Movement of biological systems on different environments

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(Received February 1978; revised October 1978)

This paper describes the movement of biological systems on different kinds of environments, which was simulated with the aid of a digital computer plotter. The biological system is represented by two areas A_a and A_d, called the feeding and discard areas respectively, and separated by a constant distance d. Each triplet (A_a, A_d, d) determines the position of M with respect to E, and the consecutive positions determine the movement of the biological system. The possible movements of the biological system vary according to the areas, the spaces to be covered, the rules that govern the succession of states, etc. The plotter makes it possible to visualize these motions, so that many different interactions between M and E may be studied. It also becomes possible to determine other important parameters, such as the general direction taken by the system M, the width of its path, the distance between successive states, the possibility of environmental regeneration, etc. Therefore, this line of work contributes not only to theoretical biomathematics, but also to a wide variety of practical applications.

Introduction

The mathematical theoretical background to this work is given elsewhere[1,2] where an environmental theory is formulated, and where the consequences of environmental stability[3] are determined for a biological system defined as being environmentally static.[1]

In the present work, some of the previous mathematical developments are shown, and then, from the method provided by the plotter itself, new elements for the analysis of biological and environmental systems are found.

The new analysis criteria may be applied to an interesting array of widely differing problems: tissue growth, the spreading of plagues, feeding strategies, problems in areography, genesis of movement in animals, evolution of societies, etc.

We wish to mention that these applications of the digital computer plotter were inspired by the enlightening paper by Ulam.[4] As it is noted there, we would emphasize that there is an endless number of rules and possibilities, which may only be tested with the aid of a computer, making it possible to infer the real circumstances of the behaviour of the system being studied.

Theoretical background

One of the elements used to develop the environmental theory[1] is the so-called environmental unity. Each of these unities is defined by a cartesian product:

$$Y_i \times \prod_{j \in J} E_i^{y_j} \quad (J = 1, 2, \ldots, n-1)$$

where Y_i is a set representing a well defined quantity of matter in a definite physical state (material physical nature), and each $E_i^{y_j}$ is a one-point set which represents a given value of extrinsic energy linked to the material physical nature Y_i. This linkage is represented mathematically by means of the cartesian product operation shown above.

0307-904X/79/020125-05/$02.00

Energies are considered to be either extrinsic or intrinsic[1] (hence the qualification of 'extrinsic' given to the energy represented by $E_i^{y_j}$ above). The intrinsic energy is strictly equivalent to a given matter in a definite physical state (material physical nature) represented by the set Y_i; and, therefore, the intrinsic energy is mathematically undefinable. The extrinsic energy is not considered to be strictly equivalent to the material, physical nature but only inherent to it. This is due to the fact that in some cases certain kinds of energy may be associated with a given matter which, however, depend on the particular state of that matter. In fact, a given material physical nature may have many kinds of extrinsic energies associated with it. Moreover, the respective values of these extrinsic energies can vary. Also, the linkage between matter and these extrinsic energies may itself be of a labile nature.

In environmental theory,[1] different kinds of environmental unities are defined. One such environmental unity is given by the domain and range of transfers between the biological system M and the environment E. These unities, together with the concept of cardinality of:

$$Y_i \times \prod_{j \in J} E_i^{y_j}$$

have led to the idea that a biological system M may be identified by means of two areas A_a and A_d, and a distance 'd' between them. Thus, A_a and A_d are drawn by the plotter within a given region representing the environment E. The area A_a represents the transfers from E to M, and the area A_d represents the transfers from M to E; for this reason, they are called the 'feeding area' and 'discard area' respectively. Thus, each triplet (A_a, A_d, d) drawn by the plotter represents a certain position of M in the region that defines E. The respective values of each A_a and A_d correspond to the cardinality of the original unities of the form:

$$Y_i \times \prod_{j \in J} E_i^{y_j}$$

The criteria used to obtain the successive positions of M arise from the principle of adequate design,[5] which states that 'the design of an organism is such that the organism performs its necessary function adequately and with a minimum expenditure of energy and material, both in the performance of the function and in the construction of the organism'. Therefore, we may infer that two successive positions of the triplet (A_a, A_d, d) are determined by minimizing the amount of energy required to carry out the movement and the amount of matter consumed in order to reproduce that very energy.

In special cases, or when particularly complex systems interact, additional factors may be involved that restrict the rules derived from the above principle.

Another theoretical element applicable to the present work is the environmentally static biological system criterion,[1] which may be expressed as: 'given the whole life of a biological system M, where each consecutive time period determines each system $M_1, M_2, \ldots, M_n$ respectively, and such that these systems are related with their respective environmental systems $E_1, E_2, \ldots, E_n$, then the biological system M will be environmentally static if each system E_j ($j \in I, I = 1, 2, \ldots, n$) has the unities defined by transfers from M_i to E_i for every $i \leqslant j, i \in I$, and E_j has the unities which must be used by transfers from E_k to M_k, for every $k \geqslant j, k \in I$'.

Thus, with the criteria mentioned above, starting from an initial triplet (A_a, A_d, d) representing M, the plotter may draw the subsequent positions of M in the region E representing the environment. These positions are drawn following a general strategy which is given not only by the movement law, but also by the values of A_a, A_d and d, their relative values, their possible variability, the general shape of the limit of E, the starting position of M with respect to this limit, and many other factors depending on the particular problem under study.

In this way, the number of triplets (A_a, A_d, d) will determine the temporal magnitude of the staticity of M, and also enable us to study the strategy to follow in order that M may become infinitely static in the region defined by E.

Method

The biological system is drawn by the plotter as two areas A_a (feeding area) and A_d (discard area), with their respective centres joined by a line representing the distance d. For example, *Figure 1* shows the movement of a biological system defined by a feeding radius R_a = 1 cm, a discard radius R_d = 0.5 cm and a distance d = 3 cm.

The environmental system is enclosed by a limit which can have different configurations. In *Figure 1*, the limit is a circumference of radius R_E = 16 cm. The movement of M within E is obtained by applying the principle of adequate design, considering that the successive positions where the system is placed are determined by expending a minimal amount of energy between them.

Although the examples do not include the concept of matter, it can be introduced, for instance, by varying the areas A_a and A_d according to the distances covered between two previous successive states of M.

It should be noted that the area A_a is a measure of the quality of the environment E for the given biological system M. For example, if A_a represents food for M, we can make the areas A_a smaller in regions of the environment where the food concentration is greater. In fact, the triplet (A_a, A_d, d) is considered to remain in each position for constant time periods Δt. Thus, as the region representing E is finite, the number of times that A_a can be inscribed in the region adopted for E will also be finite, unless a criterion for the repair of E is adopted, as in the case shown in *Figures 4–7*.

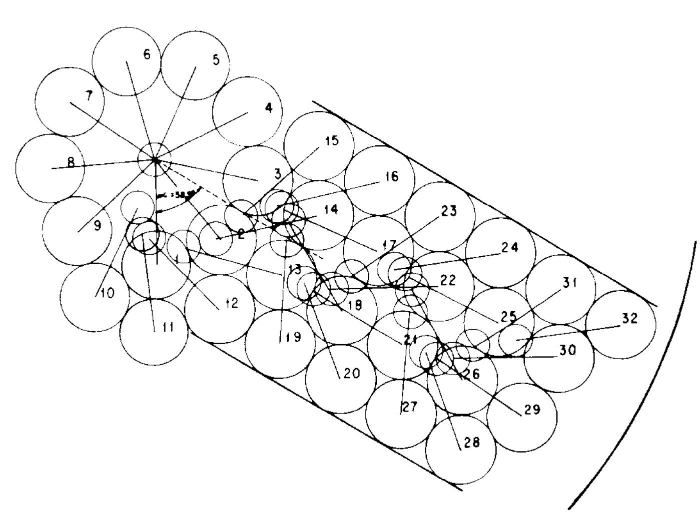

Figure 1 System E; R_E = 16 cm. System M; (R_a = 1 cm, R_d = 0.5 cm; d = 3 cm). Δ = 7.65 cm (constant); α = 58.5°; S = 3, 5, 7, 12, 5 mm

As for the discard area A_d, it may be taken to represent the action of M on all E.

Another matter for consideration are the surfaces occupied by A_a and A_d. In certain cases, depending on the problem, the surface occupied by A_d may not be used for placing A_a in subsequent times (e.g. when A_d represents contaminating products). Again, in other cases, the surface occupied by A_d may yield food of better quality or greater concentration so that for the subsequent states of M, the area A_a may be placed on a previous A_d, and the relative value of A_a may change in accordance with the improvement in the quality of the food.

Partial overlapping of A_a areas on previous A_d areas is also possible.

As we do not deal with any particular problem in this paper, the only criterion we have used to determine the movement of M in E is the principle of adequate design. Therefore, the following rule has been adopted for successive positions of the triplet (A_a, A_d, d): R1: 'The inscription of every new state i of M, (A_{a_i}, A_{d_i}, d_i), is such that A_{a_i} does not overlap other previous areas $A_{a_{i-j}}$ and $A_{d_{i-j}}$ ($j = 1, 2, \ldots, m$), and A_{a_i} is inscribed immediately beside $A_{a_{i-1}}$ so that the distance between the centres of A_{di-1} and A_{d_i} is minimal'.

Analysis of the examples

Figure 1 illustrates many of the technical conditions previously pointed out. In this, as in other figures, the areas A_a are identified by a number written inside which refers to the state of M. A straight line joining the centres of A_a and A_d determines d, and the triplet thus obtained represents M in the corresponding state.

Comparing two successive states, it is possible to see how the rule R1 works. For example, in *Figure 1*, A_{a_2} is drawn beside A_{a_1} and the distance from A_{d_1} to A_{d_2} is null because the two areas coincide. In certain cases, the new area is positioned at random by means of an instruction in the simple computer program that was used, because there are two or more positions for which the rule R1 is valid. This is the case of A_{a_2}, which could have been drawn to the left of A_{a_1}.

Continuing with the analysis of *Figure 1*, when M arrives at its state number 10, it is possible to recognize the first motion of the discard area ($A_{d_{10}}$) from the place where the previous ones were drawn.

In the 16th state, a definite orientation of M may be observed, so that the width of the path followed by M, Δ, the orientation angle with respect to the original starting position, α, and the sequence of final distances S between successive A_{d_i} can be obtained from the drawing. (In this case, the values of S between states 20, 21, 22, 23 and 24 are 3, 5, 7 and 12.5 mm, and these values are repeated for other subsequent distances between states).

Figure 2 shows the same system, but the distance d has now been increased to 5 cm. It is possible to see the changes in Δ, α and the different distances between successive areas A_d when the movement became uniform (e.g. from 30 to 37).

The initial configurations of the figures, with 'feeding areas' turning about the position of successive 'discard areas' (see, for instance, states 1 to 9 in *Figure 1*), are not always relevant to the general process of M–E interaction. Regarding the latter, other parameters may be obtained from the drawings besides Δ, α and S, by applying the present method. To mention some, it may be interesting to consider the distance between any two feeding areas (compare the distances between states 1 and 32 in *Figures 1* and *2*), the sequential movement of the discard areas (which is approximately regular), or the number of states before the environmental limit is reached (32 in the case of *Figure 1* with the environmental radius R_E = 16 cm and 37 in the case of *Figure 2* with R_E = 12.5 cm).

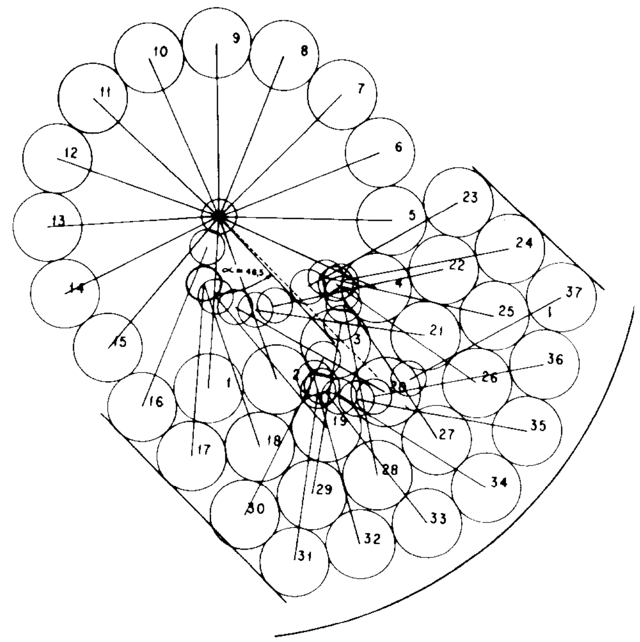

Figure 2 System E; R_B = 12.5 cm. System M; R_0 = 1 cm; R_d = 0.5 cm; d = 5 cm. $\Delta \simeq 13$ cm; $\alpha \simeq 48.5°$

Figure 3 illustrates the theoretical objective pursued in earlier developments.[2] There, the system M moves according to the principle of adequate design, within the entire region E, inscribing a total of 36 states, when M is a system that is defined as environmentally static. There M is determined by a transfer from E to M with R_a = 1 cm. (R_a = radius of feeding area), a transfer from M to E with R_d = 0.5 cm (R_d = radius of discard area) and the distance between the two areas, d = 2 cm.

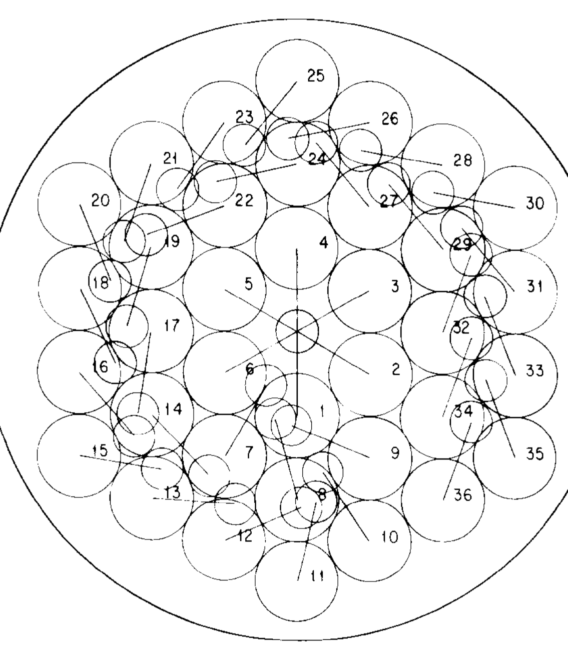

Figure 3 System E; R_B = 7.5 cm. System M; R_a = 1 cm; R_d = 0.5 cm; d = 2 cm

The staticity of M is lost for the states after 36, when the system goes beyond the environmental limit.

The same system is analysed introducing an interesting variation given by 'environmental regeneration' in the sectors occupied by areas $A_{a_{i-13}}$ and $A_{d_{i-13}}$ (i is the last state in the sequence). *Figures 4*, *5* and *6* show the regeneration of the environment in the places where previous states were inscribed, and how the system orients itself differently compared with *Figure 3*. In *Figure 7*, the movement of the system is shown up to its 52nd state, which is not, however, the last state for which M is environmentally static.

Discussion

This work gives rise to interesting comments from many points of view, because of the diversity of the disciplines to which it is relevant.

From the point of view of theoretical biomathematics, in which this work was originally inspired, it is a useful tool for the study of feeding area regeneration strategies, so that the system remains enclosed within the geographical limits of its original environment. This would involve the relative variation of A_a, A_d and d, the rules for erasing A_a and A_d, the environmental limit, the position of M's initial state with respect to the limit, etc. In this line of analysis, it is interesting to see how the principle of adequate design may be applied in order to explain the tendency of many different biological systems to turn in the same direction in a repeated sequence (e.g. in *Figure 2*, M turns, moving several times in the same direction from state 24 to state 29, and then to the other side from state 31 to state 36). This problem is found in areographical studies.

Many new and more complex analyses are now possible due to the visualization afforded by the use of the digital computer plotter. The drawings have led to new criteria

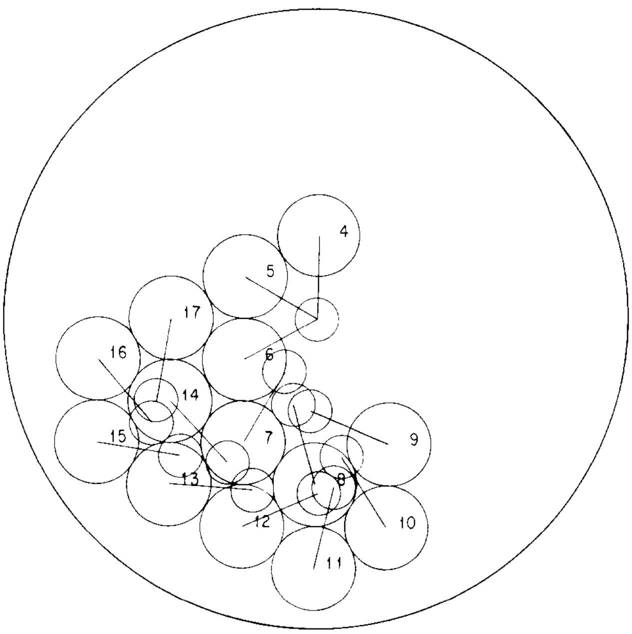

Figure 4 First 13th steps (same case as *Figure 3*)

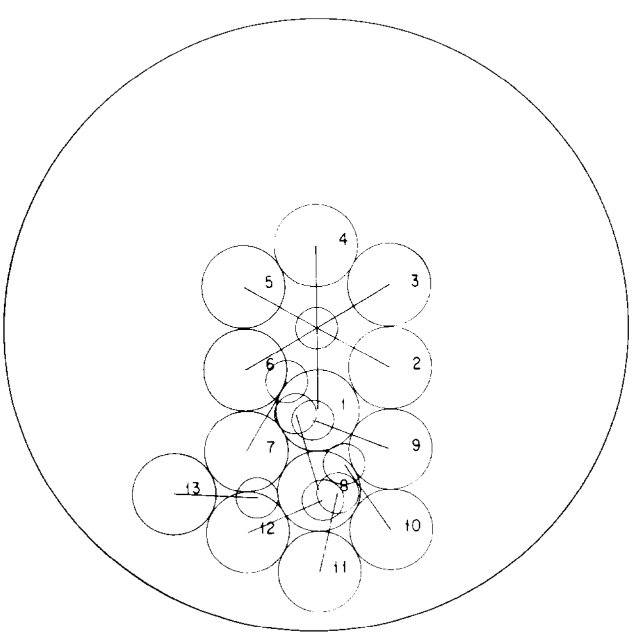

Figure 5 Steps previous to step 4 are erased (same case as *Figure 3*)

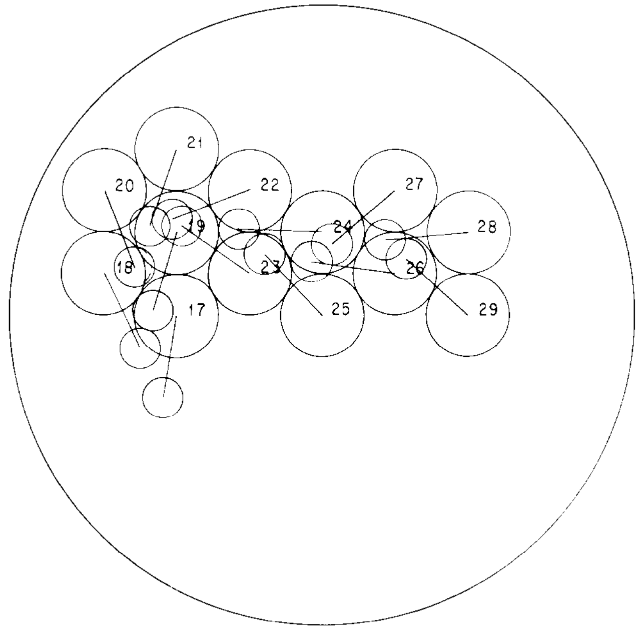

Figure 6 Steps previous to step 17 are erased (same case as *Figure 3*)

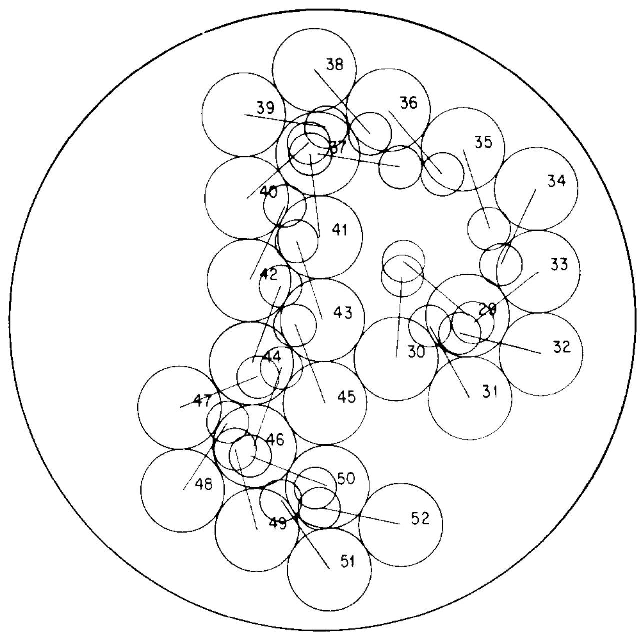

Figure 7 Movement of environmentally static M on a regenerative environment (same case as *Figures 4, 5* and *6*)

of analysis regarding, for example, the width of the path Δ the orientation angle α, the distance between a state and the initial state, and the sequence S between areas A_a. It is worth noting that S may or may not be related to the definitive movement of M. The 'learning of different and larger movements' due to environmental variations is also connected with this criterion for movement. (This is the case in *Figure 3*, where the limit of E provokes a change from state 13 in sequence S obtained between states 7 and 12.)

From the point of view of the biological applications, many practical cases may be analysed with the aid of the method described in this paper or another similar one (e.g. working only with areas A_a).

We may also mention the possibility of agricultural studies, by linking the parameters α and Δ with the areographic development of plagues in crops. The study of a possible strategy for maintaining M permanently within the bounds of E could be useful for cattle raising, with the variables given by the number of animals, kind and quantity of food, rainfall, etc.

On another biological level, we may also mention studies concerning the generation and destruction of cellular tissues.

Acknowledgement

This work is part of the research programme carried out under a joint agreement for research in biomathematics between the Comisión Nacional de Energía Atómica and the Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Argentina.

References

1 Leguizamón, C. A. *Bull. Math. Biol.*, 1975, 37, 675
2 Leguizamón, C. A. (To appear)
3 Leguizamón, C. A. (To appear)
4 Ulam, S. *Proc. Symp. Appl. Math.*, Providence, 14, 215, 1962
5 Rashevsky, N. In 'Foundations of Mathematical Biology', Vol. III (Ed. R. Rosen), 1973