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Citation: *AIP Advances* **1**, 042146 (2011); doi: 10.1063/1.3664297

View online: <http://dx.doi.org/10.1063/1.3664297>

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## Full correspondence between asymmetric filling of slits and first-order phase transition lines

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(Received 13 September 2011; accepted 24 October 2011; published online 14 November 2011)

Adsorption on single planar walls and filling of slits with identical planar walls are investigated in the frame of the density functional theory. In this sort of slits the external potential is symmetric with respect to its central plane. Calculations were carried out by applying both the canonical and grand canonical ensembles (CE and GCE, respectively). The behavior is analyzed by varying the strength of the adsorbate-substrate attraction, the temperature  $T$ , and the coverage  $\Gamma_\ell$ . Results obtained for physisorption of Xe on alkaline surfaces are reported in the present work. Prewetting (PW) lines and wetting temperatures,  $T_w$ , are determined from the analysis of adsorption on single walls. The filling of slits is analyzed for temperatures  $T > T_w$ . It is found that whenever for a given Xe-substrate combination the adsorption on a single wall exhibits a first-order wetting transition then asymmetric profiles that break the left-right symmetry of the external potential appear in the filling of an equivalent slit. These spontaneously symmetry breaking (SSB) solutions occur in a restricted range of  $\Gamma_\ell$  with a  $T$ -dependent width. In the case of closed slits analyzed in the CE scheme, the obtained asymmetric profiles exhibit lower Helmholtz free energies than the symmetric species and, therefore, could be stabilized in this geometry. For open slits, the GCE scheme yields all the symmetric and SSB states in the corresponding convex regimes of the free energy. It is shown that both the CE and the GCE frames yield three coexistent states, two symmetric and one asymmetric twofold degenerate. Both a PW line and the related SSB effect terminate at the same temperature. For rather strongly attractive surfaces reentrant SSB states are found at a fixed value of  $T$ . Copyright 2011 Author(s). This article is distributed under a Creative Commons Attribution 3.0 Unported License. [doi:10.1063/1.3664297]

### I. INTRODUCTION

The study of the adsorption of fluids on solid walls is of considerable current interest. An entire volume including reviews and discussions of theoretical and experimental investigations on this subject was recently published.<sup>1</sup> That set of articles indicates that properties of systems involving fluids at interfaces, like wetting, spreading and filling, are not only interesting from the fundamental physics point of view, but have also numerous technological applications. Namely, these systems consist of a liquid phase ( $l$ ) adsorbed on solid substrates ( $s$ ) in the presence of a vapor atmosphere

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(v). Its behavior depends on: the strength of the fluid-fluid ( $f$ - $f$ ) attraction,  $\varepsilon_{ff}$ , the well depth of the substrate-fluid ( $s$ - $f$ ) interaction,  $\mathcal{W}_{sf}$ , the temperature,  $T$ , and the chemical potential,  $\mu$ .

A systematic classification of the adsorption on attractive planar surfaces, which exhibit an infinite extent in the  $x$  and  $y$  directions, was performed by Pandit, Schick, and Wortis.<sup>2</sup> In practice, at a fixed  $T$  the adsorption properties are mainly determined by the relative strength  $\varepsilon_r = \mathcal{W}_{sf}/\varepsilon_{ff}$ . By varying this parameter it is possible to sweep a variety of physical phenomena like drying, critical wetting, prewetting, wetting, and layer formation. Given a value of  $\varepsilon_r$ , the results are usually summarized as a phase diagram in the  $(T, \mu)$  plane, typical examples are depicted in Fig. 1 of Ref. 3. For moderate substrates (i.e., when  $\varepsilon_r$  is slightly bigger than one) there is a first-order wetting transition at the point  $[T_w, \mu_w = \mu_0(T_w)]$  between the triple point temperature,  $T_t$ , and the critical one,  $T_c$ . In this case  $T_w$  is characterized by the appearance of coexisting thin and very thick adsorbed fluid films, while for  $T < T_w$  the coverage of adsorbed films is finite (incomplete wetting). Under these conditions for  $T \geq T_w$  there is an associated prewetting (PW) line which extends away from  $[T_w, \mu_w]$  into the region of pressures below the corresponding bulk saturation value  $P_0(T)$  and terminates at the critical prewetting (CPW) point  $[T_{cpw}, \mu_{cpw}]$ , see Fig. 1(b) in Ref. 3. It is usually assumed that in this regime the density profile of the fluid only depends on the coordinate  $z$  perpendicular to the substrate,  $\rho(\mathbf{r}) = \rho(z)$ . A PW transition is marked by a jump in the excess surface density (coverage), often expressed in nominal layers  $\ell$  as

$$\Gamma_\ell = (1/\rho_l^{2/3}) \int_0^\infty dz[\rho(z) - \rho_B], \quad (1)$$

where  $\rho_B$  is the asymptotic bulk vapor density and  $\rho_l$  the liquid density at saturation for a given temperature. The discontinuity in  $\Gamma_\ell$  vanishes at  $T_{cpw}$ , where the coexisting thin and thick films become identical. For slightly more attractive substrates the phase diagram presents several convergent PW lines like in Fig. 1(d) of Ref. 3. In the case of strong substrates the coverage increases from zero, either in monolayer steps or continuously.

If a fluid is adsorbed on a single planar wall the physisorption potential does not exhibit any symmetry along the  $z$  axis perpendicular to the substrate. On the other hand, when the confinement is produced by a planar slit with identical walls the  $s$ - $f$  interaction becomes symmetric with respect to the plane located at the center of the slit. Under such conditions it is quite reasonable to expect that  $\rho(z)$  would respect the symmetry of the external potential. However, Sikkenk *et al.*<sup>4,5</sup> have found by carrying out molecular dynamics calculations that asymmetric density profiles of the fluid may appear in the process of filling a planar slit built up of moderately attractive identical walls. This phenomenon is known as spontaneous symmetry breaking (SSB) effect and disappears above a critical temperature  $T_{sb}$ . The authors of Ref. 4 explained the occurrence of structures with different symmetries in terms of the balance of substrate-liquid  $\gamma_{sl}$ , substrate-vapor  $\gamma_{sv}$ , and liquid-vapor  $\gamma_{lv}$  surface tensions. The SSB effect has been analyzed over more than two decades. In most of the studies, a Canonical Ensemble (CE) statistics was applied to examine filling of closed slits with classical gases.<sup>6-12</sup> Recently, Berim and Ruckenstein<sup>13</sup> have also investigated the occurrence of SSB for quantum <sup>4</sup>He confined in planar slits.

Furthermore, this kind of asymmetric solutions were also found by Merkel and Löwen in open systems.<sup>14</sup> By using both computer simulations and density functional (DF) theory in the frame of the Grand Canonical Ensemble (GCE) these authors demonstrated that under certain conditions a system confined by a symmetric potential can reach a state where the fluid shows a liquid-like density on one side and vapor-like density on the other side. Further evidence for symmetry breaking may be found in the works reported by Rżysko *et al.*<sup>15</sup> who carried out Monte Carlo simulations and by Edison and Monson who applied a dynamic mean field theory.<sup>16</sup>

Wetting and filling are intimately related phenomena. In a recent simultaneous study of wetting and filling we found some evidence for a correspondence between  $T_{cpw}$  of a single PW line obtained in the case of one wall adsorption and  $T_{sb}$  determined for the disappearance of SSB in slits.<sup>10,12</sup> Moreover, Rżysko *et al.*<sup>15</sup> have also suggested that the SSB is a result of the first-order nature of the transition. If the transition were of the second order, SSB would not occur. It is the aim of the present work to report a systematic search for establishing a general relation between PW and SSB. The behavior as a function of  $\varepsilon_r$  and  $T$  was examined. It was found that: whenever for a given

adsorbate-substrate combination the adsorption on a single planar wall exhibits a first-order wetting transition then asymmetric profiles appear by filling an equivalent slit, and both these phenomena PW and SSB terminate at the same temperature.

The theoretical background is outlined in Sec. II. Section III is devoted to present and analyze the results of our calculations performed in the frames of both the CE and GCE statistics. A summary of the findings is provided in Sec. IV.

## II. THEORETICAL FRAME

To establish a reliable correspondence between wetting and filling both kind of systems are treated in exactly the same approach. The present calculations have been carried out in the frame of the DF theory recently applied for studying physical adsorption of Ne and Ar.<sup>10-12,17,18</sup> This theory is based on the existence of a grand canonical functional  $\Omega(T, \mu, [\rho(\mathbf{r})])$  of the one-particle density  $\rho(\mathbf{r})$  in the (one-component) system, which depends parametrically on the two thermodynamic variables  $T$  and  $\mu$ .<sup>19</sup>

$$\Omega(T, \mu, [\rho(\mathbf{r})]) = F_{\text{DF}}(T, [\rho(\mathbf{r})]) - \mu N, \quad (2)$$

where  $F_{\text{DF}}$  is the functional for the Helmholtz free energy and  $N$  the number of adatoms

$$N = \int \rho(\mathbf{r}) \mathbf{d}\mathbf{r}. \quad (3)$$

The equilibrium density profile  $\rho_e(\mathbf{r})$  of the adsorbed fluid is obtained by solving the Euler-Lagrange (E-L) equation derived by minimizing (at fixed  $T$  and  $\mu$ ) the grand thermodynamic potential

$$\left. \frac{\delta \Omega(T, \mu, [\rho])}{\delta \rho(\mathbf{r})} \right|_{\rho=\rho_e(\mathbf{r})} = 0. \quad (4)$$

The value of the functional at equilibrium,  $\Omega(T, \mu, [\rho = \rho_e])$  is the real equilibrium grand canonical free energy  $\Omega_e$ .

### A. Density functional

In the DF theory there is no recipe for an exact expression of the intrinsic free energy  $F_{\text{DF}}$  corresponding to a fluid immersed in an external potential  $U_{sf}(\mathbf{r})$ .<sup>20</sup> We adopted the following form

$$\begin{aligned} F_{\text{DF}}(T, [\rho(\mathbf{r})]) = & \nu_{\text{id}} k_B T \int \mathbf{d}\mathbf{r} \rho(\mathbf{r}) \{ \ln[\Lambda^3 \rho(\mathbf{r})] - 1 \} \\ & + \int \mathbf{d}\mathbf{r} \rho(\mathbf{r}) \Delta f_{\text{HS}}[\bar{\rho}(\mathbf{r}); d_{\text{HS}}] \\ & + \frac{1}{2} \int \int \mathbf{d}\mathbf{r} \mathbf{d}\mathbf{r}' \rho(\mathbf{r}) \rho(\mathbf{r}') \Phi_{\text{attr}}(|\mathbf{r} - \mathbf{r}'|) \\ & + \int \mathbf{d}\mathbf{r} \rho(\mathbf{r}) U_{sf}(\mathbf{r}). \end{aligned} \quad (5)$$

The first term is the ideal gas free energy, where  $k_B$  is the Boltzmann constant and  $\Lambda$  the de Broglie thermal wavelength. Factor  $\nu_{\text{id}}$  was introduced by Ancilotto and collaborators in Eq. 4 of Ref. 21 (in a standard theory it is equal to unity). The second term is the repulsive  $f$ - $f$  interaction approximated by a hard-sphere (HS) fundamental measure theory (FMT) functional taken from Kierlik and Rosinberg<sup>22</sup> (KR), which has proven to be very successful even in highly inhomogeneous situations. Here  $d_{\text{HS}}$  is the HS diameter. The KR version is completely equivalent to the original FMT devised by Rosenfeld.<sup>23</sup> Notice that in the literature there are more recent versions of the FMT formulation like, for instance, the White Bear proposal<sup>24</sup> where the Mansoori-Carnahan-Starling-Leland<sup>25</sup> bulk equation of state was included. This fact makes the White Bear version more adequate for studying binary mixtures, but in the case of one-component HS fluid the difference with the original Rosenfeld functional is negligible as depicted in Fig. 3 of Ref. 24. The third term is the

attractive  $f$ - $f$  interaction treated in a mean-field approximation (MFA), which is written in terms of a recently proposed version<sup>17</sup> of the separation of the spherically symmetric Lennard-Jones (LJ) 12-6 potential originally introduced by Weeks, Chandler, and Andersen<sup>26</sup> (WCA)

$$\Phi_{\text{attr}}^{\text{WCA}}(r) = \begin{cases} -\tilde{\epsilon}_{ff}, & r \leq r_m \\ 4\tilde{\epsilon}_{ff} \left[ \left( \frac{\tilde{\sigma}_{ff}}{r} \right)^{12} - \left( \frac{\tilde{\sigma}_{ff}}{r} \right)^6 \right], & r > r_m, \end{cases} \quad (6)$$

where  $r = |\mathbf{r} - \mathbf{r}'|$  and  $r_m = 2^{1/6}\tilde{\sigma}_{ff}$  is the position of the LJ minimum. We set  $d_{\text{HS}} = \tilde{\sigma}_{ff}$ . The last contribution to Eq. (5) is due to the adsorbate-substrate interaction.

The well depth  $\tilde{\epsilon}_{ff}$  and interaction size  $\tilde{\sigma}_{ff}$  are considered as free parameters because the use of its standard bare values  $\epsilon_{\text{XeXe}}/k_B = 221$  K and  $\sigma_{\text{XeXe}} = 4.1$  Å (see, e.g., Ref. 27) overestimates the experimental result  $T_c = 289.74$  K. The three adjustable parameters (namely,  $v_{rn}$ ,  $\tilde{\epsilon}_{ff}$ , and  $\tilde{\sigma}_{ff}$ ) were simultaneously determined by imposing that on the  $l$ - $v$  coexistence curve of Xe at a fixed  $T$  the data of  $\rho_l(T)$ ,  $\rho_v(T)$ , and  $P_0(T) = P(\rho_l) = P(\rho_v)$  listed in Table XXIII of Ref. 28 be reproduced. In addition, since two bulk phases can coexist at a given  $T$  if and only if their chemical potentials are equal at  $P_0$ , i.e.,

$$\mu_l(T, P_0) = \mu_v(T, P_0) = \mu_0(T), \quad (7)$$

therefore, we also imposed this condition. For determining the optimal  $\tilde{\sigma}_{ff}$  all the contributions to the Helmholtz free energy containing this parameter, i.e., all the corresponding terms in both the  $\Delta f_{\text{HS}}[\bar{\rho}(\mathbf{r}); d_{\text{HS}} = \tilde{\sigma}_{ff}]$  and  $\Phi_{\text{attr}}^{\text{WCA}}(r; \tilde{\epsilon}_{ff}, \tilde{\sigma}_{ff})$  energies were considered. The obtained parameter  $v_{\text{id}}$  is essentially unity near  $T_l$  and decreases to about 0.9 in the regime close to  $T_c$  where there is a well known departure from the ideal gas behavior.<sup>29</sup> Since  $v_{\text{id}}$  is not strictly unity the Henry's law for adsorption at very low densities<sup>30</sup> is slightly violated in the present formulation. All in all, the present DF version should be considered as a relatively easy phenomenological approach with parameters introduced to account for experimental bulk data.

## B. Euler-Lagrange equations

The equilibrium density profile  $\rho(\mathbf{r})$  of the fluid is determined from the variational Eq. (4) written as

$$\frac{\delta}{\delta\rho(\mathbf{r})} \left[ F_{\text{DF}}[\rho(\mathbf{r}')] - \mu \int d\mathbf{r}' \rho(\mathbf{r}') \right] = 0. \quad (8)$$

This functional minimization leads to

$$\frac{\delta[F_{\text{DF}}(T, [\rho(\mathbf{r}')])]}{\delta\rho(\mathbf{r})} = \mu. \quad (9)$$

This variational condition yields the following E-L equation for a planar geometry

$$v_{\text{id}} k_B T \ln[\Lambda^3 \rho(z)] + Q(z) = \mu, \quad (10)$$

with

$$\begin{aligned} Q(z) &= \Delta f_{\text{HS}}[\bar{\rho}(z); d_{\text{HS}}] \\ &+ \int_0^L dz' \rho(z') \frac{\delta \Delta f_{\text{HS}}[\bar{\rho}(z'); d_{\text{HS}}]}{\delta \bar{\rho}(z')} \frac{\delta \bar{\rho}(z')}{\delta \rho(z)} \\ &+ \int_0^L dz' \rho(z') \bar{\Phi}_{\text{attr}}(|z - z'|) + U_{sf}(z). \end{aligned} \quad (11)$$

Here  $L$  is the size of the box adopted for solving the E-L equation. In addition, the number of particles by wall's area  $A$  becomes

$$n = N/A = \int_0^L \rho(z) dz. \quad (12)$$

### 1. Grand canonical ensemble

In an experimental setup devised to investigate adsorption, the fluid is usually exposed to a reservoir at fixed  $T_{bath}$  and  $P_{bath}$  (i.e.,  $\mu_{bath}$ ) allowing an unrestricted particle exchange with the environment. Under such conditions, the system at thermodynamic equilibrium exhibits fixed  $V$ ,  $T = T_{bath}$ , and  $\mu = \mu_{bath}$ , hence, the adequate procedure for studying such a phenomenon is to apply the GCE statistics. In such a scheme the E-L equation should be solved by fixing  $\mu$  and searching for the optimal solutions for  $\rho(z)$  and  $n = N/A$ .

Given a value of the independent variable  $\mu$  the density profile may be evaluated by inverting Eq. (10)

$$\rho(z) = \frac{1}{\Lambda^3} \exp\left(\frac{\mu - Q(z)}{v_{id} k_B T}\right), \quad (13)$$

and the self-consistent solution is obtained iterating this relation up to convergence. The number of particles per unit area is calculated with the expression

$$n = \frac{1}{\Lambda^3} \exp\left(\frac{\mu}{v_{id} k_B T}\right) \int_0^L dz \exp\left(-\frac{Q(z)}{v_{id} k_B T}\right). \quad (14)$$

### 2. Canonical ensemble

However, in practice, it is usual<sup>7,8,35</sup> to solve the adsorption problem by fixing  $n$  together with  $V$  and  $T$ , that is by applying the CE scheme, which is much easier to manage numerically than the GCE one. Notice that  $\mu$  and  $n$  are conjugate Legendre variables. In the CE statistics (strictly valid for closed systems)  $\mu$  is treated as a Lagrange multiplier  $\mu_{ce}$  to be determined from the solutions together with  $\rho(z)$ . So, in this case one should solve

$$\frac{\delta[F_{DF}(T, [\rho(z')])]}{\delta\rho(z)} = \mu_{ce}, \quad (15)$$

which leads to the E-L equation

$$v_{id} k_B T \ln[\Lambda^3 \rho(z)] + Q(z) = \mu_{ce}. \quad (16)$$

Then, the density profile  $\rho(z)$  can be written as

$$\rho(z) = \rho_0 \exp\left(-\frac{Q(z)}{v_{id} k_B T}\right), \quad (17)$$

with

$$\rho_0 = \frac{1}{\Lambda^3} \exp\left(\frac{\mu_{ce}}{v_{id} k_B T}\right). \quad (18)$$

Since in the CE scheme the calculations are performed at a fixed  $n$ , whilst  $\mu_{ce}$  is an unknown quantity, then Eq. (18) is used to evaluate  $\mu_{ce}$  and  $\rho_0$  is obtained from the integrated density profile

$$n = \rho_0 \int_0^L dz \exp\left(-\frac{Q(z)}{v_{id} k_B T}\right), \quad (19)$$

which leads to

$$\rho_0 = \frac{n}{\int_0^L dz \exp\left(-\frac{Q(z)}{v_{id} k_B T}\right)}. \quad (20)$$

Equating expressions (17), (18) and (20) one gets

$$\rho(z) = \frac{n}{\int_0^L dz \exp\left(-\frac{Q(z)}{v_{id} k_B T}\right)} \exp\left(-\frac{Q(z)}{v_{id} k_B T}\right) \quad (21)$$

for the density profile and

$$\mu_{ce} = -v_{id} k_B T \ln \left[ \frac{1}{n \Lambda^3} \int_0^L dz \exp \left( -\frac{Q(z)}{v_{id} k_B T} \right) \right] \quad (22)$$

for the chemical potential. In this scheme,  $\rho(z)$  is determined by iterating Eq. (21) until the required convergence is achieved, then  $\mu_{ce}$  is calculated with Eq. (22).

### III. ANALYSIS OF THE ADSORPTION

Before entering into the analysis of physisorption, we validated the obtained parameters by evaluating the surface tension of the free liquid-vapor interface,  $\gamma_{lv}$ . For such a purpose we set  $U_{sf}(z) = 0$  and followed the procedure outlined in previous works. Along the calculations dimensionless variables  $z^* = z/\tilde{\sigma}_{ff}$  and  $L^* = L/\tilde{\sigma}_{ff}$  for distances,  $n^* = N\tilde{\sigma}_{ff}^2/A$  for areal density, and  $\rho^* = \rho\tilde{\sigma}_{ff}^3$  for volume density are used.<sup>17,18</sup> Our results for  $\gamma_{lv}$  agree very well (in a similar way as for Ne and Ar in the latter references) with experimental data<sup>31</sup> over the entire temperature range  $T_l (= 161.3 \text{ K}) \leq T \leq T_c$ .

For the analysis of physisorption we adopted in all the cases the *ab initio* potentials of Chismeshya, Cole, and Zaremba (CCZ),  $U_{CCZ}(z)$ , given by Eq. (3) in Ref. 32 with the potential parameters listed in Table 1 therein. We report here results for adsorption of Xe on surfaces of Na, Li, and Mg. This set of substrates with  $\varepsilon_r = \mathcal{W}_{sXe}/\varepsilon_{XeXe} = 2.1, 2.8, \text{ and } 5.5$ , respectively, displays different wetting scenarios. Let us mention that Sinanoğlu and Pitzer<sup>33</sup> suggested that the strength of the pair *f-f* potential is modified by interactions with solid substrates. This effect is discussed in detail in Ref. 34. In the present paper, we neglect the adsorption-induced interactions and, along with other earlier authors,<sup>13,21,35-37</sup> adopt for studying physisorption the pair *f-f* potential of Eq. (6) with the empirical parameters  $\tilde{\varepsilon}_{XeXe}$  and  $\tilde{\sigma}_{XeXe}$  determined in the previous section.

In a first step we shall study adsorption on a single wall in order to determine PW lines and  $T_w$  temperatures. In so doing methods of analysis will be outlined, which will be also utilized for examining filling of pore slits.

#### A. Adsorption on a single wall

In this geometry the E-L equation was solved for  $U_{sf}(z) = U_{CCZ}(z)$  in a box of size  $L^* = 40$ . Adsorption isotherms at fixed temperatures, i.e., the reduced chemical potential  $\Delta\mu(T) = \mu(T) - \mu_0(T)$  and  $\Gamma_\ell$  were calculated using both the GCE and CE schemes.

Let us outline the treatment of the obtained results by describing the analysis of the adsorption isotherm for Xe deposited on Na at the highest temperature at which a van der Waals loop is observed. So, data calculated at  $T = 245 \text{ K}$  are displayed in Fig. 1. The solid lines stand for results obtained with both the GCE and CE calculations. Between the spinodal points  $S_1$  and  $S_2$  (characterized by  $d\mu/dn = 0$ ) there is a region marked by a dashed curve where  $d\mu/dn < 0$ . In that regime  $F_{DF}$  is not convex and the data were obtained from CE calculations. The GCE scheme only provides solutions with positive slope  $d\mu/dn$  guaranteeing convexity of  $F_{DF}$ . Notice, that this property is also exhibited by variational microscopic calculations performed in the framework of the paired-phonon analysis in conjunction with the hyper-netted chain expansion.<sup>38-41</sup>

In practice, to get convergence in the CE scheme one needs a smaller number of iterations than in the GCE case. This feature is due to the fact that at each step of the CE procedure  $\rho(z)$  is normalized to the required number of particles. Therefore, the CE scheme is widely used in the literature to study adsorption. A theoretical justification for the broad use of CE was provided by White *et al.*<sup>42</sup> These authors have shown that for stable or metastable solutions the results of the GCE and CE statistics may differ only if one considers situations of extreme confinement with a small number of particles.

The CE scheme provides  $\Delta\mu(T)$  for all values of  $n$ . When in this frame the data exhibit van der Waals loops, the two coexistent stable states located at  $n_{min}$  and  $n_{max}$  (i.e., at  $\Gamma_{\ell, min}$  and  $\Gamma_{\ell, max}$ ,

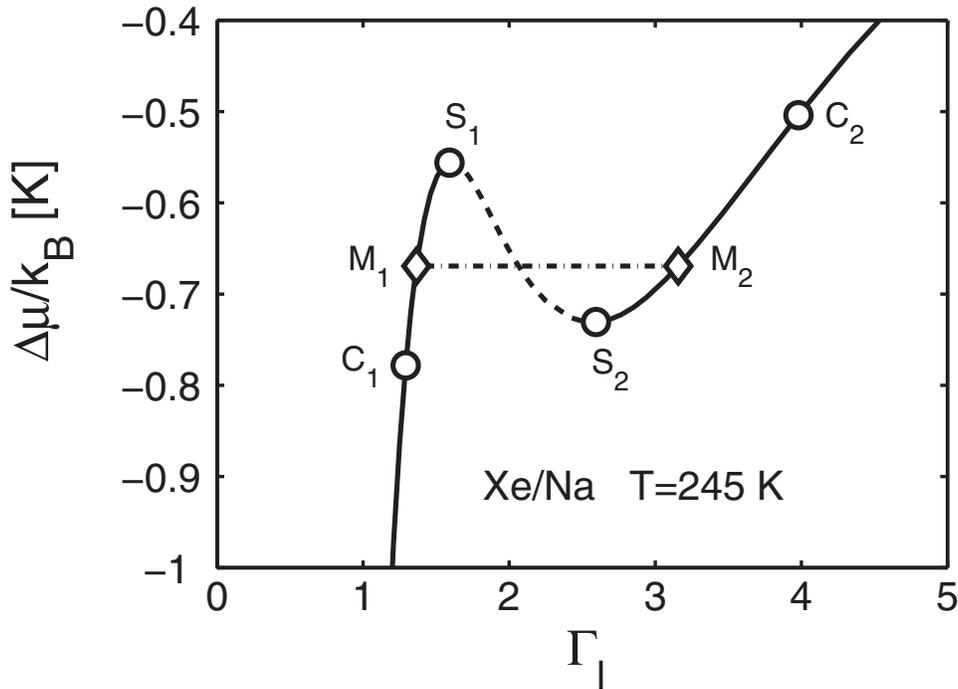


FIG. 1. Adsorption isotherm for Xe adsorbed on a single wall of Na at  $T = 245$  K. Solid curves were obtained with both the GCE and CE schemes, while the dashed line is obtained in the CE scheme only. Labels  $S_1$  and  $S_2$  show the location of spinodal points, while  $C_1$  and  $C_2$  are two generic stable states in the thin and thick film regimes, respectively.  $M_1$  and  $M_2$  stand for the coexistent thin and thick films determined from Maxwell construction indicated by the dash-dotted line.

respectively) which satisfy

$$\mu(n_{min}) = \mu(n_{max}) = \mu_e(\Omega_e/A) \quad (23)$$

are determined by applying the Maxwell's rule of equal areas as in Refs. 17, 35 and 43

$$\begin{aligned} \int_{n_{min}}^{n_{max}} [\mu(T) - \mu_0(T)] dn \\ = \Delta\mu_{pw}(T)[n_{max} - n_{min}] . \end{aligned} \quad (24)$$

For  $n \leq n_{min}$  the system grows continuously. In this case, for  $n \geq n_{max}$  (i.e., for  $\Gamma_\ell \geq \Gamma_{\ell, max}$ ) the film's growth does not present any further jump in coverage, for stronger substrates multiple jumps may occur (see below results for the Xe/Li system).

In the case of the GCE scheme, it is impossible to apply the equal area Maxwell's rule given by Eq. (24) due to the lack of data in the non convex region. Hence, some other method should be utilized to get the coexistent states. In the following lines we shall outline an adequate procedure.

The thermodynamics of a surface system is specified by the differential form for the free energy<sup>40,41</sup>

$$dF = d(E - TS) = -S dT + \gamma dA + \mu dN . \quad (25)$$

Here  $E$  is the total internal energy and  $\gamma$  the total surface tension (or surface energy per unit area). At a fixed  $T$  and constant  $A$  one gets

$$\mu = \left( \frac{\partial F}{\partial N} \right)_{T,A} = f + n \frac{\partial f}{\partial n} , \quad (26)$$

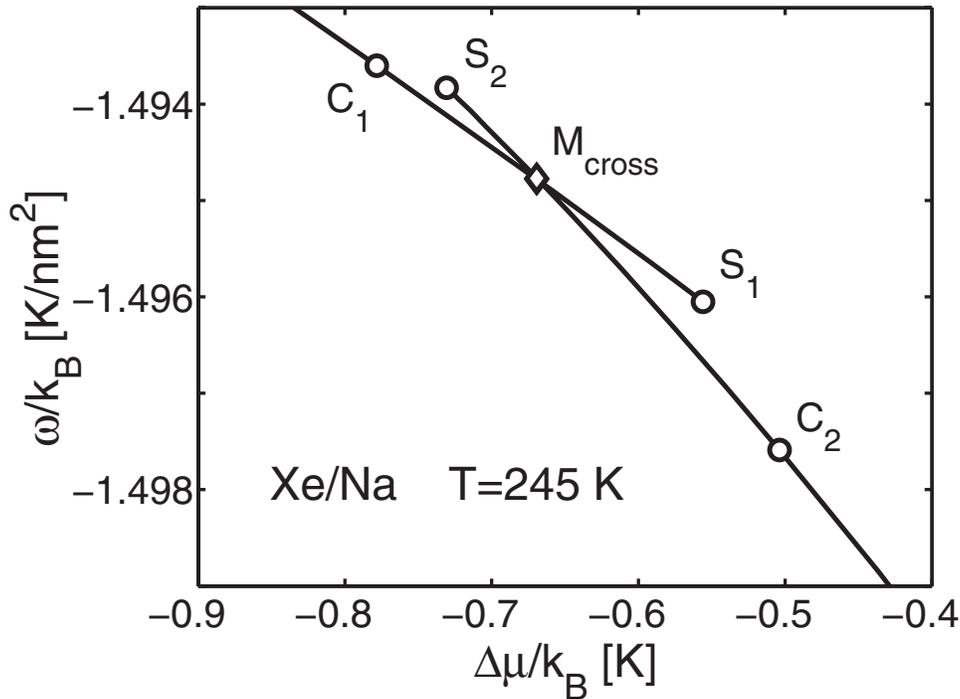


FIG. 2. Grand potential per unit area as function of the chemical potential for Xe adsorbed on a single wall of Na at  $T = 245$  K. Data were obtained in the GCE scheme. The curve containing the  $C_1$  and  $S_1$  points correspond to thin films, while that with  $S_2$  and  $C_2$  points to thick films. The crossing point labeled by  $M_{cross}$  indicate the coexistent thin and thick films.

and at constant  $N$

$$\begin{aligned} \gamma &= \left( \frac{\partial F}{\partial A} \right)_{T,N} = \frac{\partial f}{\partial n^{-1}} = -n^2 \frac{\partial f}{\partial n} = n(f - \mu) \\ &= \frac{F - \mu N}{A} = \frac{\Omega}{A} = \omega. \end{aligned} \quad (27)$$

The equilibrium conditions for a single-component fluid at  $T$  are that  $\gamma$  and  $\mu$  be uniform throughout the system. Therefore, two phases can coexist at a given  $T$  if and only if as required by Eq. (23) for some  $\mu_e$  both phases have the same  $\gamma_e$  (in other words equal  $\omega_e$ ). From a set  $(n, \mu, f_{DF})$  obtained in the convex regime of  $F_{DF}$ , i.e. along the sectors  $C_1$ - $S_1$  and  $S_2$ - $C_2$  of the curves traced in Fig. 1, one can evaluate  $\omega$ , and in turn these data can be used to perform the “so called” tangent Maxwell construction.<sup>44</sup> In practice, knowing  $\mu$  and  $\omega$  as a function of  $n$  one can plot  $\omega$  vs.  $\Delta\mu$  and determine  $\mu_e$  and  $\omega_e$  at coexistence. See, for instance, the crossing diagram plotted in Fig. 3 of Ref. 40 and the zigzag constructions displayed in Fig. 5(b) of Ref. 16 and in Fig. 2 of Ref. 43. Our data are plotted in Fig. 2, where the crossing point labeled by  $M_{cross}$  determines  $\Delta\mu_{pw}(T)$  and  $\omega_{pw}$  at coexistence. At this crossing point there is a knee characteristic of a first-order transition. Of course, the values of  $\Delta\mu_{pw}(T)$  obtained in Figs. 1 and 2 coincide. Moreover, the points  $M_1$  and  $M_2$  merge into  $M_{cross}$ . The states along the  $C_1$ - $M_{cross}$ - $C_2$  line are stable, while that lying in the sectors  $M_1$ - $S_1$  and  $S_2$ - $M_2$  are metastable. According to Hansen and McDonald, this kind of metastable states could be reached experimentally if sufficiently care is taken to prevent formation of the thermodynamically stable phase (see Chap. 5.6 in Ref. 29).

Figure 3(a) shows adsorption isotherms for Xe/Na evaluated at several temperatures above  $T_w$ . Horizontal lines indicate  $\Delta\mu_{pw}(T)$  determined from Maxwell constructions described above. One may observe the evolution of the PW phenomenon characterized by the jump in coverage  $\Delta\Gamma_\ell = \Gamma_{\ell, max} - \Gamma_{\ell, min}$  (or equivalently  $\Delta n = n_{max} - n_{min}$ ) at  $\Delta\mu_{pw}(T)$ . The largest jump occurs close to  $T_w$ , it shrinks for increasing  $T$  and eventually disappears at  $T_{cpw}$  where the thin and thick films merge. The temperature dependence of  $\Delta\mu_{pw}(T)$  is shown in Fig. 3(b). This kind of PW line corresponds to

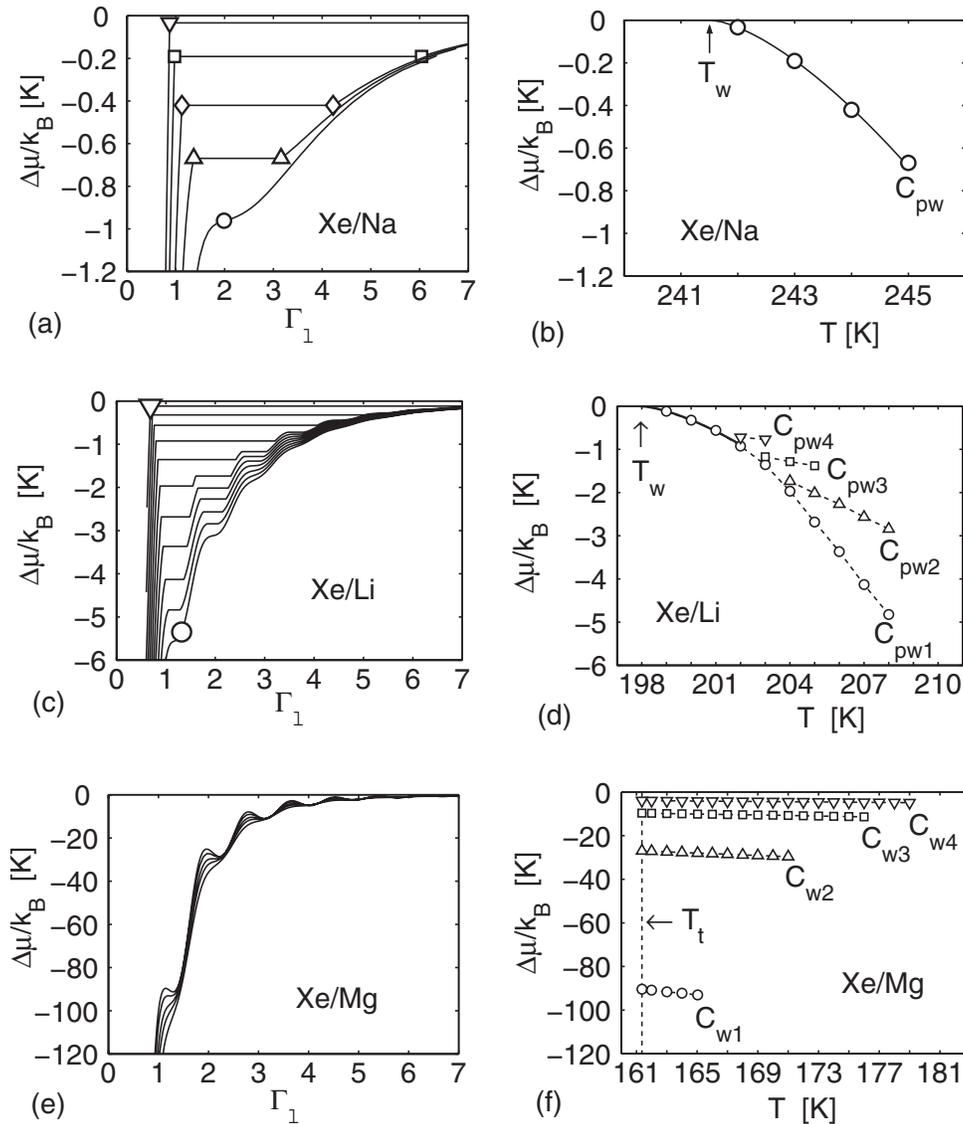


FIG. 3. (a) Adsorption isotherms for Xe adsorbed on a single wall of Na as a function of coverage  $\Gamma_l$  at temperatures from 242 K (triangle-down) to 246 K (circle) in steps of 1 K. (b) PW line for the Xe/Na system. The solid curve is the tangential fit. (c) Same as (a) for Xe/Li at  $T$  from 199 K (triangle-down) to 209 K (circle). (d) Phase diagram for Xe/Li. (e) Same as (a) for Xe/Mg at several values of  $T$  from  $T_t$  to 184 K. (f) Phase diagram for Xe/Mg.

the cases depicted in panel (a) of Fig. 20 in Ref. 2 and in panel (b) of Fig. 1 in Ref. 3 and it has been measured in several systems.<sup>1</sup> This quantity approaches tangentially zero at  $T_w$ , thermodynamic arguments<sup>3</sup> leads to the form

$$\Delta\mu_{pw}(T) = \mu_{pw}(T) - \mu_0(T) = a_{pw}(T - T_w)^{3/2}, \quad (28)$$

with  $a_{pw}$  being a model parameter. In practice, this expression is used to fit data of  $\Delta\mu_{pw}(T)$  for determining  $T_w$ . In the case of Xe/Na we obtained  $T_w = 241.6$  K and  $a_{pw}/k_B = -0.12$  K<sup>-1/2</sup>. In addition, Fig. 3(a) fixes an upper limit for the CPW point at  $T_{cpw} \lesssim 246$  K.

Adsorption isotherms for the Xe/Li system are displayed in Fig. 3(c). In this case the equal area Maxwell constructions yielded multiple jumps in coverage for a fixed  $T$ . In turn, this feature led to a series of merging PW lines. These lines are plotted in Fig. 3(d), the exhibited features are similar to that shown in panel (c) of Fig. 20 in Ref. 2 and in panel (d) of Fig. 1 in Ref. 3. One

may observe a series of surface triple points which determine a line of coalescence. The fit of this line to Eq. (28) yielded  $T_w = 198.0$  K and  $a_{pw}/k_B = -0.11$  K<sup>-1/2</sup>. The present  $T_w$  is lower than the value  $T_w = 225$  K quoted in Table I of Ref. 27 obtained from GCMC calculations. In this context, it becomes important to notice that our calculations<sup>17</sup> predict wetting of Rb by Ne in agreement with the experimental result of Hess, Sabatini, and Chan,<sup>45</sup> while the GCMC result reported in Table I of Ref. 27 suggests drying for Ne/Rb indicating that in such GCMC calculations the wetting properties are shifted toward higher  $T$ .

Adsorption isotherms calculated for Xe/Mg are displayed in Fig. 3(e). Since Maxwell constructions cannot be separated on the scale of the drawing they are not shown. These data don't exhibit a "standard" PW pattern because Mg is significantly more attractive than the alkali metals. A layer-by-layer film growth for  $T \geq T_i$  may be observed. The corresponding phase diagram is plotted in Fig. 3(f) being similar to that depicted in Fig. 16(a) of Ref. 2 for strong substrates.

## B. Filling a planar slit pore

In the slit geometry Xe atoms are confined by two identical solid walls separated by a distance  $L$ . Hence, the  $s$ - $f$  interaction becomes  $U_{sf}(z) = U_{CCZ}(z) + U_{CCZ}(L - z)$  and repulsion at the walls leads to  $\rho(z=0) = \rho(z=L) = 0$ . The reported solutions of the E-L equations correspond to a slit with  $L^* = 40$ . This width is wider than  $L^* = 29.1$  adopted for the pioneering molecular dynamics calculations<sup>4</sup> guaranteeing that the direct pair interaction between two atoms located close to opposite walls be negligible. First, we shall deal with closed systems and next to discuss the results for open slits.

### 1. Closed slits - Canonical ensemble

We shall now describe the filling of closed slits, i.e. systems at constant  $n$ , performed at fixed  $T$  and  $V$ . In this case, the equilibrium states were determined by adopting the CE statistics. Symmetric and asymmetric solutions were obtained along PW lines found in Sec. III A.

Let us first look at the results for a slit of Na at  $T = 245$  K. The adsorption isotherm is displayed in Fig. 4. Symmetric solutions indicated by the curve running through the points  $C_{S1}, S_{S1}, S_{S2}, C_{S2}$  exhibit the same pattern like that displayed in Fig. 1 for a single wall adsorption. Moreover, as expected the same values of the reduced chemical potential  $\Delta\mu$  are obtained for abscissas  $\Gamma_\ell(\text{slit}) = 2 \Gamma_\ell(\text{wall})$ . In addition, there are asymmetric solutions along the curve determined by the points  $S_{S1}, S_{A1}, S_{A2}$ . The difference between the free energies per particle,  $f_{DF} = F_{DF}/N$ , of asymmetric and symmetric solutions obtained at  $T = 245$  K is displayed as a function of  $\Gamma_\ell$  in Fig. 5. The asymmetric solutions have lower free energies than the symmetric ones over the entire range  $S_{S1}$ - $S_{A2}$ . This feature is obtained at all temperatures in the range  $T_w < T < T_{sb}$ . All internal states between the spinodal points  $S_{S1}$  and  $S_{S2}$  with minimum free energy could be stabilized in a closed slit pore.<sup>43</sup>

In order to facilitate comparisons between results for different absorbers, the main findings are depicted in a compact manner in Fig. 6. Panels (a)-(e) show that the asymmetric  $\rho(z)$  appear after a thin dense layer was adsorbed on both walls, similarly to the profile plotted in Fig. 3 of Ref. 6.

Figure 6(a) shows the asymmetric filling of a slit of Na at  $T = 245$  K, data for various  $\Gamma_\ell$  are displayed. The asymmetry of the density profiles is measured by the quantity

$$\Delta_N(\%) = \frac{100}{n} \int_0^{L/2} dz [\rho(z) - \rho(L - z)] . \quad (29)$$

In fact, there are two energetically equivalent asymmetric solutions which satisfy  $\rho_{asym2}(z) = \rho_{asym1}(L - z)$  yielding  $\Delta_N[\rho_{asym2}] = -\Delta_N[\rho_{asym1}]$ . The obtained values of  $\Delta_N$  for  $T \geq T_w$  are displayed in Fig. 6(f). A look at this drawing indicates that, at a fixed  $T$ , asymmetric  $\rho(z)$  appear between a lower and an upper coverage limit and this jump diminishes with increasing temperatures. Furthermore, we can state that the SSB effect extends along the single PW line displayed in Fig. 3(b) terminating at a critical  $T_{sb}$ , which coincides with  $T_{cpw}$ .

In the case of Xe/Li we show results obtained at  $T = 205$  K. This value of  $T$  was selected because it is the highest temperature exhibiting three first-order transition lines. Panels (b), (c), and (d) of Fig. 6 show the asymmetric formation of the second, third, and fourth layer. These structures

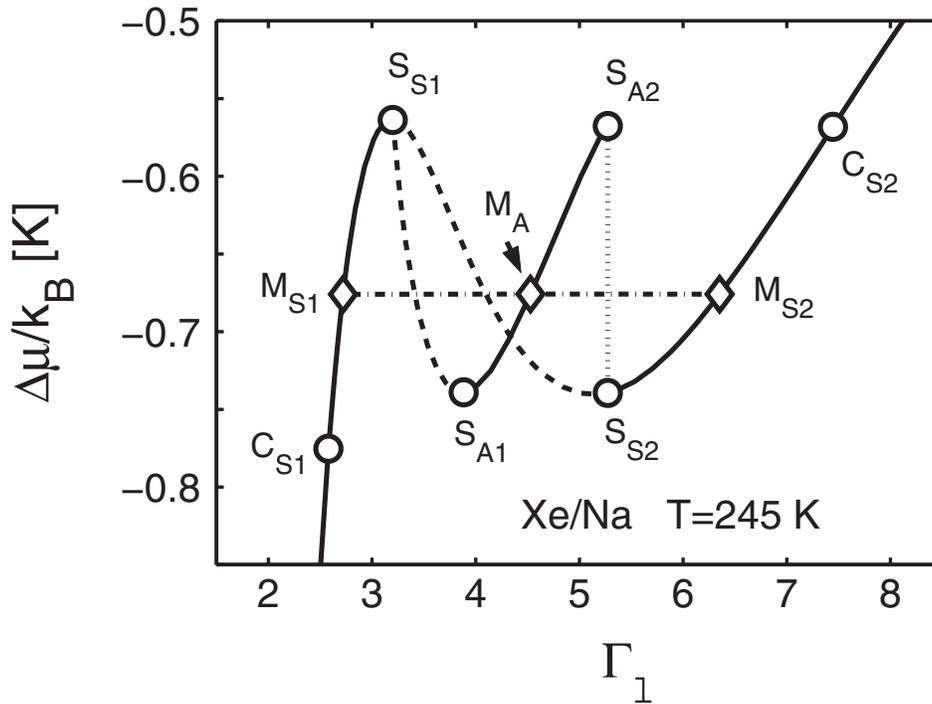


FIG. 4. Adsorption isotherm for symmetric and asymmetric solutions plotted as the reduced chemical potential versus coverage  $\Gamma_\ell$ . The symmetric solutions go through the set of points  $(C_{S1}, S_{S1}, S_{S2}, C_{S2})$ , while the asymmetric ones join the points  $(S_{S1}, S_{A1}, S_{A2}, S_{S2})$ . The horizontal line is the Maxwell construction joining the symmetric stable coexistent states  $M_{S1}$  and  $M_{S2}$ , whilst  $M_A$  stands for the asymmetric stable coexistent state (see text).

can be related, respectively, to the PW lines denoted by  $C_{pw1}$ ,  $C_{pw2}$ , and  $C_{pw3}$  in the  $(T, \Delta\mu)$  plane depicted in Fig. 3(d). A comparison of the asymmetry coefficients displayed in Fig. 6(g) and the first-order transition lines plotted in Fig. 3(d) indicates that the corresponding values of  $T_{sb}$  and  $T_{cpw}$  coincide. The chemical potential for both kind of solutions is displayed in Fig. 6(h). These van der Waals loops indicate that asymmetric solutions begin at a relative maximum of  $\mu_{sym}$  and finish at the next relative minimum. So, asymmetry appears just in the region where the slope of  $\mu_{sym}$  is negative implying instability of the symmetric solutions, whilst the slope of  $\mu_{asym}$  is positive over a large portion of that region satisfying the stability condition there. Figure 6(i) shows a comparison of  $f_{DF}$  of symmetric (circles) and asymmetric (triangles) states for  $n = 0.165$  [corresponding to  $\Gamma_\ell = 2.33$  where  $\Delta_N$  reaches its maximum at  $T = 208$  K, see Fig. 6(g)], the data correspond to temperatures  $T = 205, 206, 207,$  and  $208$  K. To get a comparison on the same scale the data were shifted by a constant  $f_{const}(T)$  at each temperature, hence, we plotted  $\Delta f_{DF} = f_{DF} - f_{const}(T)$ . It is clear that the  $\Delta f_{DF}$  of the asymmetric solutions is lower than that of symmetric one. This behavior is extended to other values of  $\Gamma_\ell$ . A crude estimation of an energy barrier between asymmetric to symmetric states was performed by assuming at fixed  $T$  and  $n$  the path  $\rho(z) = (1 - \alpha)\rho_{asym}(z) + \alpha\rho_{sym}(z)$  with  $\alpha$  running from 0 to 1. Both the free energy,  $\Delta f_{DF}$ , and the asymmetry coefficient,  $\Delta_N$ , were evaluated as a function of  $\alpha$ . These results are also displayed in Fig. 6(i). The barrier diminishes for increasing  $T$  and disappears at  $T = 209$  K.

For Xe/Mg the asymmetric solutions also occur along the first-order transition lines displayed in Fig. 3(f). Figure 6(e) shows the asymmetric formation of the second layer in a slit of Mg at  $T_t$ . The asymmetry parameter for  $T_t$ ,  $T_{nb}$  (normal boiling  $T$ ) = 165.03 K, 171 K, and 176 K is plotted in Fig. 6(j) as a function of coverage. This picture indicates that, as before, the values of  $T_{sb}$  coincide with the end temperatures of the corresponding  $C_{wi}$  lines.

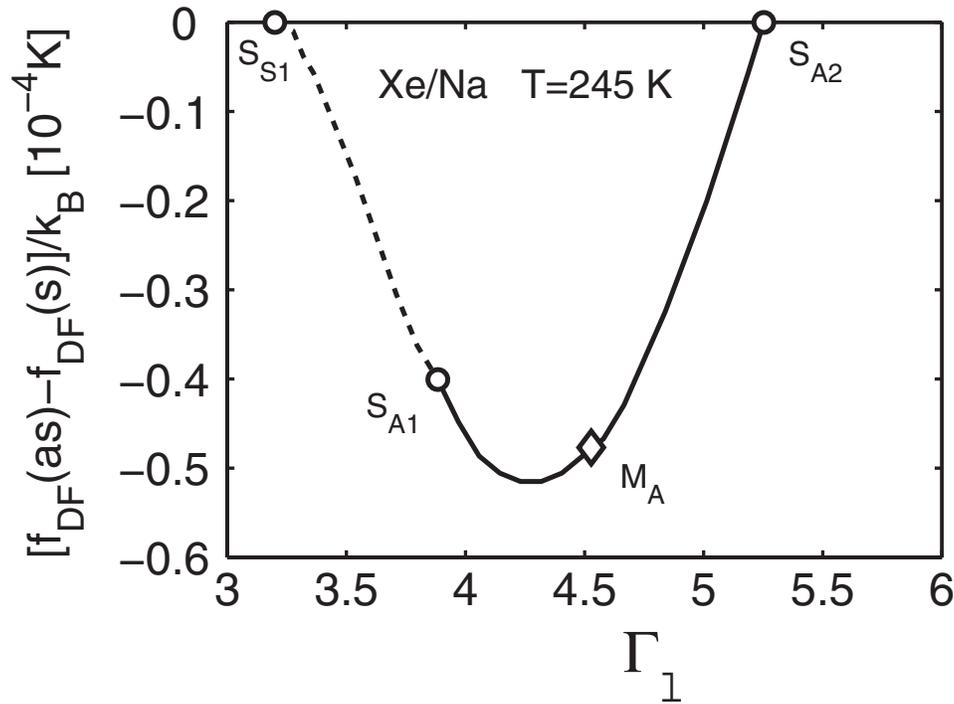


FIG. 5. Difference between free energies per particle of asymmetric and symmetric solutions obtained in the CE frame as a function of coverage  $\Gamma_\ell$ . Labels  $S_{S1}$ ,  $S_{A1}$ ,  $S_{A2}$  and  $M_A$  have the same meaning as in Fig. 4. In the regime delimited by  $S_{A1}$  and  $S_{A2}$  the chemical potential of asymmetric solutions exhibits a positive slope and these results are also obtained in the GCE scheme.

## 2. Grand canonical ensemble

Open slits are also in thermodynamic equilibrium with an environment at fixed  $T$ , but in this case the confined system is exposed to an unrestricted fluid exchange with the reservoir at fixed  $\mu$ . Under such circumstances, the adsorption occurs at constant  $V$ ,  $T$ , and  $\mu$ . Therefore, as mentioned before the adequate procedure for studying this phenomenon is to use GCE.

Let us now comment the results for the same system analyzed in detail in the previous section, i.e. Xe/Na at  $T = 245$  K. All the states marked with solid lines in Fig. 4 were also obtained from GCE calculations. Hence, GCE **does** provide both symmetric as well as asymmetric solutions in the regimes where the corresponding Helmholtz free energy,  $F_{DF}$ , is convex yielding positive  $d\mu/dn$ . The lack of data in the non convex region makes it necessary to apply the crossing method for searching the coexistent states. Figure 7 shows  $\mu$  as a function of  $\omega$  for both symmetric and asymmetric solutions corresponding to data displayed in Fig. 4. The crossing of the lines labeled by  $C_{S1}$ - $S_{S1}$  and  $S_{S2}$ - $C_{S2}$  at  $M_{cross}$  in Fig. 7 determines the coexistent symmetric solutions  $M_{S1}$  and  $M_{S2}$  already quoted in Fig. 4. Furthermore, the line  $S_{A1}$ - $S_{A2}$  corresponding to asymmetric solutions crosses the symmetric ones also just at  $M_{cross}$ . So, the state  $M_A$  included in Fig. 4 has the same  $\omega_e$  like the coexistent symmetric states  $M_{S1}$  and  $M_{S2}$ . This means that  $M_{cross}$  is a triple point determined by the intersection of three lines, where the three phases in equilibrium satisfy

$$\mu_{S1}(T, \omega_e) = \mu_{S2}(T, \omega_e) = \mu_A(T, \omega_e) = \mu_e(T, \omega_e). \quad (30)$$

The density profile of the state  $M_A$  is marked with a fat line in Fig. 6(a). In passing, let us mention that for completeness this state was also included in Fig. 5. The density profiles of the states  $M_{S1}$ ,  $M_A$ , and  $M_{S2}$  are compared in Fig. 8. In the symmetric cases  $M_{S1}$  and  $M_{S2}$  the film formed at each wall coincides, respectively, with the thin and thick structures got at coexistence for adsorption on a single wall. On the other hand, for  $M_A$  one may observe at the right wall the coexistent thick film

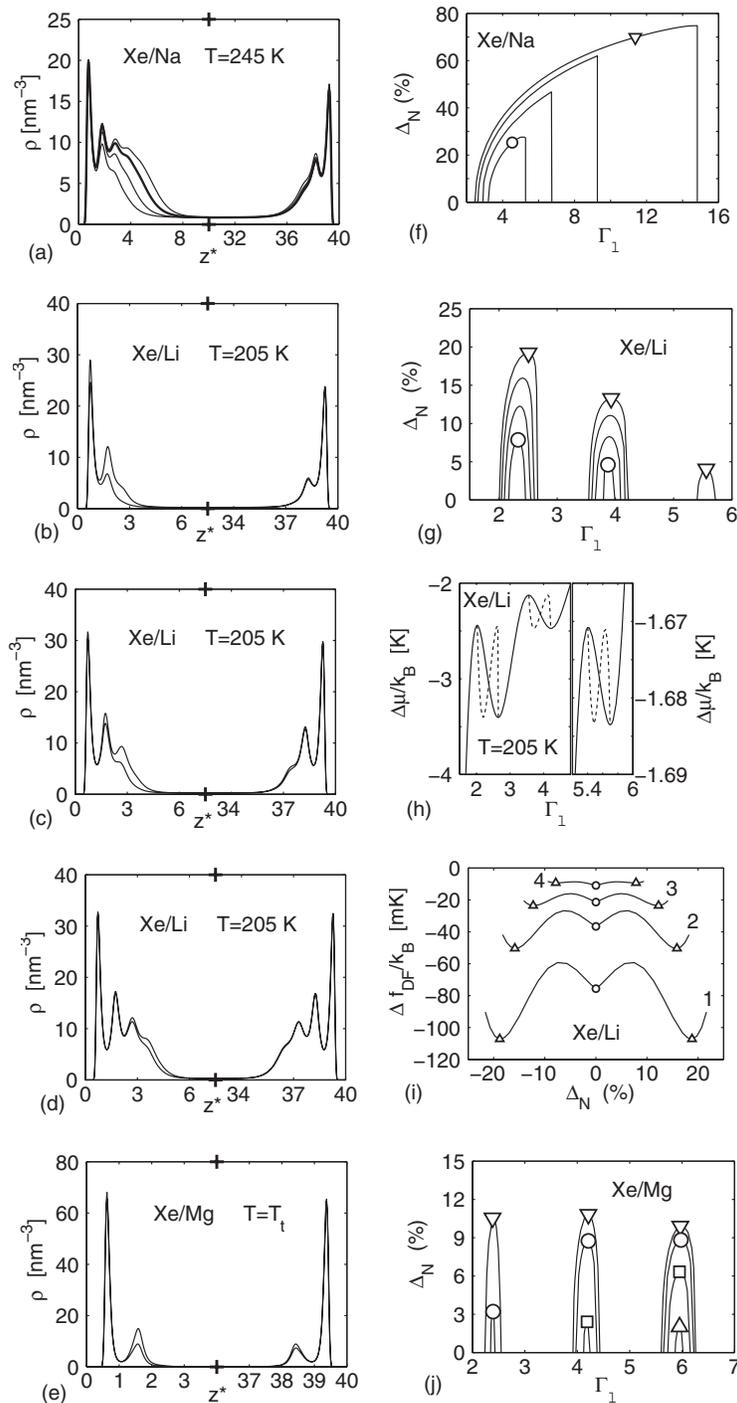


FIG. 6. (a) Asymmetric density profiles of Xe near to walls of a closed slit of Na at  $T = 245$  K for coverages  $\Gamma_\ell = 3.23, 3.79, 4.53$  and  $5.19$ . (b) Same as (a) for Xe/Li at  $T = 205$  K for  $\Gamma_\ell = 2.04$  and  $2.43$ . (c) Same as (b) for  $\Gamma_\ell = 3.55$  and  $3.85$ . (d) Same as (b) for  $\Gamma_\ell = 5.44$  and  $5.58$ . (e) Same as (a) for Xe/Mg at  $T = T_t$  for  $\Gamma_\ell = 2.24$  and  $2.42$ . (f) Asymmetry parameter for Xe/Na as a function of coverage at temperatures from  $242$  K (triangle-down) to  $245$  K (circle) in steps of  $1$  K. (g) Same as (f) for Xe/Li at  $T$  from  $205$  K (triangle-down) to  $208$  K (circle). Data correspond to the asymmetric filling of the first four layers. (h) Chemical potential for Xe/Li at  $T = 205$  K as a function of coverage. Solid and dashed curves correspond to symmetric and asymmetric solutions, respectively. (i) Relative free-energy per particle for Xe/Li showing energy barriers between symmetric (circles) and asymmetric (triangles) states at  $T = 205, 206, 207,$  and  $208$  K labeled by 1, 2, 3, and 4, respectively. (j) Same as (f) for Xe/Mg at  $T = T_t$  (triangle-down),  $T_{nb}$  (circle),  $171$  K (triangle-up), and  $176$  K (square).

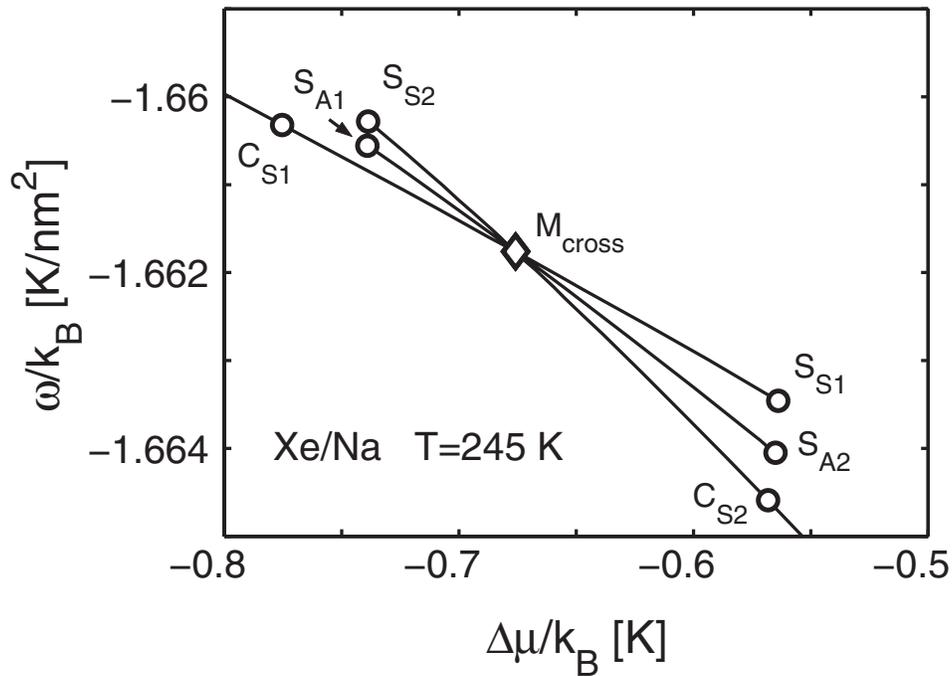


FIG. 7. Grand free energy per unit one wall area as a function of the reduced chemical potential for symmetric and asymmetric solutions. The meaning of points ( $C_{S1}$ ,  $S_{S1}$ ,  $S_{A1}$ ,  $S_{A2}$ ,  $S_{S2}$ ,  $C_{S2}$ ) is the same as in Fig. 4. The symmetric solutions lay on the curves  $C_{S1}$ - $S_{S1}$  and  $S_{S2}$ - $C_{S2}$ , while the asymmetric determine the line  $S_{A1}$ - $S_{A2}$ . The point  $M_{cross}$  indicates the location of the crossing the three lines determining the coexistence of symmetric and asymmetric solutions labeled, respectively, by  $M_{S1}$ ,  $M_{S2}$ , and  $M_A$  in Fig. 4.

and at the other wall the coexistent thin film. Notice that the asymmetric solution is twofold, it can also present a thin film at the right wall and a thick film at the left one.

It is important to notice that data plotted in Fig. 7 indicate that, away from  $M_{cross}$  for each value of  $\Delta\mu$  the asymmetric metastable states have always lower grand potential energy,  $\omega$ , than the symmetric metastable ones. On the other hand, above  $T_w$  for a given  $\mu$  the states with lowest grand potential lay on the curves denoted by  $C_{S1}$ - $M_{cross}$ - $C_{S2}$  and their structures are uniform films along the  $(x,y)$  plane. Therefore, no structure breaking symmetry on the  $(x,y)$  plane would have lower  $\omega$  at  $\mu_e$  than the coexistent symmetric and asymmetric states obtained at  $M_{cross}$ .

At lower values of  $T$  the system Xe/Na exhibits similar features to that found at  $T = 245$  K. One also gets triple points where symmetric and asymmetric states coexist. Figure 9 shows the envelope of coexistence for temperatures  $T > T_w$ . These data were obtained in both the CE and GCE frames. One may realize that the coverage  $\Gamma_\ell$  corresponding to the symmetric thick films as well as to the asymmetric ones goes to infinity when the temperature approaches  $T_w$ . On the other hand, both symmetric and the asymmetric one merge at  $T_{cpw}$  located slightly below  $T = 246$  K. Above this temperature only one sort of symmetric states are obtained.

The results obtained for the isotherms for Xe/Na analyzed above can be extended to any other series van der Waals loops found for systems Xe/Li and Xe/Mg examined in Sec. III B 1. Moreover, all the features remain unchanged when the width of the slit is enlarged. The results for the system Ar/Li reported in Ref. 12 give support to the latter statement.

#### IV. SUMMARY

Adsorption of Xe on alkaline planar surfaces was investigated within the frame of a DF theory. Namely, physisorption on single walls and filling of symmetric slit pores were analyzed. Both the CE and GCE statistics were applied.

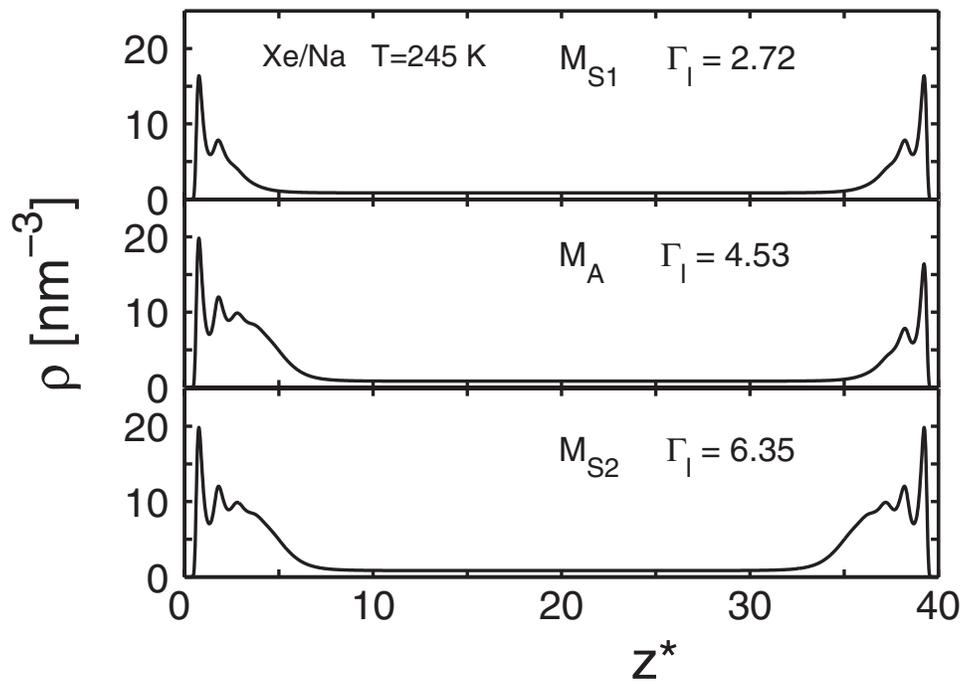


FIG. 8. Density profiles of Xe confined in a slit of Na as a function of  $z$ . These data correspond to coexistent systems in the GCE statistics at  $T = 245$  K. The labels  $M_{S1}$  and  $M_{S2}$  indicate symmetric solutions, while  $M_A$  indicates the asymmetric one.

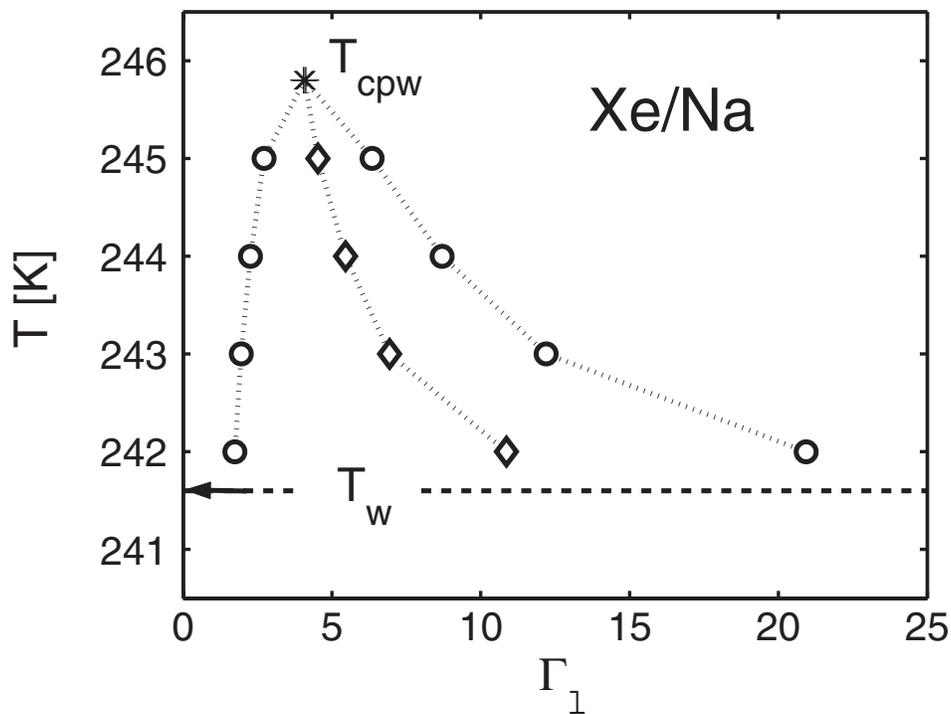


FIG. 9. Coexistent states of Xe confined in a slit of Na at temperatures between  $T_w$  and  $T_{cpw}$ . These data were obtained with both the CE and GCE statistics. Circles are symmetric states, while diamonds are asymmetric ones. The dotted lines are plotted to guide the eye, all these lines merge at  $T_{cpw}$  indicated by the asterisk.

Calculations for adsorption on a single wall were performed in order to identify first-order phase transitions and determining PW lines. Equilibrium chemical potential  $\mu_e$  at which stable thin and thick films coexist were evaluated by applying Maxwell constructions (equal area procedure for CE and crossing technique for GCE). The obtained results at coexistence with CE and GCE are equal. PW lines were used to determine wetting temperatures  $T_w$ .

The filling of closed slits of identical walls was studied at temperatures higher than  $T_w$  by applying the CE scheme. Solutions breaking the left-right symmetry of the adsorption potential exerted by the slit appear in the range of  $\Gamma_\ell$  corresponding to  $\Delta\mu$  of PW lines appear. Between spinodal points of the symmetric solutions the asymmetric profiles exhibit a lower Helmholtz free energy than the symmetric ones for the same number of particles. Therefore, these SSB species could be stabilized in a closed system.

Open slits were studied within the GCE frame. All the symmetric and asymmetric states with positive  $d\mu/dn$  provided by the CE scheme were also obtained when applying GCE. The crossing Maxwell construction yielded three coexistent states with the same  $\omega$  and  $\mu$ , i.e. two symmetric and one asymmetric twofold degenerate. The symmetric solutions exhibit at both walls either the thin or the thick film obtained at coexistence in the adsorption on a single wall. While the asymmetric state shows at the left wall the coexistent thick film and at the right wall the corresponding thin one, or vice versa. These results indicate that given a sort “coexistent film” at one wall it is no matter whether at the other wall one has the “thin” or the “thick coexistent film”. This finding is a realization in a realistic system of a behavior qualified as conceivable by Merkel and Löwen in the introduction of Ref. 14.

On the basis of the present DF calculations we conjecture that for  $T \geq T_w$  there is a one-to-one correspondence between any first-order phase transition line and the occurrence of asymmetric filling. In the case of complex phase diagrams like those of Fig. 3(d) and 3(f) corresponding to layering transitions, a multiple entrance of SSB at a fixed  $T$  was found. Moreover, the coincidence of the CPW points where the PW lines terminate and the critical temperatures  $T_{sb}$  at which asymmetric solutions disappear is clearly established.

Below  $T_w$  one expects that asymmetric profiles with lowest free energy shall also break the symmetry in the  $(x,y)$  plane exhibiting structures like those obtained in Refs. 9 and 16.

To the best of our knowledge these findings supporting a very close relation between wetting and filling have not been reported before. Moreover, this kind of SSB might appear in other theoretical frames which look symmetrical. Since the present work deals with realistic systems, the properties described above are sufficiently relevant to encourage experimental verification. Perhaps, such SSB profiles could be observable in confined colloidal suspensions.

## ACKNOWLEDGMENTS

This work was supported in part by grants PICT 31980/05 from ANPCyT, PIP 0546/09 from CONICET, and X099 from Universidad de Buenos Aires, Argentina.

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