Course title:

" Quantum entanglement in continous variable (CV) systems"

Dates: October 19 to November 6 of 2015.

Summary of the 9 lectures.

Dr. Fabricio Toscano

Profesor Adjunto del Instituto de Física
de la Universidad Federal de Rio de Janeiro (UFRJ),
Grupo de Óptica y Información Cuántica.

Lecture 0:

- Quantum States in finite Hilbert spaces.
  - Pure and mixed states state. Properties.
  - Qubits. Bloch sphere.
  - Schmidt decomposition theorem. Reduced density matrixes.
- Quick introduction to entanglement in finite Hilbert spaces.
  - Bipartite entanglement for pure and mixed states.
- Entanglement measures. General properties.
  - The entropy of entanglement for pure states.
- Communication protocols based on shared entanglement
  - An example: Quantum Teleportation of a qubit state.
  - Non-clonning theorem.
- Entanglement as a "resource" of Quantum Information tasks.
  - Using entanglement for Quantum Computing.

Lecture 1:

- Definition of a CV system.
  - Quantum cinematics
  - Position, momentum and Fock basis
  - Coherent states
- Bipartite Entanglement in CV systems.
  - Example of entangled state in CV system: the "two-mode" squeezed state.
- First example of a CV system: the quantized electromagnetic field.
- Second example of a CV system: the Transverse Spatial Degree of Freedom (STDF) of single photons propagating in the paraxial approximation.
- Generating entangled photons in the "STDG": the Spontaneous Parametric Down Conversion (SPDC)

Lecture 2:

- The Phase space representation of a CV system: quasi-probability distributions

- The classical group of translation and reflexions.
- The unitary representation of the group of translations and reflexions.
- Displacement operator in Quantum Optics and Coherent states.
- The Weyl-Wigner representation of operators (one mode). The Chord and the Wigner functions.
- Pure and mixed states in the Wigner representation.
- Mean values of observables in the Wigner representation.
- The Weyl-Wigner representation of operators for several modes.
- Reduced states in the Wigner representation.
- Marginal probability distributions of the Wigner function.
- The Wigner function of a n-mode Coherent state.

- Others quasi-probability distributions.
  - S-ordered characteristics functions and quasi-probability distributions of a quantum n-mode state.
  - Symmetric characteristic function and quasi-probability distribution of a quantum n-mode state.
  - Moments with the s-ordered characteristic functions.
  - Moments with the s-ordered quasi-probability distributions.
  - Smoothing the s-parameterized quasi-probability distributions.
  - The Q-function (Husimi function) of a quantum n-mode state.
  - The P-function of a quantum n-mode state.
  - The Optical Equivalence Theorem.
  - "Classicality" or "Non-classicality" of a quantum state of a CV system.
  - Brief Summary of characteristic function and quasi-probability distribution.

Lecture 3:

- Review of separable states in multimode CV systems
- Local unitary operations and entanglement
- Quantum information protocols with CV systems
- Quadratic evolution in CV systems: the symplectic group and its unitary representation the metaplectic group
  - Symplectic structure in CV systems
  - Real symplectic group $Sp(2n, R)$:
    - $Sp(2n, R)$ in classical mechanics
    - Complex and real form of the real symplectic group $Sp(2n, R)$.
    - The Lie algebra of $Sp(2n, R)$.
    - Uniparametric subgroups of $Sp(2n, R)$.
    - Factorisations of the symplectic group $Sp(2n, R)$.

    - The Euler of Bloch-Messiah factorisation of $Sp(2n, R)$.
  - The unitary representation of $Sp(2n, R)$: the metaplectic group $Mp(2n, R)$:
    - The metaplectic algebra
    - The exponential map in the metaplectic group $Mp(2n, R)$
    - The composition law of the metaplectic group.
    - The Lie algebra of $Mp(2n, R)$ in the complex form: complex form of quadratic Hamiltonians
    - Generators of the Lie algebra of the metaplectic group
    - The Euler or Bloch-Messiah factorisation of the metaplectic group $Mp(2n, R)$

- Example of the generators of the real metaplectic group in quantum optics (i.e. CV system of the quadratures of the electromagnetic field)
  - Passive generators:
    - Free evolution of the fields: the harmonic oscillators.
    - The beam splitter and phase shifters
    - Coherent state in a beam splitter. Metaplectic Covariance
  - Active generators:
    - Degenerate and non-degenerate parametric amplifier
    - The Optical Parametric Oscillator (OPO)
  - The balance homodyne detection
  - Basic tools of Gaussian operations in CV Quantum Information

- Example of the generators of the real metaplectic group in the CV system of traverse degrees of freedom of single photons.

Lecture 4:

- Gaussian States: definition
- Pure state condition for Gaussian states
- Example of gaussian states: a multimode coherent state
- Elimination of First moments in the Covariance matrix of an arbitrary state
- Invariant approach to uncertainty relations for quantum systems: General framework
- The Multimode Schrödinger-Robertson uncertainty principle: "bona fide" covariance matrix.
  - The Schrödinger-Robertson uncertainty principle for one mode systems
- Metaplectic invariance of the multimode Schrödinger-Robertson Uncertainty principle
- One mode Gaussian states: pure states
- One mode Gaussian states: mixed states

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Universidad de Buenos Aires
Facultad de Ciencias Exactas y Naturales

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VISTO:

la nota presentada por la Dra. Andrea Bragas, Directora Adjunta del Departamento de Física, en la que se eleva información y el programa del curso de posgrado **Quantun entanglement in continuous variable (CV) systems**, que será dictado por los Dres. Fabricio Toscano y Diego Wisniacki del 19 de octubre al 6 de noviembre de 2015,

CONSIDERANDO:

lo actuado en la Comisión de Doctorado

lo actuado en la Comisión de Postgrado,

lo actuado por este cuerpo en Sesión Ordinaria realizada en el día de la fecha,

en uso de las atribuciones que le confiere el Artículo Nº 113 del Estatuto Universitario,

EL CONSEJO DIRECTIVO DE LA FACULTAD DE
CIENCIAS EXACTAS Y NATURALES
RESUELVE

Artículo 1°: Autorizar el dictado del curso de posgrado **Quantun entanglement in continuous variable (CV) systems** de 31,5 hs de duración.

Artículo 2°: Aprobar el programa del curso de posgrado **Quantun entanglement in continuous variable (CV) systems** obrante a fs 5 a 11 del expediente de la referencia.

Artículo 3°: Aprobar un puntaje máximo de dos (2) puntos para la Carrera del Doctorado.

Artículo 4°: Comuníquese a la Dirección del Departamento de Física y a la Biblioteca de la FCEN (con fotocopia del programa incluida). Comuníquese a la Dirección de Alumnos y a la Secretaría de Posgrado (sin fotocopia del programa). Cumplido, archívese

RESOLUCION CD N° 2831
SP GA 30/10/2015

Dr. JUAN CARLOS REBOREDA
DECANO

Dr. PABLO J. PAZOS
Secretario Adjunto de Posgrado
FCEyN - UBA