WMAP 5-year constraints on time variation of $\alpha$ and $m_e$
in a detailed recombination scenario

Claudia G. Scóccola\textsuperscript{a,b,*},1, Susana J. Landau\textsuperscript{c,2}, Hector Vucetich\textsuperscript{a}

\textsuperscript{a} Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque S/N 1900 La Plata, Argentina
\textsuperscript{b} Instituto de Astrofísica La Plata – CONICET, Argentina
\textsuperscript{c} Departamento de Física, FCEyN, Universidad de Buenos Aires, Ciudad Universitaria – Pab. 1, 1428 Buenos Aires, Argentina

\section*{ARTICLE INFO}

Article history:
Received 12 August 2008
Accepted 27 September 2008
Available online 1 October 2008

Editor: L. Alvarez-Gaumé

\section*{ABSTRACT}

We study the role of fundamental constants in an updated recombination scenario. We focus on the
time variation of the fine structure constant $\alpha$, and the electron mass $m_e$ in the early Universe, and put
bounds on these quantities by using data from CMB including WMAP 5-yr release and the 2dFGRS power
spectrum. We analyze how the constraints are modified when changing the recombination scenario.
© 2008 Elsevier B.V. All rights reserved.

Time variation of fundamental constants is a prediction of theo-
ries that attempt to unify the four interactions in nature. Many
efforts have been made to put observational and experimen-
tal constraints on such variations. Primordial light elements abund-
dances produced at Big Bang nucleosynthesis (BBN) and cosmic
microwave background radiation (CMB) are the most powerful
tools to study the early universe and in particular, to put bounds
on possible variations of the fundamental constants between those
early times and the present.

Previous analysis of CMB data (earlier than the WMAP five-year
release) including a possible variation of $\alpha$ have been performed
byRefs. [1–4] and including a possible variation of $m_e$ have been
performed by Refs. [3–5]. In March 2008, WMAP team released
data collected during the last five years [6]. Moreover, new pro-
cesses relevant during the recombination epoch have been taken
into account. Indeed, in the last years, helium recombination has
been studied in great detail [7–10], revealing the importance of
these considerations on the calculation of the recombination his-
these transitions relevant during the recombination epoch have been taken
into account. This has enabled its implementation on numerical

codes such as RECFAST, since the complete calculations done by
Switzer and Hirata take a large amount of computational time.
Another improvement in the recombination scenario is the inclu-
sion of the semi-forbidden transition $2^2p \rightarrow 1^1s$, the feedback from
spectral distortions between $2^1p \rightarrow 1^1s$ and $2^3p \rightarrow 1^1s$ lines, and
the radiative line transfer.

The release of new data from WMAP brings the possibility of
updating the constraints on the time variation of fundamental con-
stants. In this Letter we study the variation of $\alpha$ and $m_e$ in the
improved recombination scenario. It could be argued that $m_e$
is not a fundamental constant in the same sense as $\alpha$ is and that
constraints on the Higgs vacuum expectation value ($\langle 11 \rangle$) are
more relevant than bounds on $m_e$. However, in the recombination
scenario the only consequence of the time variation of $\langle 11 \rangle$ is a varia-
tion in $m_e$.

The effect of a possible variation of $\alpha$ and/or $m_e$ in the rec-
ombination scenario and in the CMB temperature and polarization
spectra has been analyzed previously [5,12–14]. Here we focus in
the effect of the variation of $\alpha$ and $m_e$ on the improved recombi-
nation scenario.

The recombination equations implemented in RECFAST in the
detailed recombination scenario [15] including the fitting formul-
ae of [11] are:

\begin{equation}
H(z)(1+z) \frac{dx_p}{dz} = \left( x_{HeII} n_{HeII} \alpha_{HeII} - \beta_{HeII}(1-x_p)e^{-h_{11s}/kT} \right) C_{HeII},
\end{equation}

\begin{equation}
H(z)(1+z) \frac{dx_{HeII}}{dz} = \left( x_{HeII} n_{HeII} \alpha_{HeII} - \beta_{HeII}(f_{HeII} - x_{HeII})e^{-h_{11s}/kT} \right) C_{HeII}
+ \left( x_{HeII} n_{HeII} \alpha_{HeII} - \frac{g_{HeII}}{S_{HeII}} \right) \beta_{HeII}(f_{HeII} - x_{HeII}) \times e^{-h_{11s}/kT}.
\end{equation}
so their dependencies are known (see for example [4]).

As remarked in [15], the dependence on the fundamental constants is the same, as an example):

where

\[ C_H = \frac{1 + K_H A_H n_H (1 - x_p)}{1 + K_H (A_H + \beta_H) n_H (1 - x_p)} , \]

\[ C_{\text{Hel}} = \frac{1 + K_{\text{Hel}} A_{\text{Hel}} n_{\text{Hel}} (f_{\text{He}} - x_{\text{He}}) e^{\hbar \nu_p / kT_m}}{1 + K_{\text{Hel}} (A_{\text{Hel}} + \beta_{\text{Hel}}) n_{\text{Hel}} (f_{\text{He}} - x_{\text{He}}) e^{\hbar \nu_p / kT_m}} , \]

\[ C_{t} = \frac{1 + K_t \beta_{t} n_{t} (f_{\text{He}} - x_{\text{He}}) e^{\hbar \nu_p / kT_m}}{1 + K_t \beta_{t} n_{t} (f_{\text{He}} - x_{\text{He}}) e^{\hbar \nu_p / kT_m}} . \]

The last term in Eq. (2) accounts for the recombination through the triplets by including the semi-forbidden transition 2p → 1s. As remarked in [15], \( A_{t} \) is fitted with the same functional form used for the \( K_{t} \) singlets, with different values for the parameters, so the dependences on the fundamental constants are the same, being proportional to \( \alpha^2 m_e^{-3/2} \). The two photon transition rates \( \Lambda_H \) and \( \Lambda_{\text{Hel}} \) depend on the fundamental constants as \( \alpha^4 m_e \). The photoionization coefficients \( \beta \) are calculated as usual from the recombination coefficients \( \alpha \). (with c standing for H, HeI and HeII), so their dependencies are known (see for example [4]).

The \( K_H \), \( K_{\text{Hel}} \) and \( K_{t} \) quantities are the cosmological redshifts of the HLy \( \alpha \), HeI 2p → 1s and HeII 2p → 1s transition line photons, respectively. In general, \( K \) and the Sobolev escape probability \( p_{S} \) are related through the following equations (taking Hel as an example):

\[ K_{\text{Hel}} = \frac{\sigma_{\text{Hel},2p}^{1s}}{\sigma_{\text{Hel},2p}^{1p}} \frac{1}{n_{\text{Hel},1p} A_{\text{Hel},2p-1p} p_{S}} \]  

\[ K_{t} = \frac{\sigma_{t}^{1s}}{\sigma_{t}^{1p}} \frac{1}{n_{t} A_{t}^{2p-1p} p_{S}} \]

where \( A_{\text{Hel},2p-1p} \) and \( A_{t}^{2p-1p} \) are the Einstein A coefficients of the Hel 2p → 1p transitions, respectively. To include the effect of the continuum opacity due to H, based on the approximate formula suggested by Ref. [11], \( p_{S} \) is replaced by the new escape probability \( p_{\text{esc}} = p_{S} + p_{\text{con,H}} \) with

\[ p_{\text{con,H}} = \frac{1}{1 + \alpha_{t} \gamma_{H}} \]

and

\[ \gamma' = \frac{\sigma_{t}^{1s} A_{\text{Hel}}^{2p-1p} f_{\text{He}} - x_{\text{Hel}} \gamma_{H} c^2}{8 \pi^3 / 2 \sigma_{t}^{1s} (v_{\text{Hel},2p}^{1s} v_{\text{Hel},2p}^{1p} + \Delta v_{D,2p}^{1p}) (1 - x_{p})} \]

where \( \sigma_{t}^{1s} \) is the H ionization cross section at frequency \( v_{\text{Hel},2p} \) and \( \Delta v_{D,2p}^{1p} = v_{\text{Hel},2p} \sqrt{2} kT / m_{\text{He}} c^2 \) is the thermal width of the Hel 2p → 1p line. The cross section for photo-ionization from level n is [16]:

\[ \alpha(n, \nu) = \frac{2 \alpha \pi A_{n}^2}{3 \sqrt{3} \ Z^2} (1 + n^2 \epsilon - 3 \ g_{H}(n, \epsilon)) \]

where \( g_{H}(n, \epsilon) \approx 1 \) is the Gaunt–Kramers factor, and \( a_0 = \hbar / (m_e c \alpha) \) is the Bohr radius, so \( \sigma_{t}^{1s} \) is proportional to \( \alpha^{-4} m_e^{-2} \).

The transition probability rates \( A_{\text{Hel,2p-1p}} \) and \( A_{t}^{2p-1p} \) can be expressed as follows [17]:

\[ A_{t}^{2p-1p} = \frac{4 \alpha \pi A_{n}^2}{3 \sqrt{3}} |\langle \psi_{1} | r_{1} + r_{2} | \psi_{2} \rangle |^2 \]

where \( \omega_{ij} \) is the frequency of the transition, and \( i(j) \) refers to the initial (final) state of the atom. First we will analyze the dependence of the bra-ket. To first-order in perturbation theory, all wavefunctions can be approximated to the respective wavefunction of hydrogen. Those can be usually expressed as \( \exp(-\omega r / a_0) \) where \( a_0 \) is the Bohr radius and \( q \) is a number. It can be shown that any integral of the type of Eq. (11) can be solved with a change of variable \( x = r / a_0 \). If the wave functions are properly normalized, the dependence on the fundamental constants comes from the operator, namely \( r_1 + r_2 \). Thus, the dependence of the bra-ket goes as \( a_0 \).

On the other hand, \( \omega_{ij} \) is proportional to the difference of energy levels and thus its dependence on the fundamental constants is \( \omega_{ij} \approx m_e \alpha^2 \). Consequently, the dependence of the transition probabilities of Hel on \( \alpha \) and \( m_e \) is

\[ A_{t}^{2p-1p} \approx m_e \alpha^5 \]

In Fig. 1 we show how a variation in the value of \( \alpha \) at recombination affects the ionization history, moving the redshift at which recombination occurs to earlier times for larger values of \( \alpha \). The
difference between the functions when the two different recombination scenarios are considered, for a given value of \( \alpha \), is smaller than the difference that arise when varying the value of \( \alpha \). Something similar happens when varying \( m_\text{e} \).

With regards to the fitting parameters \( a_{\text{He}} \) and \( b_{\text{He}} \), since detailed calculation of their values are not available yet, it is not possible to determine the effect that a variation of \( \alpha \) or \( m_\text{e} \) would have on these new parameters. Wong et al. [15] have shown that they must be known at the 1% level for future Planck data. In this Letter, however, we are dealing with the 5 yr data from WMAP satellite and this accuracy is not required. To come to this conclusion, we have calculated the temperature, polarization and cross correlation CMB spectra, allowing the parameters \( a_{\text{He}} \) and \( b_{\text{He}} \) to vary at the 50% level. We found that for the temperature and polarization spectra, the variation is always lower than the observational error (1% for temperature and almost 40% in polarization). The largest variations occur in the cross correlation CMB spectra \( \langle C_{\ell}^{T E} \rangle \). In this case, we have calculated the observational errors divided by the value of the \( C_{\ell} \)’s of all measured \( C_{\ell}^{T E} \) and compared them with the relative variation in the \( C_{\ell} \)’s induced when changing \( a_{\text{He}} \) and \( b_{\text{He}} \) by a 50%. In all of the cases the first quantity is several orders of magnitude greater than the variation of the \( C_{\ell} \)’s. Therefore, in order to analyze WMAP5 data, there is no need to modify these parameters.

To put constraints on the variation of \( \alpha \) and \( m_\text{e} \) during recombination time in the detailed recombination scenario studied here, we introduced the dependencies on the fundamental constants explicitly in the latest version of Recfast code [18], which solves the recombination equations. We performed our statistical analysis by exploring the parameter space with Monte Carlo Markov chains generated with the publicly available CosmoMC code of Ref. [19] which uses the Boltzmann code CAMB [20] and Recfast to compute the CMB power spectra. We modified them in order to include the possible variation of \( \alpha \) and \( m_\text{e} \) at recombination. We ran eight Markov chains and followed the convergence criterion of Ref. [21] to stop them when \( R - 1 < 0.0180 \). Results are shown in Table 1 and Fig. 2.

The observational set used for the analysis was data from the WMAP 5-year temperature and temperature-polarization power

**Table 1**

Mean values and 1σ errors for the parameters including \( \alpha \) and \( m_\text{e} \) variations. NS stands for the new recombination scenario, and PS stands for the previous one.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>wmap5 + NS</th>
<th>wmap5 + PS</th>
<th>wmap3 + PS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega_b h^2 )</td>
<td>0.02241 ± 0.00084</td>
<td>0.02242 ± 0.00086</td>
<td>0.0218 ± 0.0010</td>
</tr>
<tr>
<td>( \Omega_{\text{CDM}} h^2 )</td>
<td>0.1070 ± 0.0078</td>
<td>0.1071 ± 0.0080</td>
<td>0.106 ± 0.011</td>
</tr>
<tr>
<td>( \Theta )</td>
<td>1.033 ± 0.023</td>
<td>1.032 ± 0.024</td>
<td>1.032 ± 0.028</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.0870 ± 0.0081</td>
<td>0.0863 ± 0.0084</td>
<td>0.090 ± 0.004</td>
</tr>
<tr>
<td>( \Delta \alpha/\alpha_0 )</td>
<td>0.004 ± 0.015</td>
<td>0.003 ± 0.015</td>
<td>0.023 ± 0.025</td>
</tr>
<tr>
<td>( \Delta m_\text{e}/(m_\text{e} 0) )</td>
<td>−0.0193 ± 0.049</td>
<td>−0.017 ± 0.051</td>
<td>0.036 ± 0.078</td>
</tr>
<tr>
<td>( n_\text{s} )</td>
<td>0.963 ± 0.014</td>
<td>0.963 ± 0.015</td>
<td>0.970 ± 0.019</td>
</tr>
<tr>
<td>( A_s )</td>
<td>3.052 ± 0.042</td>
<td>3.052 ± 0.043</td>
<td>3.054 ± 0.073</td>
</tr>
<tr>
<td>( H_0 )</td>
<td>70.3 ± 5.9</td>
<td>70.3 ± 6.0</td>
<td>70.4 ± 6.6</td>
</tr>
</tbody>
</table>

**Fig. 2.** Marginalized posterior distributions obtained with CMB data, including the WMAP 5-year data release plus 2dFGRS power spectrum. The diagonal shows the posterior distributions for individual parameters, the other panels shows the 2D contours for pairs of parameters, marginalizing over the others.
spectrum [6], and other CMB experiments such as CBI [22], ACBAR [23], and BOOMERANG [24,25], and the power spectrum of the 2dFGRS [26]. We have considered a spatially-flat cosmological model with adiabatic density fluctuations, and the following parameters:

\[ P = \left( \Omega_B h^2, \Omega_{CDM} h^2, \Theta, \tau, \Delta \alpha / \alpha_0, \Delta m_e / (m_e)_0, n_s, A_s \right) , \]  

where \( \Omega_{CDM} h^2 \) is the dark matter density in units of the critical density, \( \Theta \) gives the ratio of the comoving sound horizon at decoupling to the angular diameter distance to the surface of last scattering, \( \tau \) is the re-ionization optical depth, \( n_s \) the scalar spectral index and \( A_s \) is the amplitude of the density fluctuations.

In Fig. 2 we show the marginalized posterior distributions for the cosmological parameters, \( \Delta \alpha / \alpha_0 \) and \( \Delta m_e / (m_e)_0 \), which are the variation in the values of those fundamental constants between recombination epoch and the present. The three successively larger two-dimensional contours in each panel correspond to the 68%- , 95%- and 99%-probability levels, respectively. In the diagonal, the one-dimensional likelihoods show the posterior distribution of the parameters.

In Table 1 we show the results of our statistical analysis, and compare them with the ones we have presented in Ref. [4], which were obtained in the standard recombination scenario (i.e. the one described in [27], which we denote PS), and using WMAP3 [28,29] data. The constraints are tighter in the current analysis, which is expectable since that data set is more accurate. For the fundamental constants, the contours notably shrink. Moreover, the constraints are shifted to a region of the parameter space closer to that of null variation in the case of \( \alpha \). On the other hand, limits on the variation of \( m_e \) are shifted to negative values, but still consistent with null variation. From the one-dimensional likelihoods we see that the peak of the likelihood has moved for \( \Omega_B h^2 \). The obtained results for the cosmological parameters are in agreement within 1\( \sigma \) with the ones obtained by the WMAP Collaboration [30], without considering variation of fundamental constants.

In Fig. 3 we compare the probability distribution for \( \Delta \alpha / \alpha_0 \) and also for \( \Delta m_e / (m_e)_0 \), in different scenarios and with different data sets.

In Fig. 4 we compare the 95%-probability contour level for the parameters, and their one-dimensional distributions, for two different analysis in the standard recombination scenario, namely the one with WMAP5 data (dashed lines) and the one with WMAP3 data (solid lines). The contours are smaller in the former case, which is expectable since data set is more accurate. For the fundamental constants, the contours notably shrink. Moreover, the constraints are shifted to a region of the parameter space closer to that of null variation in the case of \( \alpha \). On the other hand, limits on the variation of \( m_e \) are shifted to negative values, but still consistent with null variation. From the one-dimensional likelihoods we see that the peak of the likelihood has moved for \( \Omega_B h^2 \). The obtained results for the cosmological parameters are in agreement within 1\( \sigma \) with the ones obtained by the WMAP Collaboration [30], without considering variation of fundamental constants.
Fig. 4. Comparison between the 95%-confidence levels of WMAP3 (solid line) with those of WMAP5 (dashed line). In the diagonal, we compare the one-dimensional likelihoods in these two cases.

References