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Large-scale inhomogeneities in modified Chaplygin gas cosmologies

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Abstract

We extend the homogeneous modified Chaplygin cosmologies to large-scale perturbations by formulating a Zeldovich-like approximation. We show that the model interpolates between an epoch with a soft equation of state and a de Sitter phase, and that in the intermediate regime its matter content is simply the sum of dust and a cosmological constant. We then study how the large-scale inhomogeneities evolve and compare the results with cold dark matter (CDM), Λ CDM and generalized Chaplygin scenarios. We find that unlike that like the latter, our models resemble Λ CDM.

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1. Introduction

According to increasing astrophysical indicia, the evolution of the Universe seems to be largely governed by dark energy with negative pressure together with pressureless cold dark matter (see [1] for the latest review) in a two to one proportion. However, little is known about the origin of either component, which in the standard cosmological model would play very dif-

ferent roles: dark matter would be responsible for matter clustering, whereas dark energy [2] would account for accelerated expansion. Several candidates for dark energy have been proposed and confronted with observations: a purely cosmological constant, quintessence with a single field (see [3] for earliest papers) or two coupled fields [4], *k*-essence scalar fields, and phantom energy [5]. Interestingly, a bolder alternative presented recently suggests that an effective dark energy-like equation of state could be due to averaged quantum effects [6].

The lack of information regarding the provenance of dark matter and dark energy allows for speculation

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with the economical and aesthetic idea that a single component acted in fact as both dark matter and dark energy. The unification of those two components has risen a considerable theoretical interest, because on the one hand model building becomes considerable simpler, and on the other hand such unification implies the existence of an era during which the energy densities of dark matter and dark energy are strikingly similar.

One possible way to achieve that unification is through a particular k -essence fluid, the generalized Chaplygin gas [7], with the exotic equation of state

$$p = -\frac{A}{\rho^\beta}, \quad (1)$$

where constants β and A satisfy respectively $0 < \beta \leq 1$ and $A > 0$. Using the energy conservation equation and the Einstein equation $3H^2 = \rho$ one obtains the evolution

$$3H^2 = \left(A + \frac{B}{a^{3(1+\beta)}} \right)^{1/(1+\beta)}, \quad (2)$$

where as usual a is the scale factor, $H = \dot{a}/a$ and $B > 0$ is an integration constant. This model interpolates between a $\rho \propto a^{-3}$ evolution law at early times and $\rho \simeq \text{const}$ at late times (i.e., the model is dominated by dust in its early stages and by vacuum energy in its late history). In the intermediate regime the matter content of the model can be approximated by the sum of a cosmological constant and a fluid with a soft equation of state $p = \beta\rho$. The traditional Chaplygin gas [8,9] corresponds to $\beta = 1$ (stiff equation of state).

Another possibility which has emerged recently is the modified Chaplygin gas (MCG) [10]. It is characterized by

$$\rho = \left(A + \frac{B}{a^3} \right)^{\alpha/(\alpha-1)}, \quad (3)$$

$$p = \frac{1}{\alpha-1} (\rho - \alpha A \rho^{1/\alpha}), \quad (4)$$

with $\alpha > 0$ a constant.

Alternatively, such evolution can be seen as coming from a modified gravity approach, along the lines of the Dvali–Gabadadze–Porrati [11], Cardassian [12] and Dvali–Turner [13] models. In those works the present acceleration of the universe is not attributed to an exotic component in the Universe, but to modifications in gravitational physics at subhorizon scales. Following the proposal by [14], an evolution like that

arising from (3) in standard gravity, could alternatively be obtained in the modified gravity picture for a pure dust configuration under the modification

$$3H^2 = (A + \rho_m)^{\alpha/(\alpha-1)}, \quad (5)$$

where $\rho_m \propto a^3$.

Modified Chaplygin cosmologies with $\alpha > 1$ are transient models which interpolate between a $\rho \propto a^{-3\alpha/(\alpha-1)}$ evolution law at early times and a de Sitter phase at late times, but interestingly the matter content at the intermediate stage is a mixture of dust and a cosmological constant. The sound speed for the modified Chaplygin gas [10] becomes

$$c_s^2 = \frac{1}{\alpha-1} [1 - A\rho^{(1-\alpha)/\alpha}]. \quad (6)$$

The observational tests of traditional and generalized Chaplygin models are numerous. Several teams have analyzed the compatibility of those models with the Cosmic Microwave Background Radiation (CMBR) peak location and amplitude [15], supernovae data [16] and gravitational lensing statistics [17]. The main results of those papers can be summarized as follows: models with $\beta > 1$ and some small curvature (positive or negative) are favored over the Λ CDM model, and Chaplygin cosmologies are much likelier as dark energy models than as unified dark matter models.

In what regards the modified Chaplygin gas, it has only been tested observationally in [18], using the most updated and reliable compilation of supernovae data so far: the Gold dataset by Riess et al. [19]. By means of a statistical test which depends not only on χ_{\min}^2 (as in usual procedures) but also on the number of parameters of the parametrization of the Hubble factor as a function of redshift, it was concluded that the modified Chaplygin gas cosmologies give better fits than usual and generalized Chaplygin cosmologies.

In this Letter we shall be concerned with the evolution of large-scale inhomogeneities in modified Chaplygin cosmologies. This is an issue of interest because candidates for the dark matter and dark matter unification will only be valid if they ensure that initial perturbations can evolve into a deeply nonlinear regime to form a gravitational condensate of super-particles that can act like cold dark matter. Here we will follow the covariant and sufficiently general Zeldovich-like non-perturbative approach given in [20], because it can be

adapted to any balometric or parametric equation of state. Our results indicate that our model fits well in the standard structure formation scenarios, and we find, in general, a fairly similar behavior to generalized Chaplygin models [7].

2. The model

For the modified Chaplygin gas described by Eqs. (3) and (4) the effective equation of state in the intermediate regime between the dust dominated phase and the de Sitter phase can be obtained expanding Eqs. (4) and (3) in powers of Ba^{-3} , we get

$$\rho = A^{\alpha/(\alpha-1)} + A^{1/(\alpha-1)} \frac{\alpha B}{(\alpha-1)a^3} + \mathcal{O}\left(\frac{B^2}{a^6}\right), \quad (7)$$

$$p = -A^{\alpha/(\alpha-1)} + \mathcal{O}\left(\frac{B^2}{a^6}\right), \quad (8)$$

which corresponds to a mixture of vacuum energy density $A^{\alpha/(\alpha-1)}$, pressureless dust and other perfect fluids which dominate at the very beginning of the universe. In the intermediate regime the modified Chaplygin gas behaves as dust at the time where the energy density satisfies the condition $\rho = A^{\alpha/(\alpha-1)}$. At very early times the equation of state parameter $w \equiv p/\rho$ becomes

$$w \simeq c_s^2 \simeq \frac{1}{\alpha-1}, \quad (9)$$

so that for very large α the dust-like behavior is recovered.

The next step is to investigate what sort of cosmological model arises when we consider a slight inhomogeneous modified Chaplygin cosmologies. For a general metric $g_{\mu\nu}$, the proper time $d\tau = \sqrt{g_{00}} dx^0$, and $\gamma \equiv -g/g_{00}$ as the determinant of the induced 3-metric, one has

$$\gamma_{ij} = \frac{g_{i0}g_{j0}}{g_{00}} - g_{ij}. \quad (10)$$

In the first approximation it will be interesting to investigate the contribution of inhomogeneities introduced in the modified Chaplygin gas through the expression

$$\rho = \left(A + \frac{B}{\sqrt{\gamma}}\right)^{\alpha/(\alpha-1)}. \quad (11)$$

The latter result suggests that the evolution of inhomogeneities can be studied using the Zeldovich method through the deformation tensor [20,23,24]:

$$D_i^j = a(t) \left(\delta_i^j - b(t) \frac{\partial^2 \varphi(\vec{q})}{\partial q^i \partial q^j} \right), \quad (12)$$

where $b(t)$ parametrizes the time evolution of the inhomogeneities and \vec{q} are generalized Lagrangian coordinates so that

$$\gamma_{ij} = \delta_{mn} D_i^m D_j^n, \quad (13)$$

and h is a perturbation

$$h = 2b(t)\varphi_{,i}{}^i. \quad (14)$$

Hence, using the equations above and Eqs. (7) and (8), it follows that

$$\rho \simeq \bar{\rho}(1 + \delta), \quad (15)$$

$$p \simeq \frac{1}{\alpha-1} (\bar{p} - A\alpha\bar{\rho}^{1/\alpha} + \delta(\bar{p} - A\bar{\rho}^{1/\alpha})), \quad (16)$$

where $\bar{\rho}$ is given by Eq. (3) and the density contrast δ is related to h through

$$\delta = \frac{h}{2}(1 + w), \quad (17)$$

where $w \equiv \bar{p}/\bar{\rho}$. Finally, after some algebra we get

$$\bar{p} = \bar{\rho} \left(w + \frac{(1+w)\delta}{\alpha} \right). \quad (18)$$

Now, the metric (13) leads to the following 00 component of the Einstein equations:

$$\begin{aligned} -3\frac{\ddot{a}}{a} + \frac{1}{2}\ddot{h} + H\dot{h} \\ = 4\pi G\bar{\rho} \left(1 + 3w + \left(1 + \frac{3(1+w)}{\alpha} \right) \delta \right), \end{aligned} \quad (19)$$

where the unperturbed part of this equation corresponds to the Raychaudhuri equation

$$-3\frac{\ddot{a}}{a} = 4\pi G\bar{\rho}(1 + 3w). \quad (20)$$

Using the Friedmann equation for a flat spacetime $H^2 = 8\pi G\bar{\rho}/3$, one can rewrite Eq. (19) as a differential equation for $b(a)$:

$$\begin{aligned} \frac{2}{3}a^2b'' + (1-w)ab' \\ - (1+w) \left(1 + \frac{3(1+w)}{\alpha} \right) b = 0, \end{aligned} \quad (21)$$

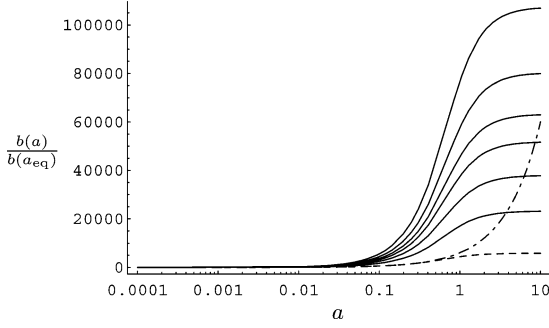


Fig. 1. Evolution of $b(a)/b(a_{\text{eq}})$ for the modified Chaplygin gas for $\alpha = 10, 11, 12, 12, 15, 20$ (continuous lines) as compared with Λ CDM (dashed line) and CDM (dashed-dotted line). Lower curves correspond to higher values of α .

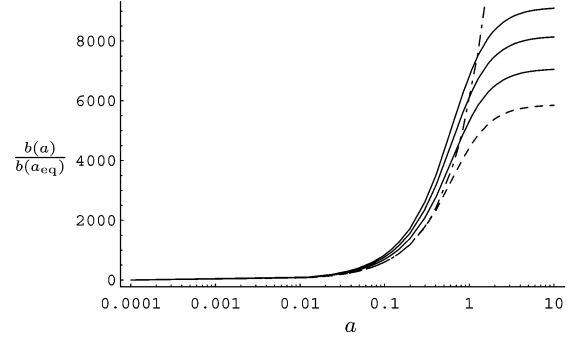


Fig. 2. Evolution of $b(a)/b(a_{\text{eq}})$ for the modified Chaplygin gas for $\alpha = 60, 80, 140$ (continuous lines) as compared with Λ CDM (dashed line) and CDM (dashed-dotted line). Lower curves correspond to higher values of α .

where the primes denote derivatives with respect to the scale-factor, a .

An expression for w as a function of the scale-factor can be derived from Eqs. (3) and (4):

$$w(a) = \frac{B - (\alpha - 1)Aa^3}{(\alpha - 1)(B + Aa^3)}. \quad (22)$$

The latter must be conveniently recast in terms of the fractional vacuum and matter energy densities. This can be done by using

$$\lim_{\alpha \rightarrow \infty} \rho = A + \frac{B}{a^3}, \quad (23)$$

combined with

$$H^2 = H_0^2 \left(\Omega_{m0} \left(\frac{a_0}{a} \right)^3 + \Omega_{\Lambda 0} \right), \quad (24)$$

where H_0 and a_0 are, respectively, the current value of the Hubble and scale factor, and $\Omega_{\Lambda 0}$ and Ω_{m0} are, respectively, the fractional vacuum and matter energy densities today. Setting $a_0 = 1$ we obtain

$$w(a) = \frac{\Omega_{m0} - (\alpha - 1)\Omega_{\Lambda 0}a^3}{(\alpha - 1)(\Omega_{m0} + \Omega_{\Lambda 0}a^3)}, \quad (25)$$

and consistently

$$\lim_{\alpha \rightarrow \infty} w(a) = -\frac{\Omega_{\Lambda 0}a^3}{\Omega_{m0} + \Omega_{\Lambda 0}a^3}. \quad (26)$$

We have used this expression to integrate Eq. (21) numerically, for different values of α , and taking $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.73$ [25]. We have set $a_{\text{eq}} = 10^{-4}$ for matter-radiation equilibrium (while keeping $a_0 = 1$ at present), and our initial condition is $b'(a_{\text{eq}}) = 0$. Our results are shown in Figs. 1 and 2.

We find that modified Chaplygin scenarios start differing from the Λ CDM only recently ($z \simeq 1$) and that, in any case, they yield a density contrast that closely resembles, for any value of $\alpha > 1$, the standard CDM before the present. Notice that Λ CDM corresponds effectively to using Eq. (23) and removing the factor $(1 + 3(1 + w)/\alpha)$ in Eq. (21). Figs. 1 and 2 show also that, for any value of α , $b(a)$ saturates as in the Λ CDM case.

In what regards the density contrast, δ , using Eqs. (14), (17) and (25) one can deduce that the ratio between this quantity in the modified Chaplygin and the Λ CDM scenarios is simply given by

$$\frac{\delta_{\text{mChap}}}{\delta_{\Lambda\text{CDM}}} = \frac{b_{\text{mChap}}}{b_{\Lambda\text{CDM}}} \frac{\alpha}{\alpha - 1}, \quad (27)$$

and its behavior is depicted in Fig. 3. We find that it asymptotically evolves to a constant value.

Now, in Fig. 4, we have plotted δ as a function of a for different values of α . As happens in the traditional [20,26] and generalized Chaplygin models, in our models the density contrast decays for large a also.

3. Discussion and conclusions

Using a Zeldovich-like approximation, we have studied the evolution of large-scale perturbations in a recently proposed theoretical framework for the unification of dark matter and dark energy: the so-called modified Chaplygin cosmologies [10], with equation

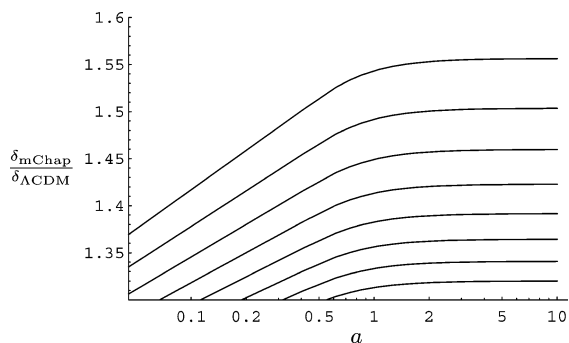


Fig. 3. Evolution of $\delta_{mChap}/\delta_{\Lambda CDM}$ for the modified Chaplygin gas for $\alpha = 60, 65, 70, 75, 80, 85, 90, 95$ (continuous lines) as compared with Λ CDM (dashed line). Lower curves correspond to higher values of α .

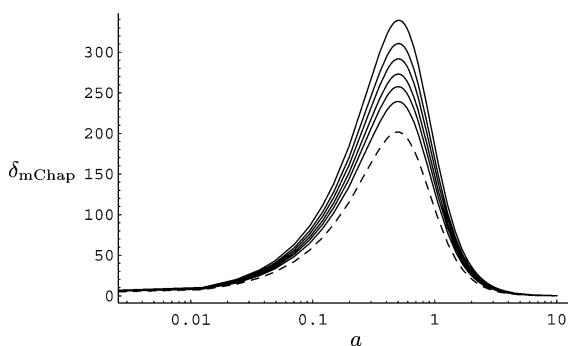


Fig. 4. Evolution of δ_{mChap} for the modified Chaplygin gas for $\alpha = 50, 60, 70, 85, 105, 150$ (continuous lines) as compared with Λ CDM (dashed line). Lower curves correspond to higher values of α .

of state

$$p = \frac{1}{\alpha - 1} (\rho - \alpha A \rho^{1/\alpha}),$$

with $\alpha > 1$. This model evolves from a phase that is initially dominated by nonrelativistic matter to a phase that is asymptotically de Sitter. The intermediate regime corresponds to a phase where the effective equation of state is given by $p = 0$ plus a cosmological constant. We have estimated the fate of the inhomogeneities admitted in the model and shown that these evolve consistently with the observations as the density contrast they introduce is smaller than the one typical of CDM scenarios.

On general grounds, the pattern of evolution of perturbations follows is similar to the one in the Λ CDM

models and in generalized Chaplygin cosmologies, and therefore our represent plausible alternatives.

As usual, in modified Chaplygin cosmologies, the equation of state parameter w can be expressed in terms of the scale factor and a free parameter α , and the value of the latter can be chosen so that the model resembles as much as desired the Λ CDM model.

It would be very interesting to deepen in the comparison between modified and generalized Chaplygin models, particularly from the observational point of view (as already done in [18]). It would also be worth generalizing our study by going beyond the Zeldovich approximation, to incorporate the effects of finite sound speed. This can be done by generalizing the spherical model to incorporate the Jeans length as in [21]. Alternatively, following [22] one could investigate whether the modified Chaplygin admits an unique decomposition into dark energy and dark matter, and if that were the case then study structure formation and show that difficulties associated to unphysical oscillations or blow-up in the matter power spectrum can be circumvented. We hope this will be addressed in future works.

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