Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Physics Letters B 578 (2004) 139–149
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Two hadron production in e^+e^- annihilation to next-to-leading order accuracy ☆

D. de Florian, L. Vanni

Departamento de Física, FCEYN, Universidad de Buenos Aires, (1428) Pabellón 1 Ciudad Universitaria, Capital Federal, Buenos Aires, Argentina

Received 30 July 2003; accepted 13 October 2003

Editor: G.F. Giudice

Abstract

We discuss the production of two hadrons in e^+e^- annihilation within the framework of perturbative QCD. The cross section for this process is calculated to next-to-leading order accuracy with a selection of variables that allows the consideration of events where the two hadrons are detected in the same jet. In this configuration we contemplate the possibility that the hadrons come from a double fragmentation of a single parton. The double-fragmentation functions required to describe the transition of a parton to two hadrons, are also necessary to completely factorize all collinear singularities. We explicitly show that factorization applies to order α_s in the case of two-hadron production.

1. Introduction

The production of one hadron in e^+e^- annihilation has been studied in much detail in perturbative QCD [1]. The corresponding cross section for the process $e^+e^- \to \gamma^*(Q) \to H(P) + X$ is usually expressed as a function of the variable

$$z = \frac{2P \cdot Q}{Q^2} \tag{1}$$

representing the energy fraction carried by the hadron. In this case the cross section can be written as a convolution of the (perturbative computable) partonic cross section σ^i and the (non-perturbative) fragmentation functions $D_i^H(x)$ giving the probability of finding a hadron in the parton with momentum fraction x, as

$$\frac{d\sigma^H}{dz} = \sum_i \sigma^i \otimes D_i^H . \tag{2}$$

☆ Partially supported by Fundación Antorchas and CONICET.
E-mail address: deflo@df.uba.ar (D. de Florian).

0370-2693
doi:10.1016/j.physletb.2003.10.047

The cross section has been computed to next-to-leading order (NLO) accuracy in [1] and to next-to-next-to-leading order (NNLO) accuracy in [2]. Furthermore, several analyses of the available data have been performed in the last years and, as a result, fragmentation functions for several hadrons have been extracted with very good precision.

Higher order QCD corrections (NLO) to the cross section for the production of two hadrons in e^+e^- annihilation have been computed in [1] in the particular case when the two hadrons H_1 and H_2 are selected from different parton jets. While a symmetric extension of the one-hadron case to two hadrons would correspond to expressing the differential cross section in terms of the momentum fractions of each hadron defined by

$$z_1 = \frac{2P_1 \cdot Q}{Q^2}, \qquad z_2 = \frac{2P_2 \cdot Q}{Q^2}, \tag{3}$$

the authors in [1] introduced a different set of variables

$$z = \frac{2P_1 \cdot q}{Q^2}, \qquad u = \frac{P_1 \cdot P_2}{P_1 \cdot Q}. \tag{4}$$

While z in Eq. (4) coincides with z_1, the momentum fraction of hadron H_1, the second variable u depends on both the momentum fraction of hadron H_2 and the angle θ_{12} between the hadrons observed from the center of mass system as

$$u = z_2 \frac{1}{2}(1 - \cos\theta_{12}), \tag{5}$$

such that u is approximately zero when the angle between the hadrons is small. Therefore, configurations where both hadrons are in the same parton jet corresponds to $u \approx 0$. Consequently, by considering events were z and u are not too small one can ensure that the two hadrons are produced from the hadronization of different partons and the cross section can be reduced to the product of the fragmentation functions D_i^H associated to each hadron [1]. In this way, the possibility of a double fragmentation from a single parton is excluded and the expression for the cross section gets simplified.

In this work we are interested in extending the calculation for the two-hadron cross section in the full phase space, including the configurations were both hadrons are produced collinearly. In order to be able to consider those events we will express the cross section in terms of the momentum fractions in Eq. (3).

With the use of these variables it is possible to contemplate simultaneously two extreme configurations: the first one corresponds to the case when the two hadrons are produced in opposite directions (or at least with a clear angular separation) and therefore belonging to different jets (Fig. 1(a)). Hadrons in this configuration can only be originated from the fragmentation of different partons. The second one corresponds to the case of both hadrons produced in the same direction, such that they are detected in the same jet (Fig. 1(b)). In the last case, hadrons could be originated from the fragmentation of two collinear partons or by the double fragmentation of the single

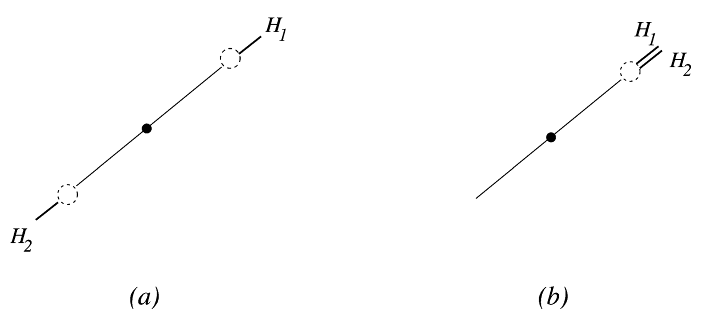

Fig. 1. (a) Represents hadrons in the first kinematical configuration. (b) Represents hadrons in the second configuration, belonging to the same jet.

parton. The price one has to pay to fully account for the second configuration is the introduction of a new set of non-perturbative phenomenological functions [3–5] to describe the possibility of the transition from a single parton to two hadrons.

One needs to introduce, then, *the double-fragmentation functions* $DD_p^{H_1 H_2}(x_1, x_2)$,[1] as the probability that a parton p fragments into the hadrons H_1 and H_2 with energy fractions x_1 and x_2. The cross section for the production of two hadrons in e^+e^- annihilation can therefore be written in the following schematic way

$$\frac{d\sigma^{H_1 H_2}}{dz_1\, dz_2} = \sum_{ij} \sigma^{ij} \otimes D_i^{H_1} \otimes D_j^{H_2} + \sum_i \sigma^i \otimes DD_i^{H_1 H_2}, \tag{6}$$

where σ^{ij} is the partonic cross section for the production of partons i and j and σ^i the cross section for parton i. The cross section is separated in two terms corresponding to the contribution of the mechanisms responsible for two-hadron production: single fragmentation of two partons, and double fragmentation of a single parton.

At leading order, the first term only contributes to the first configuration, since the two partons that undergo hadronization are produced back-to-back. At next to leading order there is one extra parton which could be emitted collinearly to one of the others, giving also origin to hadrons in the second configuration. Therefore, at order α_s and beyond, hadrons in the second configuration could be originated from any of the two fragmentation mechanisms, being not possible to separate the contribution of each term in Eq. (6), unless an additional (unphysical) scale is introduced. Only the sum of both contributions has physical sense.

The presence of collinear partons at order α_s gives origin to collinear singularities in the cross section, which are manifested in the form of poles in $\epsilon = (4 - N)/2$ when dimensional regularization is used. By means of the usual redefinition of the D_i^H functions, singularities due to collinear partons that give origin to hadrons in the first configuration can be absorbed. However, there appear singularities corresponding to hadrons belonging to the second configuration, originated from collinear partons emitted in the same direction. Since at lowest order the D_i^H functions only participate in processes associated with the first configuration, such singularities cannot be absorbed in the single fragmentation term. We will show that with the redefinition of the $DD_i^{H_1 H_2}$ functions in the double fragmentation term all singularities are factorized. In this sense, the role played by the $DD_i^{H_1 H_2}$ functions in e^+e^- annihilation is similar to the one of *fracture functions* in DIS processes [7–11]. For a formal point of view, it is possible to interpret the double-fragmentation functions as the time-like version of fracture functions.

Double-fragmentation functions $DD_i^{H_1 H_2}$ fulfill sum rules in analogy to the sum rules for the usual fragmentation functions [1]. Momentum conservation requires

$$\sum_{H_1} \int P_1^\mu \frac{d\sigma^{H_1 H_2}}{dP_1\, dP_2}\, dP_1 = \left(Q^\mu - P_2^\mu\right) \frac{d\sigma^{H_2}}{dP_2}, \tag{7}$$

being Q the initial total momentum, where the right-hand side is proportional to the total free momentum available for the production of hadron H_2. In particular, energy conservation implies [4,5]

$$\sum_{H_1} \int_0^{1-z_2} dz_1\, z_1 DD_i^{H_1 H_2}(z_1, z_2) = (1 - z_2) D_i^{H_2}(z_2), \tag{8}$$

relating the second moment of the double-fragmentation function to the single one.

[1] We have slightly modified the original notation introduced in [3,4] for the double-fragmentation functions to make more noticeable the difference with the usual ones

2. Two-hadron production in e^+e^-

In order to formalize the convolution products in Eq. (6) we define the partonic energy fractions associated to each fragmentation mechanism. For the single fragmentation term two partons fragment independently with a fraction of the parent parton energy given by

$$x_i = \frac{2p_i \cdot Q}{Q^2}, \tag{9}$$

with p_i the momentum of the parton $i = 1, 2$. At leading order both variables are fixed to one since no extra gluon radiation is allowed.

The convolution product in the single fragmentation term of Eq. (6) is expressed in terms of a double integral in x_1 and x_2 with integration intervals determined by the kinematical region allowed for the partonic process. This implies

$$0 \leqslant x_1 \leqslant 1 \quad \text{and} \quad z_1 \leqslant x_1, \qquad 1 - x_1 \leqslant x_2 \leqslant 1 \quad \text{and} \quad z_2 \leqslant x_2. \tag{10}$$

The integration zone for the single fragmentation term has to be divided into the two regions A and B indicated in Fig. 2.

In the case of the double fragmentation term only one parton fragments. We define the partonic variable as usual by

$$x = \frac{2p \cdot Q}{Q^2}, \tag{11}$$

with p being the momenta of the fragmenting parton. With this, it is possible to write the second term of Eq. (6) as a single convolution product with integration limits coming from the request $z_1/x + z_2/x \leqslant 1$.

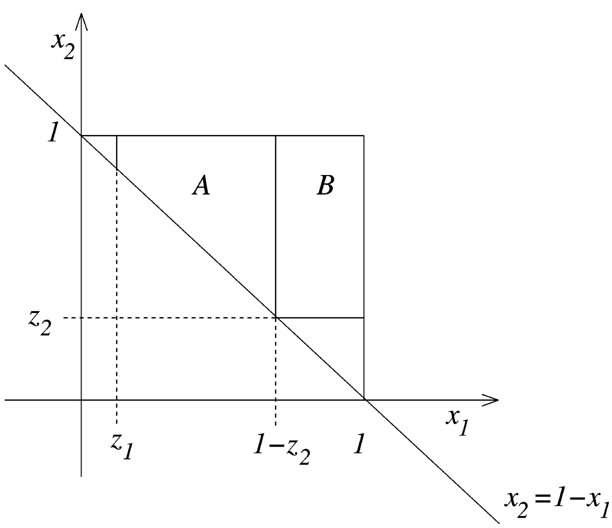

Fig. 2. Integration regions for the variables x_1 and x_2 in the single fragmentation term.

Using those conditions we can express Eq. (6) as

$$\frac{d\sigma^{H_1H_2}}{dz_1\,dz_2}=\sum_{ij}\int_{z_1}^{1-z_2}\frac{dx_1}{x_1}\int_{1-x_1}^{1}\frac{dx_2}{x_2}\frac{d\sigma^{ij}}{dx_1\,dx_2}_{A}D_i^{H_1}\left(\frac{z_1}{x_1}\right)D_j^{H_2}\left(\frac{z_2}{x_2}\right)$$
$$+\sum_{ij}\int_{1-z_2}^{1}\frac{dx_1}{x_1}\int_{z_2}^{1}\frac{dx_2}{x_2}\frac{d\sigma^{ij}}{dx_1\,dx_2}_{B}D_i^{H_1}\left(\frac{z_1}{x_1}\right)D_j^{H_2}\left(\frac{z_2}{x_2}\right)+\sum_{i}\int_{z_1+z_2}^{1}\frac{dx}{x^2}\frac{d\sigma^{i}}{dx}DD_i^{H_1H_2}\left(\frac{z_1}{x},\frac{z_2}{x}\right),\tag{12}$$

where to NLO accuracy

$$\frac{d\sigma^i}{dx}=\frac{d\sigma^{i\,(0)}}{dx}+\frac{\alpha_s}{2\pi}\frac{d\sigma^{i\,(1)}}{dx},\qquad \frac{d\sigma^{ij}}{dx_1\,dx_2}_K=\frac{d\sigma^{ij\,(0)}}{dx_1\,dx_2}_K+\frac{\alpha_s}{2\pi}\frac{d\sigma^{ij\,(1)}}{dx_1\,dx_2}_K,\tag{13}$$

and $K=A,B$ indicating the integration zone in the single fragmentation term. In Eq. (12) we have considered the case when $1-z_2\geqslant z_1$. This implies $z_1+z_2\leqslant 1$, which corresponds to the kinematical region where the double fragmentation mechanism can also contribute. If $z_1+z_2>1$ the cross section is reduced only to the second term of Eq. (12).

Some of the partonic cross sections obey symmetry relations that allow to reduce the number of independent quantities to be computed. Due to invariance under charge conjugation

$$\frac{d\sigma^{iq}}{dx_1\,dx_2}=\frac{d\sigma^{i\bar q}}{dx_1\,dx_2},\qquad \frac{d\sigma^{qi}}{dx_1\,dx_2}=\frac{d\sigma^{\bar q i}}{dx_1\,dx_2}.\tag{14}$$

To NLO accuracy it is necessary to obtain only three different partonic cross section $d\sigma^{q\bar q}/dx_1\,dx_2$, $d\sigma^{qg}/dx_1\,dx_2$ and $d\sigma^{gq}/dx_1\,dx_2$.

At leading order the only non-vanishing terms are[2]

$$\frac{d\sigma^{q\,(0)}}{dx}=e_q^2\sigma_0\delta(1-x),\qquad \frac{d\sigma^{q\bar q\,(0)}}{dx_1\,dx_2}_K=e_q^2\sigma_0\delta(1-x_1)\delta(1-x_2),\tag{15}$$

where $\sigma_0=\frac{4\pi\alpha_s^2}{3Q^2}$.

The partonic cross section at order α_s is obtained by evaluating the real and virtual diagrams indicated in Fig. 3 and integrating over the phase space of the final partons expressed in terms of x_1, x_2, such that $d\sigma^{ij}=d\sigma_R+d\sigma_V$. We compute the metric and longitudinal contributions to the partonic cross section obtained, as usual, by replacing the sum over the polarization states of the virtual photon by the corresponding projectors

$$P_{\mu\nu}^{(M)}=-g_{\mu\nu},\tag{16}$$

$$P_{\mu\nu}^{(L)}=\frac{Q^2}{(p_2\cdot Q)^2}p_{2\mu}p_{2\nu}.\tag{17}$$

The longitudinal contribution has been calculated projecting in the direction of hadron H_2. In the following, we will present in detail the results for the metric contribution, since at NLO singularities of interest occur only on that projection of the cross section. Using dimensional regularization [12,13] we obtain for the real part

$$d\sigma_R^{(M)}=e_q^2\sigma_0\frac{\alpha_s}{2\pi}C_F\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{1}{\Gamma(2-\epsilon)}\left(\frac{1-z^2}{4}\right)^{-\epsilon}x_1^{-2\epsilon}x_2^{-2\epsilon}F(x_1,x_2)\,dx_1\,dx_2,\tag{18}$$

[2] In this work we restrict the analysis to the case of pure γ^* exchange. The extension to Z-boson production can be easily performed.

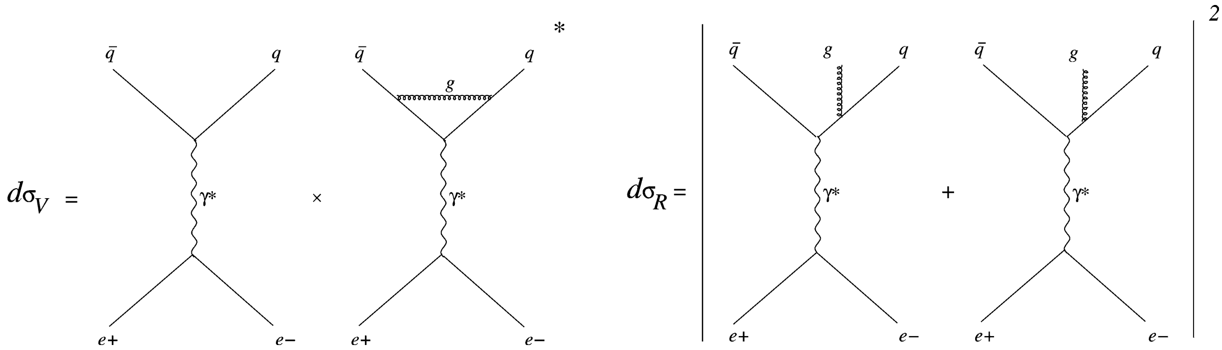

Fig. 3. Virtual and real diagrams contributing to order α_s.

with

$$F(x_1,x_2)=\left[(1-\epsilon)^2\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}-2\epsilon(1-\epsilon)\frac{(2-2x_1-2x_2+x_1x_2)}{(1-x_1)(1-x_2)}\right], \tag{19}$$

and μ being the dimensional regularization scale. The result for the virtual contribution is

$$d\sigma_V=e_q^2\sigma_0\frac{\alpha_s}{2\pi}C_F\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\left[-\frac{3}{\epsilon}-\frac{2}{\epsilon^2}-8+\pi^2\right]\delta(1-x_1)\delta(1-x_2)\,dx_1\,dx_2. \tag{20}$$

In the previous equations we have labeled the quark as parton 1 and the anti-quark as parton 2. The virtual part does not exhibit singularities beyond those already regularized in the form of poles in ϵ. In the real part the divergences appear when the denominators of the function $F(x_1,x_2)$ vanish. These infrared divergences can be regularized by means of the usual $+$ prescription, which can be easily implement by multiplying and dividing by $(1-x_i)^{1+\epsilon}$, and considering this expression as a distribution in x_i

$$(1-x_i)^{-1-\epsilon}=-\frac{1}{\epsilon}\delta(1-x_i)+\frac{1}{(1-x_i)}_{+[0,\underline{1}]}-\epsilon\left(\frac{\log(1-x_i)}{1-x_i}\right)_{+[0,\underline{1}]}+\mathcal{O}(\epsilon^2), \tag{21}$$

where $F(x)_{+[a,\underline{b}]}$ is defined as usual by

$$\int_a^b dx\,f(x)F(x)_{+[a,\underline{b}]}=\int_a^b dx\,\big[f(x)-f(b)\big]F(x).$$

The range of integration is indicated as a subscript; furthermore, the subtraction point is underlined.

For $d\sigma^{q\bar{q}(1)}/dx_1\,dx_2$ the singularities of the function $F(x_1,x_2)$ occur at $x_2=1$ in zone A, and at $x_1=1$ and $x_2=1$ in zone B. Applying the $+$ prescription as indicated, the following expression is reached

$$\frac{d\sigma^{q\bar{q}(1)}}{dx_1\,dx_2}{}_K^{(M)}=\sigma_0e_q^2\left[P_{qq}(x_1)\log\left(\frac{Q^2}{\mu^2}\right)\delta(1-x_2)+P_{qq}(x_2)\log\left(\frac{Q^2}{\mu^2}\right)\delta(1-x_1)\right.$$
$$\left.+\frac{1}{\hat{\epsilon}}P_{qq}(x_1)\delta(1-x_2)+\frac{1}{\hat{\epsilon}}P_{qq}(x_2)\delta(1-x_1)+f_{qqK}^{(M)}(x_1,x_2)\right], \tag{22}$$

where $1/\hat{\epsilon}=-1/\epsilon(4\pi)^{\epsilon}\Gamma(1-\epsilon)/\Gamma(1-2\epsilon)=(-1/\epsilon+\gamma_E-\log 4\pi)+\mathcal{O}(\epsilon)$, $P_{ij}(x)$ are the usual Altarelli–Parisi splitting kernels [14] and the functions $f_{ij\ K}^{M}$ are presented in Appendix A.

The qg partonic cross section $d\sigma^{qg(1)}/dx_1\,dx_2$ can be obtained from the $q\bar{q}$ one relabeling the parton indexes by a $x_2\to 2-x_1-x_2$ substitution in the matrix elements. In this case $F(x_1,x_2)\to F(x_1,2-x_1-x_2)$ which develops singularities at $x_1 = 1$ in zone A, and at $x_1 + x_2 = 1$ in zone B. Proceeding like in the previous case and ignoring terms containing distributions without support in the analyzed zones, we obtain

$$\frac{d\sigma^{qg(1)}}{dx_1\,dx_2}_K^{(M)} = \sigma_0 e_q^2 \bigg[\hat{P}^g_{qq}(x_1)\log\bigg(\frac{Q^2}{\mu^2}\bigg)\delta(x_1+x_2-1) + \hat{P}^q_{gq}(x_2)\log\bigg(\frac{Q^2}{\mu^2}\bigg)\delta(1-x_1) + \frac{1}{\hat{\epsilon}}\hat{P}^q_{gq}(x_1)\delta(x_1+x_2-1) + \frac{1}{\hat{\epsilon}}\hat{P}^q_{gq}(x_2)\delta(1-x_1) + f^{(M)}_{qgK}(x_1,x_2)\bigg], \tag{23}$$

where the functions $\hat{P}^k_{ji}$ are the real LO Altarelli–Parisi kernels, with the index k labeling the third particle in the vertex $i \to jk$.

In the gq partonic cross section $d\sigma^{gq(1)}/dx_1\,dx_2$, x_1 is assigned to the gluon and x_2 to the quark. Performing the substitution $x_2 \leftrightarrow x_1$ in the matrix element used in $d\sigma^{qg(1)}/dx_1\,dx_2$, $F(x_1, 2-x_1-x_2) \to F(x_2, 2-x_1-x_2)$ which develops singularities at $x_2 = 1$ and $x_1 + x_2 = 1$ in zone A, and at $x_2 = 1$ in zone B. The singularities in zone A will give origin to two different distributions, one associated to the singularity at $x_2 = 1$ and another to the singularity at $x_1 = 1 - x_2$. The result can be expressed as

$$\frac{d\sigma^{gq(1)}}{dx_1\,dx_2}_K^{(M)} = e_q^2 \sigma_0 \bigg[\hat{P}^q_{gq}(x_1)\log\bigg(\frac{Q^2}{\mu^2}\bigg)\delta(1-x_2) + \hat{P}^g_{qq}(x_2)\log\bigg(\frac{Q^2}{\mu^2}\bigg)\delta(x_1+x_2-1) + \frac{1}{\hat{\epsilon}}\hat{P}^q_{gq}(x_1)\delta(1-x_2) + \frac{1}{\hat{\epsilon}}\hat{P}^q_{gq}(x_2)\delta(x_1+x_2-1) + f^{(M)}_{gqK}(x_1,x_2)\bigg]. \tag{24}$$

This result completes the presentation of the partonic cross sections that participate in the single fragmentation term.

For the double fragmentation term, the cross sections for the production of a single parton $d\sigma^i/dx$ are required. These are exactly the same as the ones appearing in one-hadron production [1]. As a cross-check of our calculation we have re-obtained those coefficients by applying the momentum conservation relation in Eq. (7) resulting in

$$\begin{aligned}
\frac{d\sigma^{q(1)}}{dx}^{(M)} &= \frac{d\sigma^{\bar{q}(1)}}{dx}^{(M)} = e_q^2\sigma_0\bigg[P_{qq}(x)\log\bigg(\frac{Q^2}{\mu^2}\bigg) + \frac{1}{\hat{\epsilon}}P_{qq}(x) + f^{(M)}_q(x)\bigg],\\
\frac{d\sigma^{g(1)}}{dx}^{(M)} &= 2e_q^2\sigma_0\bigg[P_{gq}(x)\log\bigg(\frac{Q^2}{\mu^2}\bigg) + \frac{1}{\hat{\epsilon}}P_{gq}(x) + f^{(M)}_g(x)\bigg],
\end{aligned} \tag{25}$$

with $f^{(M)}_i$ given in Appendix A.

As indicated above, the longitudinal part does not contribute to the singular structure of the cross section to NLO accuracy. The corresponding NLO corrections to the single $f^{(L)}_{ij}$ and double $f^{(L)}_i$ fragmentation mechanism are shown in Appendix A.

3. Factorized fragmentation functions

The factorization of the bare fragmentation functions D^H_i at NLO is done in the $\overline{\mathrm{MS}}$ factorization scheme in the standard way [1]. The expression for the bare functions in terms of the factorized ones at the scale M^2 is

$$D^H_i(z) = \int_z^1 \frac{du}{u}\bigg[\delta(1-u)\delta_{ij} + \frac{\alpha_s}{2\pi}\bigg[\log\bigg(\frac{\mu^2}{M^2}\bigg) - \frac{1}{\hat{\epsilon}}\bigg]P_{ji}(u)\bigg]D^{H(\mathrm{NLO})}_j\bigg(\frac{z}{u}, M^2\bigg), \tag{26}$$

where the factorized distributions are labeled by the upper index (NLO).

It is easy to notice that not all the singularities in the partonic cross-section are canceled after the factorization of the fragmentation functions is performed. Singularities belonging to $d\sigma^{qg(1)}/dx_1\,dx_2$ and $d\sigma^{gq(1)}/dx_1\,dx_2$ in zone A still remain. They correspond to terms with $1/\epsilon$ poles proportional to $\delta(x_1+x_2-1)$, arising from the hadronization of a gluon being emitted collinear to a quark (or anti-quark) that also undergoes hadronization, giving as a final product two hadrons in the same jet. Those singularities clearly cannot be absorbed the factorization of the D_i^H functions, since a configuration with two collinear hadrons is not allowed in the single fragmentation term at the lowest order. However, this is exactly the configuration corresponding to the double fragmentation term, indicating that these singularities could be absorbed by the appropriate factorization of the $DD_i^{H_1H_2}$ functions.

The expression for the bare double-fragmentation functions $DD_i^{H_1H_2}(x,y)$ in terms of the NLO factorized ones $DD_i^{H_1H_2(\mathrm{NLO})}(x,y,M^2)$ can be obtained by requiring that all remaining collinear singularities in the partonic cross section are absorbed into the factorized distributions. The expression in the $\overline{\mathrm{MS}}$ factorization scheme, valid to $\mathcal{O}(\alpha_s)$, is

$$DD_i^{H_1H_2}(x,y)=\int_{x+y}^{1}\frac{du}{u^2}\left[\delta(1-u)\delta_{ij}+\frac{\alpha_s}{2\pi}\left[\log\left(\frac{\mu^2}{M^2}\right)-\frac{1}{\hat{\epsilon}}\right]P_{ji}(u)\right]DD_j^{H_1H_2(\mathrm{NLO})}\left(\frac{x}{u},\frac{y}{u},M^2\right)$$
$$+\frac{\alpha_s}{2\pi}\left[\log\left(\frac{\mu^2}{M^2}\right)-\frac{1}{\hat{\epsilon}}\right]\int_{z_1}^{1-z_2}\frac{du}{u(1-u)}\left[\hat{P}_{ji}^{k}(u)D_j^{H_1}\left(\frac{x}{u}\right)D_k^{H_2}\left(\frac{y}{1-u}\right)\right]. \tag{27}$$

The factorization relation in Eq. (27) contains two terms with different origins. The first one just relates the factorized and bare double-fragmentation functions, and corresponds to the standard factorization procedure for the emission of collinear partons in the double fragmentation part of the cross section, exactly as it occurs for one-hadron production. The second 'inhomogeneous' term relates the single and double-fragmentation functions and is needed to absorb the remaining singularities discussed above.

Rewriting the bare distributions in terms of the factorized ones in Eq. (12), and fixing $M^2=Q^2$, we obtain the final NLO expression for the factorized cross section for the production of two hadrons as

$$\frac{d\sigma^{H_1H_2\,(M)}}{dz_1\,dz_2}$$
$$=3\sigma_0\int_{z_1}^{1-z_2}\frac{dx_1}{x_1}\int_{1-x_1}^{1}\frac{dx_2}{x_2}$$
$$\times\sum_q e_q^2\left\{\frac{\alpha_s}{2\pi}f_{qqA}^{(M)}(x_1,x_2)\right.$$
$$\times\left[D_q^{H_1(\mathrm{NLO})}\left(\frac{z_1}{x_1},Q^2\right)D_{\bar{q}}^{H_2(\mathrm{NLO})}\left(\frac{z_2}{x_2},Q^2\right)+D_q^{H_2(\mathrm{NLO})}\left(\frac{z_2}{x_2},Q^2\right)D_{\bar{q}}^{H_1(\mathrm{NLO})}\left(\frac{z_1}{x_1},Q^2\right)\right]$$
$$+\frac{\alpha_s}{2\pi}f_{qgA}^{(M)}(x_1,x_2)\left[D_q^{H_1(\mathrm{NLO})}\left(\frac{z_1}{x_1},Q^2\right)+D_{\bar{q}}^{H_1(\mathrm{NLO})}\left(\frac{z_1}{x_1},Q^2\right)\right]D_g^{H_2(\mathrm{NLO})}\left(\frac{z_2}{x_2},Q^2\right)$$
$$\left.+\frac{\alpha_s}{2\pi}f_{gqA}^{(M)}(x_1,x_2)\left[D_q^{H_2(\mathrm{NLO})}\left(\frac{z_2}{x_2},Q^2\right)+D_{\bar{q}}^{H_2(\mathrm{NLO})}\left(\frac{z_2}{x_2},Q^2\right)\right]D_g^{H_1(\mathrm{NLO})}\left(\frac{z_1}{x_1},Q^2\right)\right\}$$
$$+3\sigma_0\int_{1-z_2}^{1}\frac{dx_1}{x_1}\int_{z_2}^{1}\frac{dx_2}{x_2}$$

$$
\begin{aligned}
&\times \sum_q e_q^2 \Bigg\{ \left[\delta(1-x_1)\delta(1-x_2) + \frac{\alpha_s}{2\pi} f_{qqB}^{(M)}(x_1,x_2) \right] \\
&\qquad \times \left[D_q^{H_1(\mathrm{NLO})}\left(\frac{z_1}{x_1}, Q^2\right) D_{\bar q}^{H_2(\mathrm{NLO})}\left(\frac{z_2}{x_2}, Q^2\right) + D_q^{H_2(\mathrm{NLO})}\left(\frac{z_2}{x_2}, Q^2\right) D_{\bar q}^{H_1(\mathrm{NLO})}\left(\frac{z_1}{x_1}, Q^2\right) \right] \\
&\qquad + \frac{\alpha_s}{2\pi} f_{qgB}^{(M)}(x_1,x_2) \left[D_q^{H_1(\mathrm{NLO})}\left(\frac{z_1}{x_1}, Q^2\right) + D_{\bar q}^{H_1(\mathrm{NLO})}\left(\frac{z_1}{x_1}, Q\right) \right] D_g^{H_2(\mathrm{NLO})}\left(\frac{z_2}{x_2}, Q^2\right) \\
&\qquad + \frac{\alpha_s}{2\pi} f_{gqB}^{(M)}(x_1,x_2) \left[D_q^{H_2(\mathrm{NLO})}\left(\frac{z_2}{x_2}, Q^2\right) + D_{\bar q}^{H_2(\mathrm{NLO})}\left(\frac{z_2}{x_2}, Q^2\right) \right] D_g^{H_1(\mathrm{NLO})}\left(\frac{z_1}{x_1}, Q^2\right) \Bigg\} \\
&+ 3\sigma_0 \int_{z_1+z_2}^1 \frac{dx}{x^2} \sum_q e_q^2 \Bigg\{ \left[\delta(1-x) + \frac{\alpha_s}{2\pi} f_q^{(M)}(x) \right] \\
&\qquad\qquad \times \left[DD_q^{H_1H_2(\mathrm{NLO})}\left(\frac{z_1}{x}, \frac{z_2}{x}, Q^2\right) + DD_{\bar q}^{H_1H_2(\mathrm{NLO})}\left(\frac{z_1}{x}, \frac{z_2}{x}, Q^2\right) \right] \\
&\qquad\qquad + 2\frac{\alpha_s}{2\pi} f_g^{(M)}(x) DD_g^{H_1H_2(\mathrm{NLO})}\left(\frac{z_1}{x}, \frac{z_2}{x}, Q^2\right) \Bigg\}. \qquad (28)
\end{aligned}
$$

The longitudinal contribution is obtained by replacing $f^{(M)} \to f^{(L)}$ and omitting the corresponding LO terms proportional to $\delta(1-x_1)\delta(1-x_2)$ and $\delta(1-x)$.

While the dependence on the momentum fractions of the double-fragmentation functions cannot be computed within perturbative QCD, the factorization scale dependence is driven by the evolution equations. As for the ordinary fragmentation functions, these equations can be obtained by requiring that the bare functions $DD_i^{H_1H_2}$ do not depend on the factorization scale

$$\frac{d}{d\log M^2} DD_i^{H_1H_2}(x,y) = 0. \qquad (29)$$

Replacing $DD_i^{H_1H_2}(x,y)$ from Eq. (27) we obtain

$$
\begin{aligned}
&\frac{d}{d\log M^2} DD_i^{H_1H_2(\mathrm{NLO})}\left(x,y,M^2\right) \\
&= \frac{\alpha_s}{2\pi} \int_{x+y}^1 \frac{du}{u^2} P_{ji}(u) DD_j^{H_1H_2(\mathrm{NLO})}\left(\frac{x}{u}, \frac{y}{u}, M^2\right) + \frac{\alpha_s}{2\pi} \int_{z_1}^{1-z_2} \frac{du}{u(1-u)} \left[\hat P_{ji}^k(u) D_j^{H_1}\left(\frac{x}{u}\right) D_k^{H_2}\left(\frac{y}{1-u}\right) \right]. \qquad (30)
\end{aligned}
$$

The first term in the right-hand side corresponds to the usual homogeneous evolution of the fragmentation functions D_i^H. It indicates that the probability of obtaining the hadrons H_1 and H_2 from the parton i is affected by the possibility of the emission of a parton j with momentum fraction u, which can produce two hadrons by a double fragmentation. The second term, on the other hand, is inhomogeneous and it is not present in the evolution equations of the D_i^H functions. It corresponds to the case of a parton i that evolves emitting the partons j and k with fractions u and $1-u$ respectively, which can also give origin to two hadrons, but now by means of the mechanism of single fragmentation form each one of them. Both terms are represented symbolically in the Fig. 4. These equations fully agree with the ones originally proposed in [4,6].

The presence of these two terms in the evolution equations evidences the fact that, within the precision of any possible detector, it is physically impossible to determine which mechanism, either single or double fragmentation, has been responsible for the production of two hadrons when they are found in the same jet. In the same way, and beyond LO accuracy, only the sum of the two terms in Eq. (6) associated to each one of the mechanisms

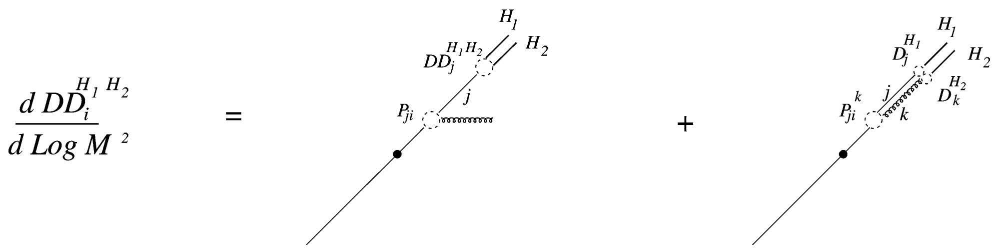

Fig. 4. Graphic representation for the evolution of the $DD_i^{H_1H_2}$. The first diagram corresponds to the first term in Eq. (30). The last one to the inhomogeneous term.

that contributes to the cross section, has a physical meaning. In this sense, the similarity with the situation of the *fracture functions* [7] in DIS appears again.

4. Conclusion

In this work, the cross section for the production of two hadrons in e^+e^- annihilation is calculated to order α_s considering events that include the possibility that both hadrons appear in the same jet. For this purpose it is necessary to extend the fragmentation model including a new type of functions, the double-fragmentation functions $DD_i^{H_1H_2}$, that describe the transition of a parton into two hadrons. These functions, along with the single-fragmentation function D_i^H, allow an unified treatment for the description of two-hadron production in e^+e^- annihilation.

While at leading order the $DD_i^{H_1H_2}$ functions are necessary to contemplate the possibility of the double fragmentation, at next-to-leading order and beyond, they are required to perform the factorization of divergences that cannot be absorbed in the single-fragmentation functions. As a result, they obey the inhomogeneous evolution equations in Eq. (30), where the two mentioned mechanisms of fragmentation are involved. We showed, for the first time, that introducing the double-fragmentation functions the usual factorization procedure can be enlarged consistently for the production of two hadrons to order α_s, reobtaining the evolution equations originally proposed in [4,6].

Appendix A

The NLO($\overline{\mathrm{MS}}$) corrections to the single fragmentation term are given by

$$
\begin{aligned}
f_{q\bar{q}A}^{(M)}(x_1,x_2) &= \frac{4}{3}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)_{+[0,\underline{1}]}} + \left[\hat{P}_{qq}(x_1)\log\big[(1-x_1)x_1\big] + \frac{4}{3}(1-x_1)\right]\delta(1-x_2),\\
f_{q\bar{q}B}^{(M)}(x_1,x_2) &= \frac{4}{3}\frac{x_1^2+x_2^2}{(1-x_1)_{+[0,\underline{1}]}(1-x_2)_{+[0,\underline{1}]}} + \frac{4}{3}\delta(1-x_1)\delta(1-x_2)\big(\pi^2-8\big)\\
&\quad + \left[\frac{4}{3}\big(1+x_1^2\big)\left(\frac{\log(1-x_1)}{1-x_1}\right)_{+[0,\underline{1}]} + \frac{4}{3}(1-x_1) + \hat{P}_{qq}(x_1)\log x_1\right]\delta(1-x_2)\\
&\quad + \left[\frac{4}{3}\big(1+x_2^2\big)\left(\frac{\log(1-x_2)}{1-x_2}\right)_{+[0,\underline{1}]} + \frac{4}{3}(1-x_2) + \hat{P}_{qq}(x_2)\log x_2\right]\delta(1-x_1),\\
f_{qgA}^{(M)}(x_1,x_2) &= \frac{4}{3}\frac{x_1^2+(2-x_1+x_2)^2}{(1-x_1)(x_1+x_2-1)_{+[\underline{1-x_2},1]}} + \left[\hat{P}_{qq}(x_1)\big[\log(1-x_1)x_1^2\big] + \frac{4}{3}(1-x_1)\right]\delta(x_1+x_2-1),
\end{aligned}
$$

$$f_{qgB}^{(M)}(x_1,x_2) = \frac{4}{3}\frac{x_1^2+(2-x_1-x_2)^2}{(1-x_1)_{+[0,\underline{1}]}(x_1+x_2-1)} + \left[\hat{P}_{gq}(x_2)\left[\log(1-x_2)x_2\right] + \frac{4}{3}x_2\right]\delta(1-x_1),$$

$$f_{gqA}^{(M)}(x_1,x_2) = \frac{4}{3}\frac{x_2^2+(2-x_1-x_2)^2}{(1-x_2)_{+[0,\underline{1}]}(x_1+x_2-1)_{+[\underline{1-x_2},1]}} + \left[\hat{P}_{gq}(x_1)\left[\log(1-x_1)x_1\right] + \frac{4}{3}x_1\right]\delta(1-x_2)$$
$$+ \left[\hat{P}_{gq}(x_1)\left[\log(1-x_1)^2x_1\right] + \frac{4}{3}x_1\right]\delta(x_1+x_2-1),$$

$$f_{gqB}^{(M)}(x_1,x_2) = \frac{4}{3}\frac{x_2^2+(2-x_1-x_2)^2}{(1-x_2)_{+[0,\underline{1}]}(x_1+x_2-1)} + \left[\hat{P}_{gq}(x_1)\left[\log(1-x_1)x_1\right] + \frac{4}{3}x_1\right]\delta(1-x_2),$$

$$f_{q\bar{q}\,A/B}^{(L)}(x_1,x_2) = \frac{8}{3}\left(\frac{x_1+x_2-1}{x_2^2}\right),$$

$$f_{qg\,A/B}^{(L)}(x_1,x_2) = \frac{16}{3}\left(\frac{1-x_2}{x_2^2}\right),$$

$$f_{gq\,A/B}^{(L)}(x_1,x_2) = \frac{8}{3}\left(\frac{1-x_1}{x_2^2}\right).$$

The corresponding corrections to double fragmentation are

$$f_q^{(M)}(x) = \frac{4}{3}\left[\left(1+x^2\right)\left(\frac{\log 1-x}{1-x}\right)_{+[0,\underline{1}]} + 2\left(\frac{1+x^2}{1-x}\right)\log x - \frac{3}{2}\frac{1}{(1-x)_{+[0,\underline{1}]}+} - \frac{3}{2}x + \frac{5}{2}\right.$$
$$\left. + \left(\frac{2\pi^2}{3} - \frac{9}{2}\right)\delta(1-x)\right],$$

$$f_g^{(M)}(x) = \hat{P}_{gq}(x)\left[\log(1-x)x^2\right],$$

$$f_q^{(L)}(x) = \frac{4}{3},$$

$$f_g^{(L)}(x) = \frac{8}{3}\left(\frac{1-x}{x}\right).$$

References

[1] G. Altarelli, R.K. Ellis, G. Martinelli, S.-Y. Pi, Nucl. Phys. B 160 (1979) 301.
[2] P.J. Rijken, W.L. van Neerven, Nucl. Phys. B 487 (1997) 233.
[3] K. Konishi, A. Ukawa, G. Veneziano, Phys. Lett. B 78 (1978) 243.
[4] K. Konishi, A. Ukawa, G. Veneziano, Nucl. Phys. B 157 (1979) 45.
[5] I. Vendramin, Nuovo Cimento A 66 (1981) 339.
[6] U.P. Sukhatme, K.E. Lassila, Phys. Rev. D 22 (1980) 1184.
[7] L. Trentadue, G. Veneziano, Phys. Lett. B 323 (1994) 201.
[8] D. Graudenz, Nucl. Phys. B 432 (1994) 351.
[9] D. de Florian, C.A. García Canal, R. Sassot, Nucl. Phys. B 470 (1996) 195.
[10] G. Camici, M. Grazzini, L. Trentadue, Phys. Lett. B 439 (1998) 382.
[11] M. Grazzini, L. Trentadue, G. Veneziano, Nucl. Phys. B 519 (1998) 394.
[12] C.G. Bollini, J.J. Giambiagi, Nuovo Cimento B 12 (1972) 20.
[13] G. 't Hooft, M. Veltman, Nucl. Phys. B 44 (1972) 189.
[14] G. Altarelli, G. Parisi, Nucl. Phys. B 126 (1977) 298.