20 January 2000

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 472 (2000) 287–294

Cosmological magnetic fields from gauge-mediated supersymmetry-breaking models

Alejandra Kandus [a], Esteban A. Calzetta [a], Francisco D. Mazzitelli [a], Carlos E.M. Wagner [b,c]

[a] Departamento de Física, Fac. Cs. Ex. y Nat. UBA, Ciudad Universitaria, (1428) Buenos Aires, Argentina
[b] Theory Division, CERN, CH-1211 Geneva 23, Switzerland
[c] High Energy Division, Argonne National Laboratory, Argonne, IL 60439, USA

Received 21 September 1999; received in revised form 15 November 1999; accepted 23 November 1999
Editor: R. Gatto

Abstract

We study the generation of primordial magnetic fields, coherent over cosmologically interesting scales, by gravitational creation of charged scalar particles during the reheating period. We show that magnetic fields consistent with those detected by observation may be obtained if the particle mean life τ_s is in the range 10^{-14} s $\lesssim \tau_s \lesssim 10^{-7}$ s. We apply this mechanism to minimal gauge-mediated supersymmetry-breaking models, in the case in which the lightest stau $\tilde{\tau}_1$ is the next-to-lightest supersymmetric particle. We show that, for a large range of phenomenologically acceptable values of the supersymmetry-breaking scale $\sqrt{F}$, the generated primordial magnetic field can be strong enough to seed the galactic dynamo. © 2000 Elsevier Science B.V. All rights reserved.

Quite homogeneous magnetic fields of intensity $B \simeq 3 \times 10^{-6}$ Gauss are present in all structures of our Universe: galaxies, galaxy clusters and hydrogen clouds [1–4]. One of the mechanisms for the generation of magnetic fields is the primordial generation of a seed field that is further amplified by gravitational collapse and/or dynamo [1]. For the origin of the seed field, several mechanisms have been proposed recently: it has been suggested that a primordial field might be produced during the inflationary period if conformal invariance is broken [5,6]; in superstring-inspired models, the coupling between the electromagnetic field and the dilaton breaks conformal invariance and might produce the seed field [7,8]; gauge-invariant couplings between the electromagnetic field and the space-time curvature also break conformal invariance, but produces in general an uninterestingly small seed field [9]; other mechanisms are based on, for example, first order cosmological phase transitions and on the existence of topological defects [10–12].

Recently, a new mechanism for cosmological magnetic field generation was proposed [13], based on the presence during inflation of a charged, minimally coupled scalar field in its invariant vacuum state [14]. When the transition to radiation takes place, quantum creation of charged particles occurs because of the release of gravitational energy. The mean electric current is zero, but stochastic fluctuations around that mean give a non-vanishing contri-

PII: S0370-2693(99)01389-1

bution. The magnetic field induced by this stochastic current was sufficient to seed the galactic dynamo. However, there remained the important issue of finding a suitable scalar particle to generate the electric current source of the magnetic field.

In this letter we address this problem in the context of gauge-mediated supersymmetry-breaking models (GMSB). In the simplest version of these models, supersymmetry-breaking is communicated to the visible sector through a set of massive fields, called messengers, which carry non-trivial quantum numbers under the gauge group [15,16]. The messengers Φ_I, $\overline{\Phi}_I$ are assumed to acquire an explicit mass M_I by the vacuum expectation value of a singlet field $\langle X_I \rangle = M_I$, via a superpotential coupling

$$W = X_I \overline{\Phi}_I \Phi_I . \tag{1}$$

A vacuum expectation value of the auxiliary component F_I of the field X_I breaks supersymmetry and induces, through the gauge interactions of the messenger fields, the supersymmetry-breaking masses in the observable sector. For the simplest case of N sets of messenger fields belonging to the fundamental representation of SU(5) and a single field X, one gets gaugino masses

$$M_i = \frac{N\alpha_i}{4\pi}\frac{F}{M}, \tag{2}$$

where $i = 1,2,3$ are associated with the gauge groups $U(1)_Y, SU(2)$ and $SU(3)_c$, respectively.

The scalar masses not affected by Yukawa couplings are given by

$$m_S^2(\mu) = \frac{2\,N c_S^i}{16\pi^2}\alpha_i^2(0)\left(\frac{F}{M}\right)^2 - \frac{2c_S^i}{b_i}M_i^2(0)\left(\frac{\alpha_i^2(\mu) - \alpha_i^2(0)}{\alpha_i^2(0)}\right), \tag{3}$$

where m_S are the supersymmetry-breaking masses for gauginos and scalars, respectively, μ is the renormalization group scale with $\mu = 0$ being identified with the messenger mass scale, c_S^i is the quadratic Casimir of the scalar particle under the i-gauge group, and α_i and b_i are the corresponding gauge coupling and MSSM beta function coefficients [1]. From the above, it is easy to see that the right-handed sleptons are the lightest scalars in the spectrum and, for $N > 1$, the lighest stau can easily become lighter than the lightest neutralino. The lightest stau can also become lighter than the lightest neutralino due to mixing effects, for moderate and large values of $\tan\beta$, for any value of N. For the characteristic values of the supersymmetry-breaking scale F, however, the lightest supersymmetric particle is the gravitino. Indeed, the gravitino mass is given by

$$m_{\tilde{G}} = \frac{F}{\sqrt{3}\,M_{\rm Pl}}, \tag{4}$$

where $M_{\rm Pl}$ is the Planck scale (we are identifying F with the fundamental supersymmetry-breaking scale F_0). Hence, the gravitino is the lightest supersymmetric particle for any messenger mass M much lower than the GUT scale.

In general, under the assumption of R-parity conservation [18], the next-to-lightest SUSY particle will decay into a gravitino and a standard particle with an inverse decay rate [19]

$$\tau = \frac{1}{\tilde{k}^2}\left(\frac{100\,\mathrm{GeV}}{m_{\rm NLSP}}\right)^5\left(\frac{\sqrt{F}}{100\mathrm{TeV}}\right)^4 3\times 10^{11}\mathrm{GeV}^{-1}, \tag{5}$$

where $m_{\rm NLSP}$ is the mass of the NLSP particle and $\tilde{k}$ is a projection factor equal to the component in the NLSP of the superpartner of the particle the NLSP is decaying into. For the case of the stau decaying into a tau and a gravitino, $\tilde{k} = 1$.

Constraints on the value of the supersymmetry-breaking scale may be obtained, for example, by the requirement that the gravitino density does not overclose the Universe. For instance, if the gravitino mass $m_{\tilde{G}} > 1$ keV, the temperature at the beginning of the radiation-dominated epoch, called *the reheat temperature* T_γ, should be much smaller than the GUT scale in order to avoid overproduction of gravitinos [20]. The exact bound on T_γ depends on the gravitino mass. For relatively large values of the gravitino mass, corresponding to $\sqrt{F} \simeq 10^9$ GeV,

[1] For more general expressions see, for instance, Ref. [17]

($M \simeq 10^{13}$ GeV), an upper bound on T_γ of the order of 10^7 GeV is obtained [2]. The bound becomes even smaller for smaller values of F. On the other hand, for $m_{\tilde{G}} < 1$ keV and for any value of the reheat temperature larger than the weak scale, the gravitinos will be in thermal equilibrium at early times and, for these range of masses, the gravitinos are sufficiently light to lead to cosmologically acceptable values of the relic density.

The reheating period can be characterized by the temperature T_I obtained by thermalization after preheating and T_γ, the temperature at the beginning of the radiation dominated epoch [22]. In Ref. [13] an inflationary model with instantaneous reheating was considered. In the more realistic case in which the reheating is extended in time, the number of particles created at the two main transitions, namely inflation-reheating and reheating–radiation, as well as during the reheating period itself should be calculated.

We will work in conformal time, which is given by $d\eta = dt/a(t)$. Defining $\tau = H\eta$, where H is the Hubble constant during inflation, and assuming that during reheating the Universe is matter-dominated [23], the scale factors for the different epochs of the Universe read

$$\text{inflation}\ \ a_I(\tau) = \frac{1}{(1-\tau)}, \tag{6}$$

$$\text{reheating}\ \ a_R(\tau) = \left[1 + \frac{\tau}{2}\right]^2, \tag{7}$$

$$\text{radiation}\ \ a_\gamma(\tau) = \left(\frac{T_I}{T_\gamma}\right)^{1/2b}\left[\tau + 2 - \left(\frac{T_I}{T_\gamma}\right)^{1/2b}\right]. \tag{8}$$

T_I and T_γ are the temperatures of the Universe at the beginning of reheating and at the beginning of radiation, respectively. We have assumed that during radiation the temperature of the Universe scales with $a(\tau)$ as $T \propto a(\tau)^{-1}$ while during reheating it goes as $T \propto a(\tau)^{-b}$, with $0 < b < 1$ [24].

[2] For large values of the gravitino mass, the non-thermal production of gravitinos tends to be dominant, and induces a tighter bound on the reheat temperature, which may be of the order of the weak scale [21].

The evolution of a charged scalar field is given by the Klein–Gordon equation. If we expand the real and imaginary parts of the field as $(2\pi)^{-3/2}\int d^3\kappa\, \phi_\kappa(\tau) e^{i\boldsymbol{\kappa}\cdot\boldsymbol{r}} +$ h.c. , the field equation reads

$$\left[\frac{\partial^2}{\partial\tau^2} + k^2 + \left(\frac{m}{H}\right)^2 a^2(\tau) - (1-6\xi)\frac{\ddot{a}(\tau)}{a(\tau)}\right] \times \phi_\kappa(\tau) = 0, \tag{9}$$

where $k = H^{-1}\kappa$ (κ being the comoving wavenumber) and where ξ is the coupling to the curvature. We will consider the mass as built up from two contributions, the zero-temperature mass $m(0) \equiv m_{\tilde{\tau}}$ and the thermal corrections, so that we have $m^2 = m^2(0) + gT^2(\tau)$, where g is of the order of the particle gauge coupling constants.

For the inflationary period, we do not need the thermal corrections, as the temperature of that period is too low to be important. However, in supergravity theories, the possible presence of a non-renormalizable coupling of the inflaton field I to the scalar fields in the Kähler potential [21]

$$K_{I,\phi} = -\frac{C_H}{3}\frac{1}{M_{\mathrm{Pl}}^2} I^\dagger I \phi^\dagger \phi \tag{10}$$

would naturally lead to a mass contribution $\delta m^2 \simeq C_H H^2$. Hence, in general, an effective mass of the order of the Hubble constant will be generated, although the coefficient C_H may be small or even zero in the case when the specific effective coupling is forbidden by symmetries of the theory [25].

The positive-frequency solution to Eq. (9) for inflation reads

$$\phi_k^I(\tau) = \frac{\sqrt{\pi}}{2}\sqrt{1-\tau}\, H_\nu^{(1)}[k(1-\tau)], \tag{11}$$

where $H_\nu^{(1)}$ are the Hankel functions, with

$$\nu = \frac{3}{2}\sqrt{1 - \frac{16}{3}\xi - \frac{4}{9}\frac{m^2}{H^2}}\,, \tag{12}$$

where for the characteristic values of $m(0)$ and H during inflation, $m^2/H^2 \simeq C_H$. We will assume throughout this article that the scalar field couples minimally to the curvature, $\xi = 0$, and that the coefficient $C_H \ll 1$; we will briefly discuss the implications of different values of these quantities at the end of this article. For reheating and radiation dominance, we propose a WKB solution

$$\phi_k^{\pm}(\tau) = \frac{1}{\sqrt{2|\omega|(\tau)}} \exp\left(\left[\pm i \int_0^{\tau} \omega(\tau')\, d\tau'\right]\right), \tag{13}$$

where

reheating $\omega(\tau)$

$$= \sqrt{k^2 + \frac{m(0)^2}{H^2} a_R^2(\tau) + g\frac{T_I^2}{H^2} a_R^{2(1-b)}(\tau) - \tfrac{1}{2} a_R^{-1}(\tau)}\ , \tag{14}$$

radiation $\omega(\tau)$

$$= \sqrt{k^2 + \frac{m(0)^2}{H^2} a_\gamma^2(\tau) + g\frac{T_\gamma^2}{H^2} a_\gamma^2(\tau_\gamma)}\ . \tag{15}$$

It is important to note that the frequency changes from imaginary to real values, at a certain time τ_c during reheating.

We match the solutions to the field equation in the different epochs at the transition between them, i.e. at the end of inflation and at the end of reheating. At both times we demand continuity of the corresponding modes and their first time derivatives. Care must be taken to match the WKB solutions through τ_c, where $\omega(\tau_c) = 0$. We obtain

$$\phi_k(\tau) = \alpha_k \phi_{k\gamma}^{+}(\tau) + \beta_k \phi_{k\gamma}^{-}(\tau), \tag{16}$$

where by $\phi_{k\gamma}$ we denote the modes during radiation and with

$$\alpha_k = -\frac{e^{i(k-\pi/4)}}{2^{3/4}} \frac{1}{k^{3/2}} \left\{ i\frac{17+2\sqrt{2}}{8} \times \exp\left(\left[\int_0^{\tau_c} |\omega|\, d\tau'\right]\right) + \left(\frac{17-2\sqrt{2}}{16}\right) \times \exp\left(\left[-\int_0^{\tau_c} |\omega|\, d\tau'\right]\right)\right\} \simeq \frac{\mathcal{O}(1)}{k^{3/2}}, \tag{17}$$

$$\beta_k = \frac{e^{i(k+\pi/4)}}{2^{3/4}} \frac{1}{k^{3/2}} \left\{ -i\frac{17+2\sqrt{2}}{8} \times \exp\left(\left[\int_0^{\tau_c} |\omega|\, d\tau'\right]\right) + \left(\frac{17-2\sqrt{2}}{16}\right) \times \exp\left(\left[-\int_0^{\tau_c} |\omega|\, d\tau'\right]\right)\right\} \simeq \frac{\mathcal{O}(1)}{k^{3/2}}. \tag{18}$$

In the above, we are ignoring the effects produced by the change of the effective Hubble constant during the inflationary period, which results in a change of the Bogoliubov coefficients in the far ultraviolet [26]. These effects, however, are small in the range of wavelengths relevant for the analysis of the generation of magnetic fields, $k \lesssim k_{\rm tod}$, where $k_{\rm tod}$ is the comoving wave number of the relevant astrophysical scale under study (see below).

In order to proceed with our phenomenological analysis, the values of T_I, T_γ and H in the previous expressions must be specified. They can be related by the age of the Universe, which can be well approximated by the duration of the matter-dominated epoch. This is given by

$$t_{\rm tod} \simeq \frac{2}{3H}\left(\frac{T_I}{T_\gamma}\right)^{3/2b}\left(\frac{T_\gamma}{T_M}\right)^2\left[\left(\frac{T_M}{T_{\rm tod}}\right)^{3/2} - 1\right], \tag{19}$$

where $T_{\rm tod} \simeq 10^{-13}$ GeV is the present temperature of the Universe, $T_M \simeq 1$ eV is its temperature at the beginning of the matter dominated epoch and for the Hubble constant during inflation, we shall assume that 10^{11} GeV $\leq H \leq 10^{13}$ GeV. From Eq. (19) we obtain

$$T_I \simeq T_\gamma^{(3-4b)/3}\left[\frac{3H}{2} T_M^{1/2} T_{\rm tod}^{3/2} \times t_{\rm tod}\right]^{2b/3} \simeq T_\gamma \left[\frac{HM_{\rm Pl}}{T_\gamma^2}\right]^{2b/3}, \tag{20}$$

where the last equality stems from $t_{\rm tod} \simeq M_{pl}/T_{\rm tod}^{3/2} T_M^{1/2}$ [23]. For $H = 10^{11}$ GeV we have

$$T_I \simeq T_\gamma \left(\frac{10^{15}\ \text{GeV}}{T_\gamma}\right)^{4b/3}. \tag{21}$$

Therefore, independently of the value of b and for the values of the cosmological parameters considered above, the relation $T_I > T_\gamma$ is fulfilled for any value of $T_\gamma < 10^{15}$ GeV.

In order to compute the magnetic field, we consider the Maxwell equation

$$\left[\frac{\partial^2}{\partial\tau^2} - \nabla^2 + \sigma(\tau)\frac{\partial}{\partial\tau}\right]\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{j}, \tag{22}$$

where $\sigma(\tau)$ is the time-dependent conductivity of the medium and $\boldsymbol{j}$ is the electric current generated by the charged scalar particles. Although $\langle \boldsymbol{j}\rangle = 0$, the two-point correlation function is different from zero and produces a non-vanishing magnetic field. This field can be expressed in terms of the two-point function of the scalar field as (see Ref. [13] for details)

$$\begin{aligned}\langle B_\lambda^2\rangle = e^2H^4\int d\tau d\tau'\int \frac{d\boldsymbol{k}d\boldsymbol{k}'}{(2\pi)^3}\mathscr{W}^2_{kk'}(\lambda)|\boldsymbol{k}\times\boldsymbol{k}'|^2 \\ \times G^{\rm ret}_{|k+k'|}(\tau_o,\tau)G^{\rm ret}_{|k+k'|}(\tau_o,\tau')\big[4G^0_{1k}(\tau,\tau') \\ \times\delta G_{1k'}(\tau,\tau') + \delta G_{1k}(\tau,\tau')\delta G_{1k'}(\tau,\tau')\big],\end{aligned} \tag{23}$$

where

$$G^0_{1k}(\tau,\tau') = \frac{\cos\Omega_k(\tau,\tau')}{\sqrt{\omega(\tau)\,\omega(\tau')}}, \tag{24}$$

$$\begin{aligned}\delta G_{1k}(\tau,\tau') &= 2\,\alpha_k\,\beta_k^*\phi^+_{k\gamma}(\tau)\,\phi^+_{k\gamma}(\tau') \\ &\quad + 2\,\alpha_k^*\beta_k\,\phi^-_{k\gamma}(\tau)\,\phi^-_{k\gamma}(\tau') \\ &\quad + 2|\beta_k|^2G^0_{1k}(\tau,\tau'),\end{aligned} \tag{25}$$

with $\Omega(\tau) = \int^\tau d\tau'\omega(\tau')$, $\phi^+_{k\gamma}$ $(\phi^-_{k\gamma})$ the positive (negative) frequency modes of the scalar field during radiation dominance and with $G^{\rm ret}_{|k+k'|}(\tau_o,\tau)$ the retarded propagator for the electromagnetic field. $\mathscr{W}(\lambda)_{kk'}$ is the window function that filters scales smaller than λ. Eq. (23) then gives the magnetic energy of a field which is homogeneous over volumes of order λ^3, the intensity of the field therefore being estimated as $\sqrt{\langle B_\lambda^2\rangle}$. From now on it will be understood $\lambda = k_{\rm tod}^{-1}$. where, as mentioned above, $k_{\rm tod}$ is the comoving wavenumber of the astrophysical scale we are interested in.

Real particle propagation can be considered as such from the moment when the frequency becomes real, i.e. from τ_c. To evaluate Eq. (23) we shall proceed in the same way as in [13], and consider only the main contribution, which originates from the last term between brackets, which is quartic in the Bogoliubov coefficients and, within this term, from the non-oscillatory contributions. We perform the k integration with the same window function used in [13], i.e. a top-hat one. We propagate the magnetic field during reheating and radiation dominance until the moment of detection with the propagator given by the equation $[\partial^2/\partial\tau^2 + k^2 + \sigma(\tau)\partial/\partial\tau\,]G^{\rm ret}_{|k+k'|}(\tau_o,\tau) = \delta(\tau_o-\tau)$, where $\sigma(\tau)$ is the electric conductivity of the Universe. After the particles decay, the field propagates conformally. We assume that during all these periods the conductivity of the Universe is given by [27]

$$\sigma(\tau) \simeq \frac{e^{-2}T}{H} = \frac{\sigma_0}{(\tau-\tau_*)^\alpha}, \tag{26}$$

where $\alpha = 2b$, $\tau_* = -2$ and $\sigma_0 = T_I/e^2H$ for reheating, and $\alpha = 1$, $\tau_* = -2 + (T_I/T_\gamma)^{1/2b}$ and $\sigma_0 = T_I^{1/2b}T_\gamma^{1-1/2b}/e^2H$ for radiation dominance.

For $\tau_o \gg \tau$, we have

$$G^{\rm ret}_{|k+k'|}(\tau_o,\tau) \simeq -\frac{(\tau-\tau_*)^\alpha}{\sigma_0}. \tag{27}$$

The Bogoliubov coefficients are the ones given in Eqs. (17) and (18). Now we are ready to evaluate the time integrals in Eq. (23). It can be checked that the contribution from reheating is negligible with respect to the one from the radiation period. Also, the mass term dominates over the thermal correction for a particle lifetime $t_{\max} > 10^{-14}$ s. We therefore consider the time integral

$$\begin{aligned}\int_{\tau_c}^{\tau_{\max}} d\tau'\frac{G^{\rm ret}(\tau',\tau_o)}{\omega(\tau')} \simeq -\frac{2e^2H^{5/2}}{T_\gamma m(0)}\left(\frac{T_\gamma}{T_I}\right)^{5/4b} \\ \times\left[t_{\max} + \frac{1}{2H}\left(\frac{T_I}{T_\gamma}\right)^{3/2b}\right]^{1/2}.\end{aligned} \tag{28}$$

Now we are ready to evaluate the magnetic field. For this purpose it is convenient to express the comoving wave number in terms of the present one as $k_{\rm tod} =$

$\kappa_{\rm tod} T_\gamma (T_I/T_\gamma)^{1/b}/HT_{\rm tod}$. Replacing everything in Eq. (23) we obtain

$$\langle B_\lambda^2 \rangle \simeq \frac{e^6 H^5}{m(0)^2 T_\gamma^2} \kappa_{\rm tod}^4 \left(\frac{T_\gamma}{T_{\rm tod}} \right)^4 \left(\frac{T_I}{T_\gamma} \right)^{7/4b} t_{\max}. \quad (29)$$

Eq. (29) gives the intensity of the field at the moment when the electric current vanishes. After that, the field propagates as $B_\lambda^{\rm phys}(t) = \sqrt{\langle B_\lambda^2 \rangle}\, a(t_{\max})^2 / a(t)^2$ (i.e. magnetic flux conservation). Using the relation $a(t) \propto 1/T$, the present value of the magnetic field is given by

$$\langle B_\lambda^{\rm phys} \rangle \simeq \sqrt{\langle B_\lambda^2 \rangle} \left(\frac{T_{\rm tod}}{T_{\max}} \right)^2, \quad (30)$$

where $T_{\max}$ is the temperature of the Universe when the particles decay, given by $T_{\max} = T_\gamma a_\gamma(\tau_\gamma)/a_\gamma(\tau_{\max}) = T_I^{3/4b} T_\gamma^{1-3/4b}/\sqrt{2Ht_{\max}}$. Replacing everything in Eq. (30) we obtain

$$\langle B_\lambda^{\rm phys} \rangle \simeq \frac{e^3 H^{7/2} \kappa_{\rm tod}^2}{m(0) T_\gamma} \left(\frac{T_I}{T_\gamma} \right)^{-5/8b} t_{\max}^{3/2}. \quad (31)$$

In the above, we have only considered the effect induced by the scalar particle and not by the charged particles resulting from its decay. This might seem surprising since, due to charge current conservation, the charged particles coming from the scalar particle decay might also contribute in a relevant way to the magnetic field generation. However, the decay of a massive scalar particle, like the stau in the case under study, will lead mostly to charged fermions (tau leptons, in this case) with wavelengths much shorter than the ones of the original scalar field. These fermions might eventually generate magnetic fields, but, due to the wavelengths involved, these fields will not be coherent in the scales of interest for our study.

In order to apply the above formalism to the case of gauge mediated supersymmetry-breaking models, we should recall Eq. (5), which gives the lifetime of the NLSP, $\tau_{\tilde\tau} \equiv t_{\max}$ as a function of the supersymmetry-breaking scale and the mass of the lightest stau. Replacing Eqs. (20) and (5) into Eq. (31), we obtain

$$\langle B_\lambda^{\rm phys} \rangle \simeq \frac{e^3 H^{7/2} \kappa_{\rm tod}^2}{T_\gamma^{1/6} [HM_{\rm Pl}]^{5/12} \times 100\,{\rm GeV}} \times \left[\frac{1}{\tilde k^2} \left(\frac{100\,{\rm GeV}}{m(0)} \right)^{17/3} \left(\frac{\sqrt{F}}{100\,{\rm TeV}} \right)^4 \times 3 \times 10^{11}\,{\rm GeV}^{-1} \right]^{3/2}. \quad (32)$$

We see that the b-dependence has disappeared, i.e. the result does not depend on the details of the reheating period. Using the equivalence $1\,{\rm GeV}^2 \simeq 10^{20}$ Gauss and the numerical estimates $H \simeq 10^{11}$ GeV, $T_\gamma \simeq 10^7$ GeV, $\kappa_{\rm tod} \simeq 10^{-38}$ GeV (for a galactic scale of the order of 1 Mpc), $m(0) \simeq 100$ GeV and $\sqrt{F/\tilde k} = 10^6$ GeV, we obtain

$$\langle B_\lambda^{\rm phys} \rangle \simeq 10^{-12}\,{\rm Gauss}. \quad (33)$$

This value of the generated magnetic field is sufficient to seed the galactic dynamo, being also consistent with the bounds imposed by the anisotropies in the CMBR and by primordial nucleosynthesis [28,29].

In the above we have given results for specific values of $\sqrt{F}$, H and T_γ, for minimal coupling and for $C_H = 0$, that is for $\nu = 3/2$. It is interesting to discuss the dependence on $\sqrt{F}$, C_H, as well as on departures from minimal coupling. In this case it can be checked that for small $k_{\rm tod}$, the Bogoliubov coefficients are given by $\alpha_k \sim \beta_k \sim \mathscr{O}(1) k^{-\nu}$, with ν given by Eq. (12). Considering a stau lifetime $\tau_{\tilde\tau} \equiv t_{\max} = 10^n\ {\rm GeV}^{-1}$ and a stau mass $m_{\tilde\tau} \simeq 100$ GeV, the value of the physical magnetic field is given by

$$\langle B_\lambda^{\rm phys} \rangle = 10^{(118/3)(\nu - 3/2) - 106/3 + 3n/2} \times {\rm Gauss} \left(\frac{10^7\,{\rm GeV}}{T_\gamma} \right)^{(7-4\nu)/6} \times \left(\frac{H}{10^{11}\,{\rm GeV}} \right)^{(25+8\nu)/12} \times \left(\frac{\kappa_{\rm tod}}{10^{-38}\,{\rm GeV}} \right)^{(5-2\nu)}. \quad (34)$$

Acceptable values of the magnetic field, consistent with the cosmological bounds [28,29] are in the range

$$10^{-9}\ \text{Gauss} \geq B_\lambda^{\text{phys}} \geq 10^{-21}\ \text{Gauss}. \tag{35}$$

For the case $\nu = 3/2$, and for the mean values of the parameters taken above, this implies $158/9 \gtrsim n \gtrsim 86/9$, or, equivalently,

$$10^{-14}\ \text{s} \lesssim \tau_{\tilde{\tau}} \lesssim 10^{-7}\ \text{s}. \tag{36}$$

The variation of the above bounds with the cosmological parameters can be easily obtained from Eq. (34).

On the other hand, for arbitrary values of ν, the following bound is obtained:

$$\tfrac{3}{2} + \tfrac{79}{118} - \tfrac{9}{236} n \geq \nu \geq \tfrac{3}{2} + \tfrac{43}{118} - \tfrac{9}{236} n. \tag{37}$$

In gauge-mediated supersymmetry-breaking models, for a stau mass $m_{\tilde{\tau}} \simeq 100$ GeV and values of the gravitino mass $m_{\tilde{G}} \lesssim 1$ keV, the stau lifetime, Eq. (5), is such that $11 \lesssim n \lesssim 16$, or equivalently

$$10^{-13}\ \text{s} \lesssim \tau_{\tilde{\tau}} \lesssim 10^{-8}\ \text{s} \quad \text{GMSB for } m_{\tilde{G}} \lesssim 1\,\text{keV}. \tag{38}$$

For a mimimal stau coupling to the curvature and $C_H \ll 1$, that is for $\nu = 3/2$, Eq. (38) is in remarkable agreement with the values required to generate an acceptable magnetic field, Eq. (36).

The bounds on n in gauge mediated supersymmetry breaking models also imply bounds on ν

$$1.72 \gtrsim \nu \gtrsim 1.25. \tag{39}$$

Comparing this expression with the value of ν for minimal coupling of the scalar field, $\nu \simeq 3/2\sqrt{1 - 4C_H/9}$, we obtain that $C_H < 0.68$ in order to generate cosmologically relevant values of the magnetic field. As follows from Eq. (34), only small modifications of the bound on C_H may be obtained for different values of the cosmological parameters.

Consider now the departure from minimal coupling. Assuming that $C_H \ll 1$ we have $\nu \simeq 3/2\sqrt{1 - 16\xi/3}$. The bounds on the lifetime of the stau are satisfied for

$$0 \leq \xi \lesssim 0.06; \tag{40}$$

we thus obtained for a non-negligible interval of coupling values, magnetic fields of an intensity sufficient for these to be cosmologically important. The upper bounds on C_H and ξ quoted above can only be obtained for values of $\sqrt{F}$ (or equivalently n) such that the gravitino mass $m_{\tilde{G}}$ is close to 1 keV.

The results given above were obtained for a reheat temperature $T_\gamma \simeq 10^7$ GeV. As we emphasized above, for the range of gravitino masses we are concentrating on, the most relevant bound on the reheat temperature comes from Eq. (20), which assures the consistency of the whole approach. Larger values of the magnetic fields may be obtained by lowering the value of the reheat temperature. However, the final result for the magnetic field, Eq. (32), depends very weakly on the value of the reheat temperature T_γ. No relevant departures from the obtained values would be obtained even if the reheat temperature were as low as $T_\gamma \simeq 10^3$ GeV.

In summary, we have shown that cosmologically relevant magnetic fields may be generated by a scalar field, minimally coupled to the curvature, so far its lifetime is bounded by Eq. (36). The bounds on the lifetime are in excellent agreement with those obtained in minimal gauge mediated supersymmetry-breaking models with the lightest stau as the next-to-lightest supersymmetric particle, for values of the supersymmetry-breaking scale such that $m_{\tilde{G}} \lesssim 1$ keV. This conclusion is very weakly dependent on the assumed values of the cosmological parameters. Moreover, contrary to many models for magnetic field generation proposed in the literature, the present one is related to the properties of the low energy effective theory and these properties can be tested in accelerator experiments in the near future [30].

Acknowledgements

We thank A. Riotto, M. Giovannini, M. Shaposhnikov, G. Chalmers, E. Poppitz and J. Russo for useful discussions. C.W. would also like to thank the Aspen Center for Physics where part of this work has been done. C.W. was supported in part by the US Department of Energy, Division of High Energy Physics, under Contract W-31-109-ENG-38. A.K., E.A.C. and F.D.M. by Conicet, University of Buenos Aires and Fundación Antorchas.

References

[1] Ya. B. Zel'dovich, A. Ruzmaikin, D. Sokoloff, Magnetic Fields in Astrophysics, Gordon and Breach, New York, 1983.
[2] P.P. Kronberg, J.J. Perry, E.L. Zukowski, Ap. J. 387 (1992) 528.
[3] A.M. Wolfe, K. Lanzetta, A.L. Oren, Ap. J. 388 (1992) 17.
[4] A.L. Oren, A.M. Wolfe, Ap. J. 445 (1995) 624.
[5] M.S. Turner, L.M. Widrow, Phys. Rev. D 37 (1988) 2743.
[6] A.D. Dolgov, Phys. Rev. D 48 (1993) 2499.
[7] B. Ratra, Ap. J. 391 (1992) L1, and preprint GRP-287/CALT-68-1751 (1991).
[8] M. Gasperini, M. Giovannini, G. Veneziano, Phys. Rev. Lett. 75 (1995) 3796.
[9] F.D. Spedalieri, F.M. Mazzitelli, Phys. Rev. D 52 (1995) 6694.
[10] B. Cheng, A.V. Olinto, Phys. Rev. D 50 (1994) 2421; G. Sigl, A. Olinto, K. Jedamzik, Phys. Rev. D 55 (1997) 4582.
[11] K. Enqvist, P. Olesen, Phys. Lett B 329 (1994) 195; T. Vachaspati, Phys. Lett B 265 (1991) 258; A. Dolgov, J. Silk, Phys. Rev. D 47 (1993) 3144; C. Hogan, Phys. Rev. Lett. 51 (1983) 1488; A.P. Martin, A.C. Davies, Phys. Lett B 360 (1995) 71.
[12] M. Giovannini, M. Shaposhnikov, Phys. Rev. Lett. 80 (1998) 22.
[13] E.A. Calzetta, A. Kandus, F.D. Mazzitelli, Phys. Rev. D 57 (1998) 7139.
[14] B. Allen, Phys. Rev. D 32 (1985) 3136.
[15] M. Dine, W. Fischler, M. Srednicki, Nucl. Phys. B 189 (1981) 575; S. Dimopoulos, S. Raby, Nucl. Phys. B 192 (1981) 353; M. Dine, W. Fischler, Phys. Lett. B 110 (1982) 227; M. Dine, M. Srednicki, Nucl. Phys. B 202 (1982) 238; M. Dine, W. Fischler, Nucl. Phys. B 204 (1982) 346; L. Alvarez-Gaumé, M. Claudson, M. Wise, Nucl. Phys. B 207 (1982) 96; C.R. Nappi, B.A. Ovrut, Phys. Lett. B 113 (1982) 175; S. Dimopoulos, S. Raby, Nucl. Phys. B 219 (1983) 479.
[16] M. Dine, A.E. Nelson, Phys. Rev. D 48 (1993) 1277; M. Dine, A.E. Nelson, Y. Shirman, Phys. Rev. D 51 (1995) 1362; M. Dine, A.E. Nelson, Y. Nir, Y. Shirman, Phys. Rev. D 53 (1996) 2658.
[17] C.E.M. Wagner, Nucl. Phys. B528 (1998) 3.
[18] M. Carena, S. Pokorski, C.E.M. Wagner, Phys. Lett. B430 (1998) 281.
[19] See, for instance, S. Dimopoulos, M. Dine, S. Raby, S. Thomas, Phys. Rev. Lett. 76 (1996) 3494; S. Ambrosanio, G.L. Kane, G.D. Kribs, S.P. Martin, S. Mrenna, Phys. Rev. D54 (1996) 5395.
[20] T. Gherghetta, G.F. Giudice, A. Riotto, Phys. Lett. B446 (1999) 28.
[21] G.F. Giudice, I. Tkachev, A. Riotto, JHEP 9908:009 (1999), hep-ph/9907510.
[22] L. Kofman, A. Linde, A.A. Starobinsky, Phys. Rev. D 56 (1997) 3258; I.I. Tkachev, Phys. Lett. B 376 (1996) 35; S. Khlebnikov, I.I. Tkachev, Phys. Rev. Lett. 77 (1996) 219.
[23] E.W. Kolb, M.S. Turner, The Early Universe, (Addison-Wesley Publ. Co., New York, 1990).
[24] D.J.H. Chung, E.W. Kolb, A. Riotto, Phys. Rev. D 60 (1999): 063504.
[25] M.K. Gaillard, H. Murayama, K.A. Olive, Phys. Lett. B355 (1995) 71.
[26] V. Kuzmin, I. Tkachev, Phys. Rev. D59:123006 (1999).
[27] B. Cheng, A. Olinto, Phys. Rev. D 50 (1994) 2421; G. Sigl, A. Olinto, K. Jedamzik, Phys. Rev. D 55 (1997) 4582; J. Ahonen, Phys. Lett B382 (1996) 40.
[28] J.D. Barrow, P.G. Ferreira, J. Silk, Phys. Rev. Lett. 78 (1997) 3610; K. Subramanian, J.D. Barrow, Phys. Rev. Lett 81 (1998) 3575; J. Adams, U.H. Danielsson, D. Grasso, H. Rubinstein, Phys. Lett. B 388 (1996) 253.
[29] B. Cheng, D.N. Schramm, J.W. Truran, Phys. Rev. D 49 (1994) 5006; A.V. Olinto, D.N. Schramm, J.W. Truran, Phys. Rev. D 54 (1996) 4714; D. Grasso, H. Rubinstein, Phys. Lett B 379 (1996) 73; P.J. Kernan, G.D. Starkman, T. Vachaspati, Phys. Rev. D 54 (1996) 7207.
[30] See, for example, Aleph Collaboration, Phys. Lett. B433 (1998) 1; Delphi Collaboration, E. Phys. J. C7 (1999) 595; G. Wolf, talk presented at SUSY99, http://fnth37.fnal.gov/program4.html.