TOF channels (left panel). Simulated signatures as a function of the TOF channel for the following species: CH$_3^+$, CH$_4^+$, CH$_5^+$, HCNH$^+$ and C$_2$H$_5^+$ (upper right panel). Number of counts as a function of the TOF channel derived from TOF data (red curve) and the TOF simulated result (black curve) (lower right panel). [Romanelli et al., 2014b].

Finally, combining the ion mass, the peak energy on the individual spectra and the flow directional information, the bulk velocity of the plasma in this event observed by Cassini (in KSO coordinates) is $V = (5.56, 6.63, -6.06)$ km/s. Taking into account the spacecraft velocity $V_{SC}$, we derive the bulk velocity of the plasma in Saturn’s reference frame from the expression: $V_{KSO} = V - V_{SC}$. The spacecraft velocity for this interval is $V_{SC}=(4.7, 2.8, 1.9)$ km/s, this yields $V_{KSO} = (0.8, 3.9, -8.0)$ km/s.
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<th>$\delta$ (°)</th>
<th>$\theta$ (°)</th>
<th>$T_s$ (UTC)</th>
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Table 6.1: CAPS analysis for flyby T17, T19 and T40; velocity determination. Columns 1 and 2 show the beginning ($T_b$) and ending ($T_a$) times of the selected interval. Columns 3 and 4 display the angles $\delta$ and $\theta$ associated to the flow direction. Columns 5 and 6 specify the time in which the energy spectra was analyzed ($T_s$) and the anode (A) with the highest flux. Columns 7 and 8 show the energy peak $E'_1$ (without the $U_{sc}$ correction) and the corresponding flux $F_1$ for the species of mass $m_1$, while columns 9 and 10 show the same properties for the species of mass $m_2$. Columns 11, 12, 13 and 14 display the values of $U_{sc}$, $E_1/E_2$, and the proportions of each species in the total mass flux ($C_1$ and $C_2$). Columns 15, 16 and 17 show the mass of each species $m_1$, $m_2$ and the average mass value < $m$ > of the flow.
6.2.4. Determination of the Local Alfvén Velocity

In this subsection we describe the different steps carried out to calculate the local Alfvén velocities associated with the previously selected time intervals (where the bulk velocity of the plasma was determined). As defined in Equation 2.56, the Alfvén velocity is \( v_A = B/\sqrt{4\pi \rho} \), being \( B \) the magnetic field, and \( \rho = \sum_s n_s m_s \) the total mass density of the charged plasma particles, with \( s \) ranging over all plasma species of mass \( m_s \) and number density \( n_s \). By considering that the electron mass \( m_e \) is much smaller than the mass of any other plasma particle, we approximate \( \rho \sim \sum_{s \neq e} n_s m_s = n_e \sum_{s \neq e} \frac{n_s}{n_e} m_s \). The electron number density \( n_e \) and the magnetic field \( B \) are derived from RPWS wave measurements and MAG data. We approximate the ratio between the numerical density of the plasma species \( s \) and the electron numerical density, \( n_s/n_e \), by the ratio between the flux of this species \( F_s \) (associated with its corresponding energy peak) and the total flux \( F_{\text{total}} = \sum_{s \neq e} F_s \) (the \( F_s \) are determined from energy spectra of CAPS-IMS measurements in addition to data provided by TOF analysis). Hereafter, we denote this ratio by \( C_s = F_s/F_{\text{total}} \propto n_s/n_e \). This allows us to determine the relative contribution of each species to the total density. Therefore, we approximate the Alfvén velocity for each selected time interval by \( v_A \approx B/\sqrt{4\pi n_e} < m > \), where the average mass of the flow is \( < m > = \sum_{s \neq e} C_s m_s \).

In the example shown in section 6.2.3, the ratios between the flux for each species and the total flux are \( C_1 = F_1/(F_1 + F_2) = 0.73 \) and \( C_2 = F_2/(F_1 + F_2) = 0.27 \). Therefore, the contributions of these populations lead to an average mass of the plasma \( < m > = 25.1 \) amu. Additionally, \( B = (1.04, 2.98, -2.44) \) nT and \( n_e = 3.8 \) cm\(^{-3} \), which leads to an Alfvén velocity that, expressed in KSO coordinates, is \( v_A = (2.4, 6.8, -5.6) \) km/s.

The same procedure is applied to other time intervals in this flyby as well as in flybys T17 and T19. The results are presented in Table 6.1.

6.2.5. Overall Wake Properties

All previous studies based on Cassini plasma measurements obtained during the T40 flyby show that the plasma under study is mainly composed of two ion species travelling at approximately the same speed: one with masses ranging from 15 to 17 amu and another one with masses ranging from 28 to 31 amu. Their relative contribution to the total mass density is shown in Table 6.1, columns 13 and 14. The proportion of the more massive population in the plasma varies between 0.48 (at 21:00:30 UTC) and 0.77 (at 21:07:30 UTC), while the complementary contribution is mainly of lighter (15-17 amu) ions. The average mass of the flow \( < m > \) varies between 22.2 and 25.5 amu. The plasma speed in Titan’s wake increases from (8.9 ± 0.9) km/s at about 3 \( R_T \) to (17.7 ± 1.8) km/s at Titan's plasma environment: Study of ionospheric plasma acceleration processes
about 5.5 \( R_T \). Moreover, in this region the electron number density is found to vary from 0.1 to a few tens of electrons per cubic centimeter.

Based on the electron number density measurements and the plasma speed determinations, we also derive two average values for the ionospheric flux flowing away from Titan. For a plasma speed of 8.9 km/s the flux is \( 2.49 \times 10^6 \) ions cm\(^{-2}\) s\(^{-1}\), while for a plasma speed of 17.7 km/s the flux is \( 4.96 \times 10^6 \) ions cm\(^{-2}\) s\(^{-1}\). These results are close to the ones reported by Sittler et al. [2010].

6.2.6. The DeHoffmann-Teller Analysis and the Walén Test

To determine whether the observed changes in the kinetic energy of the plasma can be associated with the existence of an Alfvénic structure, we perform a deHoffmann-Teller analysis and a Walén test (see Chapter 3) of the results obtained in the previous subsections. We consider CAPS/RPWS/MAG measurements obtained in regions where Cassini observed changes in the kinetic energy of the plasma (in Titan’s wake) and focus our study on locations where these changes seem to be correlated with changes in the local Alfvén velocity.

In the case of T40, we study the region between 20:59 and 21:12 UTC where the FOV coincides with the plasma flow in four different time intervals. By making use of the Equation 3.20, we find that the associated HT velocity in KSO coordinates is \( \mathbf{V}_{HT} = (0.28, -1.49, -2.05) \) km/s. Figure 6.8 shows a plot of the corresponding electric field \( \mathbf{E}_c^{(m)} \) as a function of \( \mathbf{E}_{HT}^{(m)} \) (component by component) and gives an estimate of the quality of the HT frame. This figure also shows the best linear fit for these measurements: the slope is \( S_{HT} = 1.06 \) and the y-intercept is \( O_{HT} = 0.46 \). The correlation coefficient is \( R_{HT} = 0.83 \) and the ratio \( D(V_{HT})/D(0) = 0.20 \), which indicate that the obtained \( \mathbf{V}_{HT} \) provides a reasonable approximation of the ideal HT frame.

Figure 6.9 shows the Walén plot corresponding to the analyzed observations during flyby T40. We perform a weighted linear fit (applying the linear least-squared method) that takes into account only the uncertainties in the bulk velocity determination (y-axis in the Walén plot), since the uncertainties in the determination of the Alfvén velocities are much smaller. The Walén slope is \( S_W = (0.87 \pm 0.22) \), the y-intercept is \( O_W = (-0.20 \pm 1.49) \) km/s, the correlation coefficient is \( S_W = 0.96 \) and \( R^2 = 0.91 \). These results indicate that there is a linear relationship between the bulk velocity of the plasma and the local Alfvén velocity (seen from the HT frame). The relative error in the slope \( (0.25) \) is such that the expected slope in the case of the presence of \( \mathbf{J} \times \mathbf{B} \) forces is possible. Moreover, the value of \( O_W \) indicates that the linear fit is consistent with a straight line crossing the origin of coordinates. As a result, the changes in the kinetic energy of the
plasma measured by CAPS are compatible with the work done by $J \times B$ forces.

Similar analyses are performed for flybys T17 and T19. A summary of the Walén analysis for the three flybys is presented in Table 6.2.

### 6.2.7. Minimum variance analysis (MVA)

In this section we describe the results of a MVA [Sonnerup and Scheible, 1998] of the magnetic field data during the analyzed time intervals. As stated in Chapter 3, this method provides an estimate of the normal direction ($\hat{n}$) associated with the presence of a current layer compatible with magnetic tension forces.

For the interval studied in T40, the mean magnetic field vector is $\langle B \rangle = (0.19, 2.07, -1.91)$ nT. In this case MVA yields a high $\lambda_2/\lambda_3$ ratio (11.45). Then, this interval shows a well defined-plane of intermediate/maximum variance and a well-defined minimum variance direction. As a consequence, MVA provides a good estimate for the normal
vector to this plane which, in KSO coordinates, turns out to be \( \mathbf{n} = (-0.12, 0.40, 0.91) \). These results show that the variation in the magnetic field direction (in the same time interval where CAPS observations show an acceleration of the plasma flow) is mainly restricted to a plane and can be related to the existence of a one-dimensional current layer. MVAs of MAG data is also performed for flybys T17 and T19. The results are shown and discussed in the next section.

\[
\begin{align*}
\mathbf{n} &= (-0.12, 0.40, 0.91) \ \\
\end{align*}
\]

Figure 6.9: Walén plot corresponding to flyby T40: Walén slope \( S_W = (0.87 \pm 0.22), \) y-intercept \( O_W = (-0.20 \pm 1.49) \) km/s, \( R^2 = 0.91. \) [Romanelli et al., 2014b].

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<td>19:56 - 20:08</td>
<td>5</td>
<td>(-9.39, -5.11, 2.38)</td>
<td>0.01</td>
<td>0.99</td>
<td>1.00</td>
<td>-0.03</td>
<td>-0.96</td>
<td>-0.71</td>
<td>-0.10</td>
</tr>
<tr>
<td>T19</td>
<td>17:07 - 17:20</td>
<td>6</td>
<td>(-12.00, -1.44, -7.12)</td>
<td>0.03</td>
<td>0.98</td>
<td>1.00</td>
<td>-0.01</td>
<td>-0.92</td>
<td>-0.91</td>
<td>1.38</td>
</tr>
<tr>
<td>T40</td>
<td>20:58 - 21:15</td>
<td>4</td>
<td>(0.28, -1.49, -2.05)</td>
<td>0.20</td>
<td>0.83</td>
<td>1.06</td>
<td>0.46</td>
<td>0.96</td>
<td>0.87</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

Table 6.2: Summary of the Walén Analysis for T17, T19 and T40. The columns show the flyby number, the time interval selected, the number of data points, the deHoffmann-Teller velocity, the ratio \( \frac{D_{HT}}{D_0} \), the correlation coefficient \( R_{HT} \), the deHoffmann-Teller slope \( S_{HT} \), the deHoffmann-Teller y-intercept \( O_{HT} \), the correlation coefficient \( R_W \), the Walén slope \( S_W \), the Walén y-intercept \( O_W \).
6.3. Discussion and Conclusions

In the present study we identify, characterize and interpret the progressive plasma acceleration features observed inside Titan’s induced magnetotail region. These analyses have been based on CAPS, RPWS and MAG measurements obtained during flybys T17, T19 and T40. Studies of the ion fluxes as well as their angular distribution, mass composition and energy spectra consistently show that this plasma is mainly composed of two ion populations travelling at approximately the same speed: one with masses ranging from 15 to 17 amu and another one with masses ranging from 28 to 31 amu. During all three flybys the plasma speed in Titan’s wake takes values from about 9 km/s to about 20 km/s. All these results suggest that this plasma has an ionospheric origin. Accordingly, RPWS observations show that in these cold plasma regions the electron density ranges between 0.1 and a few tens of electrons per cubic centimeter. A relative abundance of the two main groups of ions has been computed and shows that the contribution of these two ion populations varies between 1/3 and 1/2 to the escape with a spatial/temporal variations. Based on measurements of the electron number density and plasma speed, we derive average values for the ionospheric flux flowing away from Titan. In the case of fly
by T17, this flux varies between $8.72 \times 10^6$ ions cm$^{-2}$s$^{-1}$ and $9.94 \times 10^6$ ions cm$^{-2}$s$^{-1}$. For T19, it varies between $5.05 \times 10^6$ ions cm$^{-2}$s$^{-1}$ and $9.05 \times 10^6$ ions cm$^{-2}$s$^{-1}$; while for T40 it ranges between $2.49 \times 10^6$ ions cm$^{-2}$s$^{-1}$ and $4.96 \times 10^6$ ions cm$^{-2}$s$^{-1}$. The resulting fluxes are close to the ones derived by Sittler et al. [2010]. Assuming a simplified cylindrical wake with radius $\sim 2.5$ $R_T$ [Modolo et al., 2007b], the ion scope ranges between $1.1 \times 10^{25}$ ions s$^{-1}$ and $1.3 \times 10^{25}$ ions s$^{-1}$ for T17, it ranges between $6.6 \times 10^{24}$ ions s$^{-1}$ and $1.2 \times 10^{25}$ ions s$^{-1}$ for T19 and it ranges between $3.2 \times 10^{24}$ ions s$^{-1}$ and $6.5 \times 10^{24}$ ions s$^{-1}$ for T40. We note that these estimations are larger than the total plasma outflow deduced from the Voyager 1 observations [Gurnett et al., 1982] but are close to other Cassini estimates: Wahlund et al. [2005], Modolo et al. [2007b], Cui et al. [2010] and Coates et al. [2012].

To see if the observed changes in the plasma energy might be explained in terms of magnetic tension forces, we perform tests of the Walén relation. The low values of the ratios $D_{HT}/D_0$ and the proximity of the correlation coefficients $R_{HT}$ to unity (see table 6.2) allow us to identify an approximate HT frame associated for each of the 3 flybys under study. Figures 6.9, 6.10 and 6.11 display their associated Walén plots. As a result, a quasi-stationary pattern of magnetic field and plasma velocity is likely to be present in the downstream region of Titan. Therefore, the observed time variation in these events is due to the steady motion of the pattern relative to the instrument frame.

In agreement with the Walén relation, we find a linear dependence between the plasma
Figure 6.10: Wålen plot corresponding to flyby T17: Wålen slope $S_W = (-0.71 \pm 0.57)$, y-intercept $O_W = (-0.10 \pm 0.91)$ km/s, $R^2 = 0.92$. [Romanelli et al., 2014b]

bulk velocity (transformed into the HT frame) and the local Alfvén velocity. We determine that the Wålen slopes associated with flybys T17, T19 and T40 are $(-0.71 \pm 0.57)$, $(-0.91 \pm 0.17)$ and $(0.87 \pm 0.22)$, respectively. The corresponding y-intercept values are $(-0.10 \pm 0.91)$ km/s, $(1.38 \pm 1.41)$ km/s, $(-0.20 \pm 1.49)$ km/s. The difference in the sign of the slopes are in agreement with the location of Cassini in Titan’s away or toward magnetic lobes. During flybys T17 and T19 the spacecraft is located in the toward lobe where the component of the magnetic field parallel to the flow points in the opposite direction to the $X_{TIIIS}$, while in the case of T40 both vectors (magnetic field and velocity) are pointing in the same direction. The uncertainty in the derived slopes and y-intercepts are due to experimental uncertainties in the determination of the direction of the flow as well as its energy. The results from flybys T19 and T40, which show relative errors in the slopes of 0.19 and 0.25 are such that both slopes values -1 and +1 are possible. Flyby T17 has a relative error of 0.8 due to the fact that the plasma velocity in the HT reference frame is considerably lower than in the previous two cases. As a result, the relative error increases significantly. In spite of this, the results associated with T17 are in agreement with the ones from T19 and T40. As for the y-intercept values, the experimental uncertainties
allow the possibility that each straight line crosses the origin of coordinates.

We also study the fraction of the total variance present in the y measurements (of each Walén plot) that can be explained in terms of the linear fit model taking as a proxy the value of $R^2$ since

$$R^2 = 1 - \frac{\sum_{i=1}^{M}(y_i - f_i)^2}{\sum_{i=1}^{M}(y_i - \langle y \rangle)^2} = 1 - \frac{\sum_{i=1}^{M}(y_i - f_i)^2}{M(\text{SD}(y))^2}$$

(6.3)

where $\langle y \rangle = \frac{1}{M}\sum_{i=1}^{M} y_i$, SD is the standard deviation, $f_i = ax_i + b$ and the values of $a$ and $b$ are the ones obtained through the linear fits. The $R^2$ coefficients for flybys T17, T19 and T40 are 0.92, 0.81 and 0.91, respectively. Noticeably, flybys T17 and T40, characterized by $R^2$ values higher than T19, show a closer agreement between their $y$-intercept and the behavior predicted by the Walén relation.

Moreover, we calculate the correlation coefficients between the slope and the $y$-intercept derived from the linear fits. The corresponding values for flybys T17, T19 and T40 are -0.35, 0.39 and 0.35, respectively. Even though there is no reason to expect a linear dependence between both parameters (as in fact it can be seen from these values), the sign of the correlation coefficient allows us to determine if $a$ and $b$ tend to lie...
simultaneously on the same or on opposite sides of their respective means. Since flybys T17 and T19 are negatively/positively correlated, decrements in \( a \) are correlated to increments/decrements in \( b \). In the case of flyby T40 (positively correlated), increments in \( a \) are correlated to increments in \( b \). These results support the closeness between the observations and the theoretical model.

Another important aspect to take into account is that several factors could lead to a deviation of the constant of proportionality between \( \mathbf{v}' \) and \( \mathbf{v}_A \) and the expected ±1 value: effects of pressure anisotropy, inclusion of data in which the plasma has not yet interacted with the current layer, or unknown contributions to the tangential stress balance not accounted for. Because of these reasons, a proportionality in the range ±(0.8-1.0) is interpreted as a reliable indicator of the presence of an Alfvénic structure [Khrabrov and Sonnerup, 1998]. In agreement with the existence of such structure, we find that the perturbations of the magnetic field (\( \Delta \mathbf{B} = \mathbf{B} - \mathbf{B}_o \)) are close to be perpendicular to the background magnetic field (\( \mathbf{B}_o \)) during the analyzed time intervals.

Additionally, we perform a MVA of the magnetic field measurements associated with each of the three analyzed flybys to determine if they show signatures which can be associated with the existence of one-dimensional current layers. The ratio \( \lambda_2/\lambda_3 \) results 4.5, 7.26 and 11.45 for the case of flyby T17, T19 and T40, respectively. Therefore, MVA shows a well-defined minimum variance direction and a well-defined intermediate maximum variance plane for each of the three studied flybys. As a result, MVA provides a good estimate for the normal vector to this plane in the three cases: for the flyby T17, the mean magnetic field vector is \( \langle \mathbf{B} \rangle = (1.68, 2.22, -1.18) \) nT (in KSO coordinates) and \( \mathbf{n} = (0.61, 0.67, 0.43) \). In the T19 case, the mean magnetic field vector is \( \langle \mathbf{B} \rangle = (-1.06, 3.12, -5.79) \) nT and \( \mathbf{n} = (0.14, 0.19, -0.97) \). Finally, in the T40 case, the mean magnetic field vector is \( \langle \mathbf{B} \rangle = (0.19, 2.07, -1.91) \) nT and \( \mathbf{n} = (-0.12, 0.40, 0.91) \). These results show that the variation in the magnetic field direction (for MAG data corresponding to the same time interval where CAPS observations show an acceleration of the plasma flow) is mainly restricted to a plane (intermediate maximum variance plane) and therefore, they can be associated with the existence of a one-dimensional current layer, supporting the idea that the proposed acceleration mechanism is at work.

In summary, these results show that the plasma under study is mainly composed of ions also found in Titan's gravitationally bound ionosphere. According to the performed Walén tests, slope and y-intercept values are not so far from what is theoretically predicted. Moreover, MVA of the MAG data corresponding to the three flybys also show signatures that can be associated with the existence of current layers. Therefore the observed changes in the kinetic energy might be associated with magnetic tension forces.
Resumen en castellano

Este capítulo está dedicado al estudio de las propiedades del plasma alrededor de Titán (el satélite más grande de Saturno) y de los procesos de aceleración de plasma ionosférico que tienen lugar en su magnetocola inducida. Tales procesos de aceleración son de una importancia fundamental cuando se busca estimar la magnitud del escape atmosférico debido a la interacción de Titán con su entorno de plasma.

Haciendo uso de datos provistos por Cassini, hemos seleccionado 3 “flybys” que atravesaron la magnetocola de Titán y analizamos si las fuerzas de tensión magnética son responsables por las variaciones observadas en la energía cinética del plasma de dicho satélite.

Más específicamente, observaciones provistas por el instrumento de ondas de plasma y radio muestran que dichas regiones están compuestas de plasma frío con densidades de electrones entre 0.1 y algunas decenas de electrones por centímetro cúbico. Por otro lado las mediciones del espectrómetro de plasma sugieren que el plasma en esta región está compuesto principalmente de iones cuyas masas están en el rango de 15 a 17 amu y 28 a 31 amu. A partir de estas mediciones determinamos la velocidad media del plasma y la velocidad de Alfvén en la magnetocola de Titan. El test de Walén aplicado a estas mediciones sugiere que la aceleración progresiva del plasma ionosférico (tal como se observa en mediciones del espectrómetro de plasma a bordo de Cassini) pueden ser interpretada en término de fuerzas de tensión magnéticas.

Por otro lado, también estimamos valores promedio para los flujos de partículas de origen ionosférico alejándose de Titán para cada uno de estos “flybys”.

Los trabajos realizados durante esta tesis dieron lugar a las siguientes publicaciones:

Concluding Remarks

In this thesis we have presented a contribution to the study of the interaction of magnetized plasma flows with unmagnetized obstacles surrounded by atmospheres. This analysis has been based on in situ magnetic field and plasma data obtained from instruments onboard several spacecrafts, and has been complemented by fluid and kinetic theoretical interpretations.

In Chapter 4 we have developed an analytical MHD description of the magnetic field draping that, in particular, shows a dependence of the magnetotail structure on the orientation of the background magnetic field. Interestingly, such a dependence can be observed when analyzing the Martian magnetotail structure, according to MGS MAG data. Also, we have presented an observational study on the characterization of the MPB of the IM of Halley’s comet.

In Chapter 5 we have investigated the properties and rates of occurrence of proton cyclotron waves observed upstream from the Martian and Venusian bow shocks. A theoretical analysis on the generation of these waves in a collisionless regime suggest that they are compatible with the presence of linear instabilities that emerge due to the implantation of ionized exospheric hydrogen in the solar wind. We identified a trend between the occurrence rate of the PCWs at Mars and the heliocentric distance. Moreover, a similar long-term trend has been derived for the densities of exospheric hydrogen, suggesting a
correlation between the wave occurrence and the evolution of the distant Martian exosphere. Even though all the MGS pre-mapping period took place under solar minimum conditions, VEX allowed to assess the dependence of the PCWs rate on the solar activity regime. In the case of Venus, we have found more PCWs when the Sun is more active, probably as a result of a higher relative density of exospheric protons with respect to SW protons near solar maximum.

Chapter 6 has been focused on Titan. By making use of Cassini plasma and magnetic field data, we have determined that magnetic tension forces play an important role accelerating ionospheric plasma in its wake. These results then suggest that this force contributes significantly to the atmospheric erosion of this moon of Saturn.

In view of these results, we present the following observations and conclusions. The interaction of an object with a magnetized plasma wind clearly depends on properties of this flow and those of the object. A fundamental and common property of the SW and the Kronian plasma flows is that they are both collisionless (see Table 2.1). As a result, the streaming external magnetic field is frozen into these plasmas. When an object with spatial scales larger than the gyroradii of the particles of the wind interacts with these magnetized plasmas, the streaming magnetic field plays a fundamental role, giving rise to different physical processes which also depend in a fundamental way on additional properties of the body.

First, let us focus on objects such as our Moon, which does not have an atmosphere and a magnetic field and is made of insulating matter. The surface of the body directly absorbs the solar wind particles and a bow shock is not formed since their mean free path and gyroradii are much larger than the size of the obstacle. On the contrary, if the body has a conductive core, is surrounded by an atmosphere or has a dynamo-generated magnetic field, a bow shock and a magnetosheath are formed ahead of it, whenever the velocity of the external wind is higher than the magnetosonic/alfvenic velocities. Moreover, when the size and the electric conductivity of an atmospheric object are large enough so that the time for the magnetic field to diffuse through the body is larger than the timescale for the plasma wind to flow past the body (or for changes in the ambient magnetic field), then the streaming magnetic field and the plasma frozen to it are stopped by the obstacle. The magnetic field then drapes around the obstacle and a magnetotail is formed.

Both the ionosphere and the exosphere define atmospheric unmagnetized obstacles, but the relative importance of these two elements varies from one object to the other. Active comets and Venus are likely examples of a pure exospheric interaction and an interaction where the ionospheric variability controls the induced magnetosphere, respectively.
Concluding Remarks

Despite these differences, and apart from signatures deriving from the magnetosonic nature of a magnetized plasma flow, the stages in which the momentum and energy exchange from the external wind to the atmosphere takes place, appear to be spatially ordered in a similar way at all unmagnetized atmospheric objects [Bertucci et al., 2011]. For instance, the formation of the MPB presents an enhancement of the magnetic field draping signatures, a decrease in the electron temperature and a decrease in the external ion density and energy.

Also, associated with the (extended) exospheres and the collisionless nature of the SW, PCWs are the main mechanisms through which energy and momentum is transferred between different charged particle species in the upstream regions of Mars and Venus. At Mars, the density of the pick-up ion population relative to that of the background plasma is high enough to enable wave generation for all IMF cone angles. At Venus, with a relatively lower density, only the most efficient resonant mechanism (beam distribution) enables significant PCWs generation. Moreover, the maximum distance from the planet-Sun line at which PCWs are observed on both planets ranges between 6 and 7 times the bow shock stand-off distance [Delva et al., 2011a], suggesting that the size of the SW-planetary atmosphere interaction region is similar for both planets. One particular difference in the wave properties upstream from Mars and Venus is the high amplitude and coherence observed in the first case. The latter properties point to nonlinear effects, probably associated with an incomplete thermalization of the newborn planetary ions [Mazelle et al., 2004].

In addition to the atmospheric loss due to pick-up ion processes, the proper induced magnetosphere (within the MPB) is a region where a great amount of momentum and energy is transferred between the external and the local plasma population. Therefore, this is the key region when estimating planetary/satellite ion escape rates. Studies on the ionospheric escape for Mars and Venus indicate that H$^+$, O$^+$ and for Mars also O$_2^+$ are the dominant species [Barabash et al., 2007a]; [Barabash et al., 2007b]; [Lundin et al., 2009]. Loss of H$^+$ and O$^+$ is close to the stoichiometric 2:1 ratio, reflecting the ion composition of the upper ionosphere at Venus and Mars [Lundin, 2011]. Evaluations of oxygen loss fluxes on Mars and Venus vary depending on the assumptions considered, the region visited by the spacecraft, the selected data set or the solar activity. Figure 32 of Dubinin et al. [2011] shows that the escape rates are on the order of $10^{25}$ ions s$^{-1}$ and $10^{26}$ ions s$^{-1}$, for Mars and Venus, respectively. In the case of Titan we determine (Chapter 6) that the ionospheric escape rate is on the order $10^{25}$ ions s$^{-1}$.

Based on the previous conclusions, it is important to stress the significance of current space missions that are and will be very important to improve the characterization of the
plasma conditions around different IMs. For instance, the spacecraft Mars Atmosphere and Volatile Evolution (MAVEN), currently around Mars, is helping to answer questions regarding the type of atmospheric ion distribution functions and the corresponding magnetic field configuration present in the regions upstream and downstream of its bow shock. This is of great importance to foster a deeper understanding of the different physical processes taking place between newborn exospheric ions and the SW. It also helps to understand how their particle distribution function evolves from the initial implantation to the assimilation by the SW. As part of this topic and linked with Chapter 5, Crary et al. [2015] reported initial MAVEN observations of proton cyclotron waves and pick-up protons in the upper Martian exosphere. Even though proton pick-up ions have already been observed by Mars Express (MEX), and proton cyclotron waves have been observed by MGS, MAVEN is the first spacecraft able to simultaneously measure both the particle distribution functions and the magnetic field. These observations are obtained thanks to the solar wind ion analyzer (SWIA) and the magnetometer (MAG) instruments onboard this spacecraft. MAVEN MAG sampling frequency is 32 Hz, while measurement cadences as fast as 4 s for the SWIA ion instrument can be achieved. These very high cadences also allow to observe highly varying phenomena, in particular highly varying ion fluxes. As previously stated, the study of these waves with MAVEN is of great interest since they can be used to assess Mars’s exospheric structure and its loss rate. Moreover, MAVEN primary mission will take place closer to solar maximum, which also drives interesting comparisons with previous MGS results.

At the present time, MEX and MAVEN also provide an unique opportunity to have two simultaneous spacecrafts with plasma and neutral instruments in the Martian vicinity, allowing to use their measurements in a complementary way. For example, simultaneous measurements in the SW region and the Martian environment would be helpful to extend the analysis presented in Chapter 4 and to investigate the dynamics of the induced magnetotail lobes, as well as their response to different configurations in the external drivers (e.g. SW speed, density, pressure, IMF, etc), which also vary with time.

Furthermore, MEX can also be used to partially characterize the SW state while MAVEN provides information about the Martian induced magnetosphere and the atmospheric erosion associated with different time-dependent drivers. The analysis of these data together with theoretical approaches could then be used to study fundamental properties of the induced magnetosphere as well as its responses to different structures such as extreme events, providing information about the early SW-Martian interaction [Khodachenko et al., 2007]. Studies about the effects variable external conditions over the Martian IM would also be certainly helpful to interpret Cassini data obtained in the
Concluding Remarks

highly variable surroundings of Titan.

In orbit around comet 67P/Churyumov-Gerasimenko since August 2014, the Rosetta spacecraft is also improving our current knowledge about cometary IMs. This spacecraft reached the comet 67P at a distance of about 3.65 AU from the Sun. After this, both objects were heading towards the comet’s perihelion, at about 1.25 AU from the Sun. Because of this, it is expected that this mission would allow to span both low and maximum cometary activity levels. Thus, if the size and plasma pressure of the ionized atmosphere start (at some point between both activity levels) defining its boundaries, it would allow Rosetta to provide the first measurements of the “birth” of a cometary magnetosphere. To perform this task, five sensors that constitute the Rosetta Plasma Consortium (RPC) allow to measure physical properties of the nucleus, examine the structure of the inner coma, monitor cometary activity, and study the comet’s interaction with the solar wind.

Recent results based on data obtain by RPC-Ion Composition Analyzer between 3.65 AU to 2 AU show that at these distances, the comet still presents low levels of activity [Behar et al., 2015]. In this state, the atmosphere of comet 67P is permeated by the SW and the plasma boundaries such as the bow shock or the ionopause are not observed. Under these conditions, mass loading is the main mechanism through which the comet atmosphere affects the incoming SW [Behar et al., 2015]. It is important to examine such a process in detail, not only for IMs within the solar system, but also for that of other planetary systems. Indeed, mass loading is expected to be very important for planets around young stars, where the solar EUV radiation heating the upper part of an atmosphere is generally more intense than for an older star such as the Sun [Ribas et al., 2015]; [Nilsson et al., 2015]. As a result of this increase, a higher temperature and an extended scale height of the upper atmosphere would take place, and therefore, more of the upper atmosphere may thus reach into the solar wind domain. Rosetta will accompany the comet at least up to September 2016, when this mission is expected to end.

Finally, appropriate analogues of the interaction of a stellar wind with close-in exoplanets might also be studied based on VEX observations around Venus. Indeed, as shown in Zhang et al. [2009], a very rare situation in which the SW interacted with Venus under a small IMF cone angle (about 10°), showed interesting transient alterations of its induced magnetosphere. Even though this IMF cone angle is very difficult to be found in the surroundings of Venus (and even more at Mars), this configuration could be present beyond our solar system. Terrestrial exoplanets within the close-in habitable zone of dwarf stars are believed to be unmagnetized or only weakly unmagnetized. Moreover, those exoplanets orbit very close to their host stars, and thus the Parker’s spiral angle
is expected to be very small. Therefore, even though the techniques used to observe the atmospheres of close-in exoplanets are very limited, Venus might occasionally provide a natural laboratory to study their evolution.
Concluding Remarks

Resumen en castellano

En esta tesis hemos presentado una contribución al estudio de la interacción de los flujos de plasma magnetizados con objetos sin campo magnético intrínseco global rodeados por atmósferas. Este análisis se ha basado en mediciones in-situ de campo magnético y plasma obtenidas por instrumentos a bordo de varias sondas espaciales, y ha sido complementado por medio de interpretaciones teóricas fluidicas y cinéticas.

En el Capítulo 4 hemos desarrollado una descripción magnetohidrodinámica analítica del “arropeado” del campo magnético que, en particular, muestra una dependencia de la estructura de la magnetocola inducida con la orientación del campo magnético de fondo. Dicha dependencia puede ser observada analizando la estructura de la magnetocola marciana por medio de datos provistos por el magnetómetro a bordo de MGS. También hemos presentado un estudio observacional que permite caracterizar el contorno de apilamiento magnético de la magnetósfera inducida del cometa Halley.

En el Capítulo 5 hemos investigado las propiedades y las tasas de ocurrencia de ondas a la frecuencia ciclotrón del protón (PCWs) observadas “aguas arriba” de las ondas de choque de Marte y Venus. Un análisis teórico sobre la generación de estas ondas en un régimen no colisional sugiere que son compatibles con la presencia de inestabilidades lineales que surgen debido a la implantación de hidrógeno exosférico ionizado en el viento solar. Hemos identificado una tendencia entre la tasa de ocurrencia de estas ondas en Marte y la distancia heliocéntrica de dicho planeta. Además, una tendencia de largo plazo similar ha sido derivada para las densidades de hidrógeno exosférico, sugiriendo una correlación entre la ocurrencia de dichas ondas y la evolución de la exósfera marciana distante. A pesar de que todo el período de MGS previo a la fase de cartografía tuvo lugar bajo condiciones de baja actividad solar, Venus Express permitió evaluar la dependencia de la tasa de ocurrencia de PCWs con el régimen de actividad solar. En el caso de Venus hemos encontrado más PCWs cuando el Sol es más activo, probablemente como resultado de una mayor densidad relativa de protones exosféricos con respecto a protones del viento solar cerca del máximo de actividad solar.

El capítulo 6 ha sido focalizado en Titan. Haciendo uso de mediciones de plasma y campo magnético provistos por Cassini, hemos determinado que las fuerzas de tensión magnética tienen un rol importante acelerando plasma ionosférico en la estela de dicha luna. Estos resultados entonces sugieren que esta fuerza contribuye significativamente a la erosión atmosférica de Titan.

En vista de estos resultados, presentamos las siguientes observaciones y conclusiones. La interacción de un objeto con un viento de plasma magnetizado claramente depende
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de las propiedades de este flujo y de aquellas del objeto. Un propiedad fundamental que tienen en común el viento solar y el plasma Kroniano es que ambos flujos son no colisionales. Como resultado, el campo magnético externo está “congelado” a estos plasmas. Cuando un objeto con escalas espaciales más grande que el radio de giro de las partículas del viento interactúa con estos plasmas magnetizados, el campo magnético convectado juega un rol fundamental, dando lugar a diferentes procesos físicos que también dependen fuertemente de otras propiedades del objeto.

Primero, focalicémonos en objetos tales como nuestra Luna, que no posee una atmósfera o campo magnético propio y que está compuesta de materia aislante. La superficie del cuerpo absorbe directamente las partículas del viento solar y una onda de choque no se forma ya que el camino libre medio del flujo de plasma entrante es mucho mayor que el tamaño del obstáculo. Por el contrario, si el cuerpo tiene un núcleo conductor, está rodeado por una atmósfera o tiene un campo magnético intrínseco, una onda de choque y una magnetofunda se forman delante del mismo, siempre que la velocidad del flujo externo sea mayor que las velocidades magnetosónicas/alfénicas. Además, cuando el tamaño y la conductividad eléctrica de un objeto atmosférico son lo suficientemente grandes tal que el tiempo necesario para que el campo magnético difunda a través del objeto sea mayor que la escala temporal para que el plasma fluya alrededor del objeto (o para que haya variaciones en el campo magnético ambiente), el campo magnético convectado y congelado al plasma es detenido por el obstáculo. El campo magnético luego “arropa” al obstáculo y una magnetocola se forma.

La ionosfera y la exósfera definen los obstáculos atmosféricos no magnetizados pero la importancia relativa de estos dos elementos varía de un objeto a otro. Los cometas activos y Venus son probablemente ejemplos de una interacción puramente exosférica y una interacción donde la variabilidad ionosférica controla la magnetósfera inducida, respectivamente. A pesar de estas diferencias, y dejando de lado signaturas que derivan de la naturaleza supermagnetosónica del viento de plasma magnetizado, las etapas en las que el intercambio de impulso lineal y energía desde el viento externo a las atmósferas toma lugar parece estar ordenado espacialmente en forma similar en todos los objetos atmosféricos no magnetizados [Bertucci et al., 2011]. Por ejemplo, la formación del contorno de apilamiento magnético presenta un aumento de las signaturas del “arrogado” magnético, un decrecimiento en la temperatura de los electrones y un decrecimiento de la densidad y energía de los iones externos.

También asociado con las exósferas (extendidas) y la naturaleza no colisional del viento solar, las PCWs son el principal mecanismo a través del cual la energía y el impulso lineal es transferido entre las diferentes especies de partículas cargadas en la regiones “aguas
Concluding Remarks

arriba” de Marte y Venus. En Marte, la densidad de la población de iones exosféricos capturados relativa a la del plasma de fondo es suficientemente grande para permitir generación de ondas para toda orientación del campo magnético interplanetario. En Venus, con una densidad relativa menor, sólo el mecanismo resonante más eficiente (distribución de haz) permite una generación significativa de PCWs. Además, la distancia máxima de la línea planeta-Sol en la cual las PCWs son observadas en ambos planetas varía entre 6 y 7 veces la distancia de separación de la onda de choque [Delva et al., 2011a], sugiriendo entonces que el tamaño de la región de interacción entre el viento solar y la atmósfera planetaria es similar para ambos objetos. Una diferencia particular en las propiedades de las ondas “aguas arriba” de Marte y Venus es la gran amplitud y coherencia observada en el primer caso. Estas propiedades apuntan hacia efectos no lineales que resultan probablemente de una termalización incompleta de los iones planetarios recientemente creados [Mazelle et al., 2004].

Además del escape atmosférico producto de los procesos de captura de iones recién mencionados, la magnetósfera inducida (dentro del contorno de apilamiento magnético) es la región donde gran parte del momento lineal y energía es transferida entre las poblaciones del plasma externo y local. Por lo tanto, ésta es la región clave cuando se buscan estimar tasas de escape de iones planetarios/satélites. Estudios sobre el escape ionosférico de Marte y Venus indican que el H+, el O+ y también el O2+ en el caso de Marte son las especies dominantes [Barabash et al., 2007a]; [Barabash et al., 2007b]; [Lundin et al., 2009]. Las pérdidas de H+ y O+ son cercanas a la relación estequiométrica 2:1, reflejando la composición ionizada de la alta ionósfera de Venus y Marte [Lundin, 2011]. Estimaciones sobre los flujos de escape de oxígeno de Marte y Venus varían dependiendo de las hipótesis consideradas, las regiones visitadas por las sondas espaciales, el conjunto de datos utilizados y la actividad solar. La Figura 32 de Dubinin et al. [2011] muestra que las tasas de escape son del orden de 10^{25} iones s^{-1} y 10^{26} iones s^{-1}, para Marte y Venus, respectivamente. En el caso de Titán, en el Capítulo 6 determinamos que el escape ionosférico es del orden de 10^{25} iones s^{-1}.

Finalmente, es importante destacar la importancia de las misiones espaciales actuales que son y serán muy importantes para mejorar la caracterización de las propiedades del plasma alrededor de distintas magnetósferas inducidas, tales como Mars Express (MEX), y Mars Atmosphere and Volatile Evolution (MAVEN) en las cercanías de Marte y Rosetta en órbita alrededor del cometa 67P/Churyumov-Gerasimenko. Entre varios puntos, MAVEN y MEX proveen una oportunidad única de tener dos misiones espaciales simultáneas con instrumentos para medir propiedades del plasma y partículas neutras en el entorno marciano, y hacer uso de sus observaciones en forma complementaria.
Rosetta por otro lado permite medir distintas propiedades físicas del núcleo cometaario, examinar la estructura de su coma interna, monitorear la actividad del cometa y estudiar su interacción con el viento solar en la medida que 67P/Churyumov-Gerasimenko orbita alrededor del Sol en un entorno cercano a su perihelio.
The Walén relation

The consistency of the Walén relation and the MHD equations are shown here. For a similar approach, see Alfvén [1963]. Consider the Navier-Stokes and the magnetic induction equation:

\[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi \rho} - \frac{\nabla p}{\rho} \tag{A.1} \]

\[ \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{\partial \mathbf{B}}{\partial t} = 0 \tag{A.2} \]

where the displacement current has been neglected and the plasma conductivity is infinite.

The magnetic field consists of a component \( \mathbf{B}_0 \) generated by currents outside the fluid under study (\( \nabla \times \mathbf{B}_0 = 0 \)) and an induced field \( \mathbf{b} \) produced by currents observed in the system under study. The total magnetic field is \( \mathbf{B} = \mathbf{B}_0 + \mathbf{b} \). In the case of an incompressible fluid (\( \nabla \cdot \mathbf{v} = 0 \)) with constant density \( \rho \):

\[ \nabla \times (\mathbf{v} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{B} \tag{A.3} \]

Equation A.1 can be rewritten as

\[ \frac{\mathbf{B}_0}{4\pi \rho} \cdot \nabla \mathbf{b} - \frac{\partial \mathbf{v}}{\partial t} = -\frac{(\nabla \times \mathbf{b}) \times \mathbf{b}}{4\pi \rho} + (\nabla \times \mathbf{v}) \times \mathbf{v} + \frac{1}{\rho} \nabla \left[ p + \rho \frac{v^2}{2} + \frac{(\mathbf{B} \cdot \mathbf{B}_0)}{4\pi} \right] \tag{A.4} \]
while equation A.2 can be rewritten as

\[(B_0 \cdot \nabla)v - \frac{\partial b}{\partial t} = (v \cdot \nabla)b - (b \cdot \nabla)v\]  \hspace{1cm} (A.5)

In the case where the sum of the thermal pressure and the magnetic pressure is constant, an exact solution can be found where the fluid velocity and the magnetic field generated by currents inside it are related by the following equation:

\[v = \pm \frac{b}{\sqrt{4\pi \rho}}\]  \hspace{1cm} (A.6)

In this case, equations A.4 and A.5 are reduced to:

\[\left(\pm B_0 \sqrt{4\pi \rho} \cdot \nabla\right)b - \frac{\partial b}{\partial t} = 0\]  \hspace{1cm} (A.7)

Therefore, in the reference frame where

\[\frac{\partial b}{\partial t} = 0\]  \hspace{1cm} (A.8)

the plasma velocity is \(v' = v - V_{ss'}\), where \(V_{ss'} = \pm \frac{B_0}{\sqrt{4\pi \rho}}\). Thus,

\[v' = \pm \frac{b}{\sqrt{4\pi \rho}} \pm \frac{B_0}{\sqrt{4\pi \rho}} = \pm \frac{B}{\sqrt{4\pi \rho}} = \pm v_A\]  \hspace{1cm} (A.9)

Note that in this reference frame the convective electric field is zero. This equality, known as the Walén relation, states that the bulk velocity of the plasma (seen from a reference frame where the convective electric field is zero) is equal to minus or plus the local Alfvén velocity.
Consider an unbounded collisionless cold plasma made of the following three species:

- e: a background of massless electrons \( m_e \rightarrow 0, \ q_e = -e \)
- p: a background of protons \( m_p = m, \ q_p = e \)
- b: a beam of protons \( m_b = m, \ q_b = e \)

Next, we rewrite the equations describing the mass and momentum conservation for each particle species together with Maxwell’s equations under the previous assumptions:

\[
\begin{align*}
\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s v_s) &= 0 \\
m_sn_s \left[ \frac{\partial v_s}{\partial t} + v_s \cdot \nabla v_s \right] &= q_sn_s (E + \frac{v_s}{c} \times B) \\
c \nabla \times B &= 4\pi \sum_s q_sn_s v_s \\
\nabla \cdot B &= 0
\end{align*}
\]
\[ c \nabla \times E = -\frac{\partial B}{\partial t} \quad (B.5) \]
\[ \nabla \cdot E = 4\pi \sum_s q_s n_s \quad (B.6) \]

**Equilibrium state**

Same as in Chapter 2, we slightly perturb a uniform equilibrium of a plasma to study the properties of the normal modes that propagate in it. The equilibrium considered is given by \( B_o = B_o \hat{z} \) and \( v_e^{(o)} = 0 \). Hence, Ampere’s law together with the electron massless approximation imply that:

\[ E_o = -\frac{v_e^{(o)}}{c} \times B_o = 0 \quad (B.7) \]

and

\[ v_p^{(o)} = -\frac{n_b^{(o)}}{n_p^{(o)}} v_b^{(o)} \quad (B.8) \]

From Poisson’s law the condition \( n_p^{(o)} + n_b^{(o)} = n_e^{(o)} \) is also derived. Therefore, defining \( \chi \equiv \frac{n_b^{(o)}}{n_p^{(o)}} \) we obtain that:

\[ n_c^{(o)} \equiv n^{(o)}, \quad n_b^{(o)} = \chi n^{(o)}, \quad n_p^{(o)} = (1 - \chi) n^{(o)} \quad (B.9) \]

Thus \( v_p^{(o)} = -\frac{\chi}{1 - \chi} v_b^{(o)} \). Note that since we are assuming a uniform, stationary state, the mass continuity equation is trivially satisfied for all three species. Moreover, since we are considering a cold beam, \( v_b^{(o)} = v_b^{(o)} \hat{z} \). This, in turn implies that \( v_p^{(o)} \) have non-zero component only in the \( \hat{z} \) axis. Finally, we stress that this equilibrium satisfies Maxwell’s and the momentum conservation equations.

**First order parallel propagating perturbations**

We focus on our analysis on perturbations that propagate parallel to \( B_o \), i.e. \( k = k \hat{z} \). Since \( \nabla \cdot E = \nabla \cdot B = 0 \), \( \delta E \) and \( \delta B \) have non-zero components only in the plane perpendicular to \( B_o \). In other words, \( (\delta E)_z = (\delta B)_z = 0 \).

If we further assume that the plasma is incompressible, the continuity equation then requires that \( \delta n_s = 0 \forall s \). The resulting momentum conservation equation in terms of the linear perturbations \( \delta E \), \( \delta B \) and \( \delta v_s \) can be expressed as:

\[ \delta v_s = \frac{i q_s}{m_s} L_s^{-1} (\delta E + \frac{v_s^{(o)}}{c} \hat{z} \times \delta B) \quad (B.10) \]
where

\[ \mathbf{L}_s^{-1} = \begin{bmatrix} w - kv_s^{(o)} & -i\Omega_s \\ i\Omega_s & w - kv_s^{(o)} \end{bmatrix} \tag{B.11} \]

and \( \Omega_s = q_s B_o / m_s c. \)

Based on Faraday’s law we get

\[ \delta \mathbf{B} = \frac{ck}{w} \hat{\mathbf{z}} \times \delta \mathbf{E}, \tag{B.12} \]

and based on Ampere’s law we obtain

\[ c k [\hat{\mathbf{z}} \times \delta \mathbf{B}] = -4\pi i \sum_s q_s n_s \delta \mathbf{v}_s \tag{B.13} \]

Replacing Equations B.10 and B.12 into Equation B.13, it can be shown that:

\[ [c^2 k^2 I + \sum_s w^2_{ps} (w - kv_s^{(o)}) \mathbf{L}_s^{-1}] \cdot \delta \mathbf{E} = 0 \tag{B.14} \]

where \( w^2_{ps} = 4\pi q^2 n_s / m_s \) is the plasma frequency of the particle species \( s \). The dispersion relation is derived taking the determinant of Equation B.14, and can be expressed as:

\[ c^2 k^2 + \sum_s \frac{w^2_{ps} w_s}{(w_s - \sigma \Omega_s)} = 0 \tag{B.15} \]

\( w_s = w - kv_s^{(o)} \) and \( \sigma = \pm 1 \).

Applying this result to the particular case of a dilute proton beam in an electron-proton background plasma, the dispersion relations takes de form:

\[ \frac{c^2 k^2}{\Omega_{cp}} + \frac{\sigma w^2_{po}}{\Omega_{cp}} + \frac{\chi w^2_{po} (w - kv_b^{(o)})}{(w - kv_b^{(o)} - \sigma \Omega_p)} + \frac{(1 - \chi) w^2_{po} [w + \frac{\chi}{1 - \chi} kv_b^{(o)}]}{w + \frac{\chi}{1 - \chi} kv_b^{(o)} - \sigma \Omega_p} = 0 \tag{B.16} \]

where \( w_{po} = 4\pi e^2 n_{ro}^{(o)} / m_p = \frac{4\pi e^2 n_{ro}^{(o)}}{m_p} \).

The dimensionless version of Equation B.16 results

\[ \tilde{k}^2 + \tilde{w} \sigma + \frac{\chi (\tilde{w} - \tilde{k} M_b)}{\tilde{w} - \tilde{k} M_b - \sigma} + \frac{(1 - \chi) [\tilde{w} + \frac{\chi}{1 - \chi} \tilde{k} M_b]}{\tilde{w} + \frac{\chi}{1 - \chi} \tilde{k} M_b - \sigma} = 0 \tag{B.17} \]

where \( \tilde{w} = w / \Omega_{cp}, \tilde{k} = k c / w_{po}, M_b = v_b^{(o)} / v_A \) and \( v_A = B_o \sqrt{4\pi m_p n_{ro}^{(o)}} \).

As it can be seen the dispersion relation corresponds to a cubic polynomial for two branches (i.e \( \sigma = \pm 1 \)), which then implies a total of six solutions. Since this is a polynomial of third order with real coefficients, its roots must be all real or else one root is
real an the other two are complex conjugate. If $\chi = 0$, we recover the incompressible Hall-MHD solutions whose dispersion relation is displayed in Figure 2.2. Figure B.1 displays the dispersion relation of Equation B.17 (in gray) when $M_b = 10$ and $\chi = 0.01$, values close to the ones expected at Mars. It also shows the incompressible Hall-MHD dispersion relation already presented in Chapter 2 (in black). As it can be seen, the real part of the unstable three-fluid normal mode (UM) is characterized by values of $w$ and $k$ close to that of the anomalous resonant line ($w - kv_b^{(o)} = -\Omega_{cp}$) when the UM growth rate is close to its maximum value. In other words, for these values of $M_b$ and $\chi$, the UM is close to the anomalous resonant mode in which positive ions interact with right-hand polarized waves (see Chapter 2 and Chapter 5).

![Figure B.1: Cold-beam instability for $M_b = 10$ and $\chi = 0.01$.]


BIBLIOGRAPHY


BIBLIOGRAPHY


Lebreton, J., et al., 2005. An overview of the descent and landing of the Huygens probe


Lewis, G., N. André, C. Arridge, A. Coates, L. Gilbert, D. Linder, and A. Rymer (2008),
Derivation of density and temperature from the cassini–huygens CAPS electron spectrometer,

Lichtenegger, H., M. Delva, H. Gröller, and C. Bertucci (2013), The puzzling hydrogen


R. Lundin, S. Barabash, M. Holmström, H. Nilsson, M. Yamauchi, E.M. Dubinin, M.
36, L17202.

Lundin, R. (2011), Ion acceleration and outflow from Mars and Venus: An overview,

Three-dimensional multispecies MHD studies of the solar wind interaction with Mars

Mazelle, C., and Neubauer, F.M., (1993). Discrete wave packets at the proton cyclotron

Mazelle, C., J. B. Cao, G. Belmont, F. M. Neubauer, and A. J. Coates, (1997). Compressive character of low frequency waves driven by newborn ions at Comet Grigg-

interaction upstream from the Earth’s bow shock: A case study from Cluster-CIS.
Planetary and Space Science 51, 785 – 795.


BIBLIOGRAPHY


