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Classical behaviour after a phase transition

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Abstract

We analyze the onset of classical behaviour after a second-order phase transition by considering a scalar field theory in which the system-field interacts with its environment, represented both by further fields and by its own short-wavelength modes. Within our approximations we see that the long-wavelength modes have become classical by the time that the transition has been first implemented (the spinodal time).

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Cosmology and particle physics suggest strongly that phase transitions have occurred in the early Universe, in particular, at the grand unified and electroweak scales [1].

An analysis of phase transitions in quantum field theory that takes the non-equilibrium nature of the dynamics into account from first principles is very difficult, and has only begun to be addressed. In particular, the naive picture of a classical order parameter (inflaton or Higgs) field ϕ rolling down an adiabatic effective potential, that was once a mainstay of cosmological field theory modelling, has been shown to be suspect [2]. Alternatively, the suggestion by Kibble [3] that, while a non-adiabatic approach is crucial, causality alone can set saturated classical bounds on time and distance scales during a transition, has been shown to be only partly true. The issue of how a quantum system evolves into the classical theory has been addressed in Refs. [4,5]. For some models, it has been shown that classicality emerges as a consequence of profuse particle creation, whereby a non-perturbatively large occupation number of long-wavelength particles produces, on average, a diagonal density matrix. This dephasing effect occurs at late times. Here we will consider a model of an explicitly *open* system, in which classicality is an early time event, induced by the environment. The result is completely different from that of Refs. [4,5].

There are several time scales which are relevant for the description of the onset of a transition. If the quench is fast, the initial stages of a scalar transition can be described by a free field theory with inverted potential, $(\text{mass})^2 < 0$. This description is valid until the field wave functional explores the ground states of the potential at the spinodal time t_{sp}. Specifically, the field ordering after the transition is due to the growth in amplitude of its unstable long-wavelength modes. In consequence, the short-wavelength modes of the field,

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together with all the other fields χ_a with which the ϕ inescapably interacts, form an environment whose coarse-graining makes the system-field classical [6]. As a result, there is an additional time scale associated with the environment, the decoherence time t_D [7,8]. Once $t > t_D$ the order parameter becomes a classical entity [9].

In this Letter we consider a simple model of a scalar order-parameter field ϕ, whose $\mathcal{Z}_2$ symmetry is broken by a double-well potential. Specifically, we take the simplest classical action with scalar environmental fields χ_a ($\mu^2, m_a^2 > 0$)

$$\begin{aligned} S[\phi,\chi] &= S_{\rm syst}[\phi] + S_{\rm env}[\chi] + S_{\rm int}[\phi,\chi], \\ S_{\rm syst}[\phi] &= \int d^4x \left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4\right\}, \\ S_{\rm env}[\chi_a] &= \sum_{a=1}^{N}\int d^4x \left\{\frac{1}{2}\partial_\mu\chi_a\partial^\mu\chi_a - \frac{1}{2}m_a^2\chi_a^2\right\}, \\ S_{\rm int}[\phi,\chi] &= -\sum_{a=1}^{N}\frac{g_a}{8}\int d^4x\, \phi^2(x)\chi_a^2(x). \end{aligned} \qquad (1)$$

As we have already observed, the separation in (1) is not yet the separation between the ultimately classical 'system' and its 'environment' since the short-wavelength modes of the ϕ-field never become classical, and must be treated as part of the decohering environment. However, to demonstrate how an environment renders the order-parameter field classical we consider the simpler case in which the environment is taken to be composed of the N fields χ_a alone. Since the effect of environmental sources is additive, at our level of approximation, the χ_a fields alone give us an *upper* bound on t_D.

For weak couplings λ, $g_a \ll 1$ and comparable masses $m_a \simeq \mu$ we shall find that t_D is shorter than the spinodal time t_{sp}, defined as the time for which $\langle\phi^2\rangle_t \sim \eta^2 = 6\mu^2/\lambda$, the ground states of the system. In consequence, by the time that the field is ordered it can be taken to be classical.

The model has a continuous transition at a temperature T_c. The environmental fields χ_a reduce T_c and, in order that $T_c^2 = \mu^2/12(\lambda + \sum g_a) \gg \mu^2$, we must take $\lambda + \sum g_a \ll 1$. For order of magnitude estimations it is sufficient to take identical $g_a = \bar{g}/\sqrt{N}$. We will also assume $\bar{g} \simeq \lambda$. For one-loop consistency in our subsequent calculation we assume that $N \gg 1$. The effect of many weakly coupled environmental fields is that they act on the system field without it being able to act back on them. For example, the dominant hard loop contribution of the ϕ-field to the χ_a thermal masses is $\delta m_T^2 = O(\bar{g}T_c^2/\sqrt{N}) = O(\mu^2/N) \ll \mu^2$. Similarly, the two-loop (setting sun) diagram, which is the first to contribute to the discontinuity of the χ-field propagator, is of magnitude $\bar{g}^2T_c^2/N = O(\bar{g}\mu^2/N^{3/2}) \ll \delta m_T^2$, in turn. That is, the effect of the thermal bath on the propagation of the environmental χ-fields is ignorable. We stress that this is not a Hartree or large-N approximation of the type that, to date, has been the major way to proceed [5,10] for a *closed* system.

We shall assume that the initial states of the system and environment are both thermal, at a temperature $T_0 > T_c$. We then imagine a change in the global environment (e.g., expansion in the early universe) that can be characterised by a change in temperature from T_0 to $T_f < T_c$. The relevant object is the reduced density matrix $\rho_r[\phi^+,\phi^-,t] = \langle\phi^+|\hat{\rho}_r(t)|\phi^-\rangle$. It describes the evolution of the system under the influence of the environment, defined by

$$\rho_r\big[\phi^+,\phi^-,t\big] = \int \mathcal{D}\chi_a\, \rho\big[\phi^+,\chi_a,\phi^-,\chi_a,t\big],$$

where $\rho[\phi^+,\chi_a^+,\phi^-,\chi_a^-,t] = \langle\phi^+\chi_a^+|\hat{\rho}(t)|\phi^-\chi_a^-\rangle$ is the full density matrix. The environment will have the effect of making the system effectively classical once $\rho_r(t)$ is, approximately, diagonal in the field configuration basis. Quantum interference can then be ignored and we obtain a classical probability distribution from the diagonal part of $\rho_r(t)$, or equivalently, by means of the reduced Wigner functional, which is positive definite after the decoherence time (see Ref. [9] for an explicit demonstration of this for a toy model). This behaviour is essentially different from the dephasing effects [5]. Our onset of classical behaviour is an early-time event which, beneficially, allows us to use perturbation theory.

Assuming that the initial full density matrix can be factorised, the temporal evolution of the reduced one is given by

$$\begin{aligned} &\rho_r\big[\phi_f^+,\phi_f^-,t\big] \\ &\quad = \int d\phi_i^+ \int d\phi_i^-\, J_r\big[\phi_f^+,\phi_f^-,t|\phi_i^+,\phi_i^-,t_0\big] \\ &\qquad \times \rho_r\big[\phi_i^+\phi_i^-,t_0\big], \end{aligned}$$

where J_r is the reduced evolution operator

$$J_r\big[\phi_f^+,\phi_f^-,t|\phi_i^+,\phi_i^-,t_0\big] = \int_{\phi_i^+}^{\phi_f^+}\mathcal{D}\phi^+\int_{\phi_i^-}^{\phi_f^-}\mathcal{D}\phi^-\, e^{i\{S[\phi^+]-S[\phi^-]\}}F\big[\phi^+,\phi^-\big]. \quad (2)$$

The Feynman–Vernon [11] influence functional $F[\phi^+,\phi^-]$ is defined as

$$F\big[\phi^+,\phi^-\big] = \int d\chi_{ai}^+\int d\chi_{ai}^-\,\rho_\chi\big[\chi_{ai}^+,\chi_{ai}^-,t_0\big] \times\int d\chi_{af}\int_{\chi_{ai}^+}^{\chi_{af}}\mathcal{D}\chi_a^+\int_{\chi_{ai}^-}^{\chi_{af}}\mathcal{D}\chi_a^- \times\exp\Big(i\Big\{S\big[\chi_a^+\big]+S_{int}\big[\phi^+,\chi_a^+\big]\Big\} - i\Big\{S\big[\chi_a^-\big]+S_{int}\big[\phi^-,\chi_a^-\big]\Big\}\Big).$$

Given our thermal initial conditions it is not the case that the full density matrix has ϕ and χ fields uncorrelated initially, since it is the interactions between them that leads to the restoration of symmetry at high temperatures. Rather, on incorporating the hard thermal loop 'tadpole' diagrams of the χ (and ϕ) fields in the ϕ mass term leads to the effective action for ϕ quasi-particles,

$$S_{syst}^{eff}[\phi] = \int d^4x\left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2(T_0)\phi^2 - \frac{\lambda}{4!}\phi^4\right\},$$

where $m_\phi^2(T_0) = -\mu^2(1-T_0^2/T_c^2) > 0$. As a result, we can take an initial factorised density matrix of the form $\hat\rho[T_0] = \hat\rho_\phi[T_0]\hat\rho_\chi[T_0]$, where $\hat\rho_\phi[T_0]$ is determined by the quadratic part of $S_{syst}^{eff}[\phi]$ and $\hat\rho_\chi[T_0]$ by $S[\chi_a]$.

Provided the change in temperature is not too slow the exponential instabilities of the ϕ-field grow so fast that the field has populated the degenerate vacua well before the temperature has dropped to T_f. As T_c has no particular significance for the environment field, for these early times we can keep the temperature of the environment fixed at $T_\chi = T_0 = \mathcal{O}(T_c)$. Since it is the system-field ϕ field whose behaviour changes dramatically on taking T_ϕ through T_c, we adopt an *instantaneous* quench for T_ϕ from T_0 to $T_f = 0$ at time $t = 0$, in which $m_\phi^2(T)$ changes sign and magnitude instantly, concluding with the value $m_\phi^2(t) = -\mu^2$, $t > 0$.

Beginning from this initial distribution, peaked around $\phi = 0$, we follow the evolution of the system, with Hamiltonian determined from (1). From the influence functional we define the influence action $\delta A[\phi^+,\phi^-]$ by $F[\phi^+,\phi^-] = \exp i\delta A[\phi^+,\phi^-]$. After further defining $\Delta = \frac{1}{2}(\phi^{+2}-\phi^{-2})$ and $\Sigma = \frac{1}{2}(\phi^{+2}+\phi^{-2})$, the real and imaginary parts of the influence action are, in the one loop (two vertices) and large-N approximations,[1]

$$\mathrm{Re}\,\delta A = \frac{\bar g^2}{8}\int d^4x\int d^4y\,\Delta(x)K_q(x-y)\Sigma(y),$$

$$\mathrm{Im}\,\delta A = -\frac{\bar g^2}{16}\int d^4x\int d^4y\,\Delta(x)N_q(x-y)\Delta(y),$$

where $K_q(x-y) = \mathrm{Im}\,G_{++}^2(x,y)\theta(y^0-x^0)$ is the dissipation kernel and $N_q(x-y) = \mathrm{Re}\,G_{++}^2(x,y)$ is the noise (diffusion) kernel. G_{++} is the relevant closed-time-path correlator of the χ-field at temperature T_0.

The first step in the evaluation of the master equation is the calculation of the density matrix propagator J_r from Eq. (2). In order to estimate the functional integration which defines the reduced propagator, we perform a saddle point approximation

$$J_r\big[\phi_f^+,\phi_f^-,t\big|\phi_i^+,\phi_i^-,t_0\big] \approx \exp iA\big[\phi_{cl}^+,\phi_{cl}^-\big],$$

where $A[\phi^+,\phi^-] = S[\phi^+] - S[\phi^-] + \delta A[\phi^+,\phi^-]$, and $\phi_{cl}^\pm$ is the solution of the equation of motion $\delta\,\mathrm{Re}\,A/\delta\phi^+|_{\phi^+=\phi^-} = 0$ with boundary conditions $\phi_{cl}^\pm(t_0) = \phi_i^\pm$ and $\phi_{cl}^\pm(t) = \phi_f^\pm$. It is very difficult to solve this equation analytically. For simplicity, we assume that the system-field contains only one Fourier mode with $\vec k = \vec k_0$. We are motivated in this by the observation [10,12] that the exponentially growing long-wavelengths increasingly bunch about a wave-number $k_0 < \mu$, which diminishes with time initially as $k_0^2 = \mathcal{O}(\mu^2/t)$.

The classical solution is of the form $\phi_{cl}(\vec x,s) = f(s,t)\cos(\vec k_0.\vec x)$ where $f(0,t) = \phi_i$ and $f(t,t) = \phi_f$. Qualitatively, $f(s,t)$ grows exponentially with s for $t \leqslant t_{sp}$, and oscillates for $t_{sp} < s < t$ when $t > t_{sp}$. We

[1] The large-N approximation singles out the two-vertex loop in the one-loop perturbative approximation.

shall therefore approximate it, for $t \leqslant t_{\mathrm{sp}}$ as

$$f(s,t) = \phi_{\mathrm{f}} \frac{\sinh(\omega_0 s)}{\sinh(\omega_0 t)} + \phi_{\mathrm{i}} \frac{\sinh[\omega_0(t-s)]}{\sinh(\omega_0 t)},$$

where $\omega_0^2 = \mu^2 - k_0^2$. In order to obtain the master equation we must compute the final time derivative of the propagator J_{r}, and after that eliminate the dependence on the initial field configurations $\phi_{\mathrm{i}}^{\pm}$ coming from the classical solutions $\phi_{\mathrm{cl}}^{\pm}$ (see Ref. [6]).

To determine the onset of classical behaviour it is sufficient to calculate just the correction to the normal unitary evolution coming from the noise kernel. For clarity we drop the suffix 'f' on the final state fields. If $\Delta = (\phi^{+2} - \phi^{-2})/2$ for the *final* field configurations, then the relevant part of the master equation for $\rho_{\mathrm{r}}(\phi^+, \phi^-, t)$ is

$$i\,\dot{\rho}_{\mathrm{r}} = \langle \phi^+ | [H, \hat{\rho}_{\mathrm{r}}] | \phi^- \rangle - i \frac{\bar{g}^2}{16} V \Delta^2 D(k_0, t) \rho_{\mathrm{r}} + \cdots . \tag{3}$$

The volume factor V that appears in the master equation is due to the fact we are considering a density matrix which is a functional of two different field configurations, $\phi^{\pm}(\vec{x}) = \phi^{\pm} \cos \vec{k}_0.\vec{x}$, which are spread over all space. The time-dependent diffusion coefficient $D(k_0, t)$ due to each of the many external environmental fields is then given by

$$D_\chi(k_0, t) = \int_0^t ds\, u(s) \big[\mathrm{Re}\, G_{++}^2(2k_0; t-s) + 2\, \mathrm{Re}\, G_{++}^2(0; t-s) \big]. \tag{4}$$

In (4), $u(s) = \cosh^2 \omega_0 s$ when $t \leqslant t_{\mathrm{sp}}$, and is an oscillatory function of time when $t > t_{\mathrm{sp}}$.

Although G_{++} is oscillatory at all times, for times $\mu t > 1$ (until the spinodal one) the exponential growth of $u(t)$ enforces a similar behaviour on $D_\chi(k_0, t)$,

$$D_\chi(k_0, t) \sim \frac{(k_{\mathrm{B}} T_0)^2}{\mu^3} \omega_0 \exp[2\omega_0 t], \tag{5}$$

associated with the instability of the k_0 mode. For long-wavelength modes $D_\chi(t) \sim (k_{\mathrm{B}} T_0/\mu)^2 \exp[2\mu t]$. For $t > t_{\mathrm{sp}}$ the diffusion coefficient stops growing, and oscillates around $D_\chi(k_0, t = t_{\mathrm{sp}})$.

In our present model the environment fields χ_{a} are not the only decohering agents. The environment is also constituted by the short-wavelength modes of the self-interacting field ϕ. Therefore, we split the field as $\phi = \phi_< + \phi_>$, where the system-field $\phi_<$ contains the unstable modes with wavelengths longer than the critical value μ^{-1}, while the bath or environment-field contains the stable modes with wavelengths shorter than μ^{-1} (in practice, whether the separation is made at $k = \mu$ exactly or at $k \approx \mu$ is immaterial [15] by time t_{D}, when the power of the ϕ-field fluctuations is peaked at $k_0 \ll \mu$). This gives an additional contribution to the diffusion coefficient. Without the additional powers of N^{-1}, a high temperature resummation of loop diagrams [16] is essential to get a reliable G_{++}. This is beyond the scope of this Letter. It will be enough for our purposes to compute an upper bound on the decoherence time t_{D} only considering the external fields χ_{a}.

We estimate t_{D} by considering the approximate solution (3), [2]

$$\rho_{\mathrm{r}}\big[\phi_<^+, \phi_<^-; t\big] \approx \rho_{\mathrm{r}}^{\mathrm{u}}\big[\phi_<^+, \phi_<^-; t\big] \exp\left[-V\Gamma \int_0^t ds\, D(k_0, s) \right],$$

where $\rho_{\mathrm{r}}^{\mathrm{u}}$ is the solution of the unitary part of the master equation (i.e., without environment) and $D(k_0, s)$ denotes the total diffusion. It is obvious from this (and also from (3)), that the diagonal density matrix just evolves like the unitary matrix (the environment has almost no effect on the diagonal part of ρ_{r}). In terms of the dimensionless fields $\bar{\phi} = (\phi_<^+ + \phi_<^-)/2\mu$, and $\delta = (\phi_<^+ - \phi_<^-)/2\mu$, we have $\Gamma = (1/16)\bar{g}^2 \mu^4 \bar{\phi}^2 \delta^2$.

The system behaves classically when the non-diagonal elements of the reduced density matrix are much smaller than the diagonal ones. We, therefore, look at the ratio

$$\left| \frac{\rho_{\mathrm{r}}[\bar{\phi} + \delta, \bar{\phi} - \delta; t]}{\rho_{\mathrm{r}}[\bar{\phi}, \bar{\phi}; t]} \right| \approx \left| \frac{\rho_{\mathrm{r}}^{\mathrm{u}}[\bar{\phi} + \delta, \bar{\phi} - \delta; t]}{\rho_{\mathrm{r}}^{\mathrm{u}}[\bar{\phi}, \bar{\phi}; t]} \right| \exp\left[-V\Gamma \int_0^t ds\, D(k_0, s) \right]. \tag{6}$$

It is not possible to obtain an analytic expression for the ratio of unitary density matrices that appears in Eq. (6). The simplest approximation is to neglect

[2] We are following the decoherence time definition of [13].

the couplings of the system field [14]. In this case the unitary density matrix remains Gaussian at all times as

$$\left|\frac{\rho_{\rm r}^{\rm u}[\bar\phi+\delta,\bar\phi-\delta;t]}{\rho_{\rm r}^{\rm u}[\bar\phi,\bar\phi;t]}\right| = \exp\left[-\frac{T_{\rm c}}{\mu}\delta^2 p^{-1}(t)\right], \qquad (7)$$

where $p^{-1}(t)$, essentially $\mu^2\langle\phi^2\rangle_t^{-1}$, decreases exponentially with time to a value $\mathcal{O}(\lambda)$. A full numerical calculation [9] shows that $\rho_{\rm r}^{\rm u}$ becomes a non-Gaussian function (the associated Wigner function becomes non-positive). In any case, in the unitary part of the reduced density matrix the non-diagonal terms are not suppressed. [3]

The decoherence time $t_{\rm D}$ sets the scale after which we have a classical system-field configuration. According to our previous discussion, it can be defined as the solution to

$$1 \approx V\Gamma\int_0^{t_{\rm D}} ds\, D(k_0,s) \geqslant V\Gamma\int_0^{t_{\rm D}} ds\, D_\chi(k_0,s).$$

It corresponds to the time after which we are able to distinguish between two different field amplitudes (inside a given volume V).

Suppose we reduce the couplings $\bar g\sim\lambda$ of the system ϕ-field to its environment. Since, as a one-loop construct, $\Gamma\propto\bar g^2\sim\lambda^2$, we might expect that, as $\bar g$, λ decrease, then $t_{\rm D}$ increases and the system takes longer to become classical. Although this is the usual result for Brownian motion, say [7], it is not simply the case for quantum field theory phase transitions. The reason is two fold. Firstly, there is the effect that $\Gamma\propto T_0^2$, and $T_0^2\propto\lambda^{-1}$ is non-perturbatively large for a phase transition. Secondly, because of the non-linear coupling to the environment, obligatory for quantum field theory, $\Gamma\propto\bar\phi^2$. The completion of the transition finds $\bar\phi^2\simeq\eta^2\propto\lambda^{-1}$ also non-perturbatively large. This suggests that Γ, and hence $t_{\rm D}$, can be approximately independent of λ. In fact, the situation is a little more complicated, but the end result is that $t_{\rm D}$ does not increase (relative to $t_{\rm sp}$) as the couplings become uniformly weaker.

[3] This should not be confused with the observation that the unitary Gaussian density matrix does show classical correlation, whereby the Wigner functional becomes localised in phase space about its classical solutions. However, this has nothing to do with eliminating quantum interference between different field histories.

In quantifying the decoherence time V is understood as the minimal volume inside which there is no possibility for coherent superpositions of macroscopically distinguishable states for the field (i.e., there is no 'Schrödinger cat' states inside V). Thus, our choice is that this volume factor is $\mathcal{O}(\mu^{-3})$ since μ^{-1} (the Compton wavelength) sets the smallest scale at which we need to look. In particular, μ^{-1} characterises the thickness of domain boundaries (walls) as the field settles into its ground-state values. Inside this volume, we do not discriminate between field amplitudes which differ by $\mathcal{O}(\mu)$, and therefore we take $\delta\sim\mathcal{O}(1)$. For $\bar\phi$ we set $\bar\phi^2\sim\mathcal{O}(\alpha/\lambda)$, where $\lambda\leqslant\alpha\leqslant 1$ is to be determined self-consistently from the condition that, at time $t_{\rm D}$, $\langle\phi^2\rangle_t\sim\alpha\eta^2$.

Note that the diagonalisation of ρ_t occurs quickly, but not so quickly that $\mu t\ll 1$. Consequently, in order to evaluate the decoherence time in our model, we have to use Eq. (5). We obtain, for the upper bound on $t_{\rm D}$,

$$\exp[2\mu t_{\rm D}] \approx \frac{\lambda\sqrt{N}\,\bar g}{\bar g^2\alpha} = \mathcal{O}\left(\frac{\sqrt{N}}{\alpha}\right), \qquad (8)$$

whereby $\mu t_{\rm D}\simeq\ln(\eta/T_{\rm c}\sqrt{\alpha})$. The value of α is determined as $\alpha\simeq\sqrt{\mu/T_{\rm c}}$. For comparison, we find $t_{\rm sp}$, for which $\langle\phi^2\rangle_t\sim\eta^2$, given by

$$\exp[2\mu t_{\rm sp}] \approx \mathcal{O}\left(\frac{\eta^2}{\mu T_{\rm c}}\right). \qquad (9)$$

The exponential factor, as always, arises from the growth of the unstable long wavelength modes. The factor $T_{\rm c}^{-1}$ comes from the $\coth(\beta\omega/2)$ factor that encodes the initial Boltzmann distribution at temperature $T_0\gtrsim T_{\rm c}$. As a result,

$$\mu t_{\rm sp} \sim \ln\left(\frac{\eta}{\sqrt{\mu T_{\rm c}}}\right), \qquad (10)$$

whereby $1<\mu t_{\rm D}\leqslant\mu t_{\rm sp}$, with

$$\mu t_{\rm sp}-\mu t_{\rm D} \simeq \frac{1}{4}\ln\left(\frac{T_{\rm c}}{\mu}\right) > 1, \qquad (11)$$

for weak enough coupling, or high enough initial temperatures. Factors of $\mathcal{O}(1)$ have been omitted in the argument of the logarithm in (11) and previous equations, requiring that $T_{\rm c}\gg\mu$.

This is our main result, that for the physically relevant modes (with small k_0) classical behaviour has been established before the spinodal time, when the ground states have became populated. We can say more in that, for an instantaneous quench, non-linear behaviour only becomes important in an interval Δt, $\mu \Delta t = \mathcal{O}(1)$, before the spinodal time [15], and, therefore, ρ_r becomes diagonal before non-linear terms could be relevant. In this sense, classical behaviour has been achieved before quantum effects could destroy the positivity of the Wigner function W_r.

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