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Crustal Shortening, Root Spreading, Isostasy, and the Growth of Orogenic Belts: A Dimensional Analysis

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Scaling laws that describe the growth of mountain belts and the change in their regional topographic profiles with time (t) are obtained by physical arguments that account for crustal shortening, isostasy, and creeping flow at the mountain roots. An average power law rheology characterized by an exponent n is assumed, and different values of n are considered. The model predicts the formation of a localized range characterized by a height (h) and a width (a). As crust on the sides of a range is compressed, the root broadens due to buoyancy-driven creep. This process in conjunction with isostasy causes h to increase with time as $t^{1/4}$-$t^{1/3}$, and a as $t^{3/4}$-$t^{2/3}$, as n is varied from 1 (Newtonian rheology) to ∞ (plastic rheology). Accordingly, the aspect ratio a/h increases as $t^{1/2}$-$t^{1/3}$. The scaling laws depend on the rheology, but rather weakly; the aspect ratio increases with age but is nearly independent of the rate of shortening. The deformation extends gradually inland with increasing time, so that the age of tectonic deformation decreases with distance from the continental border. Uplift takes place inland from the crest while subsidence occurs at the opposite side. The ratio a/h of the Andes and Tibet is consistent with this model.

Introduction

It is generally believed that large horizontal forces, such as those associated with subduction of oceanic lithosphere, or collision of two continental blocks, produce the folding and shortening of the crust that leads to the growth of mountain belts (e.g., *Dewey and Bird* [1970]). The characteristic time for isostatic adjustment is short compared to the time scale of orogeny as attested by measurements of isostatic rebound and by the fact that isostatic compensation prevails on a regional scale (e.g., *Jacobs et al.* [1974]). However, an isostatically compensated mountain range is not in hydrostatic equilibrium and will spread and collapse laterally unless restrained by appropriate stresses. Clear indications that spreading is actually taking place despite the fact that the crust on the sides of the range is under compression have been observed by *Dalmayrac and Molnar* [1981] and *Suarez et al.* [1983] in the Andes and by *Molnar and Tapponnier* [1978] in Tibet. A satisfactory model of range growth must explain this puzzling fact.

A detailed description of the growth of a range requires knowledge of the stresses acting on the lithosphere, its structure and rheology, and the boundary conditions. These data are not completely available. In addition, such a project would run into enormous observational as well as mathematical and computational difficulties. Clearly, models that take into account the basic physics and the most relevant parameters of the problem may be of great help in clarifying the issues, even if the simplification is achieved at the price of some drastic and rough approximations.

In this paper we develop physical arguments that take into account isostasy, shortening of the crust, and a very simple rheology, all of which allow us to derive scaling laws that describe the evolution of high mountain ranges and some of their main features. Similarity and dimensional methods (see, for example, *Sedov* [1959]) are chiefly used. By comparison with observational data it is possible to constrain the rheology of the upper mantle and the crustal roots. It is not our purpose to develop a fully realistic picture, but only physical insight into the main factors that govern the growth of a range. This is a necessary first step before attempting a more detailed description beyond the scope of the present paper.

Various physical models have been used recently to understand the buildup and evolution of mountain belts. The thin viscous sheet model of *England and McKenzie* [1982] has been used by *Houseman and England* [1986] to calculate the deformation of continental lithosphere produced by the collision with a rigid indenter, and the results have been compared with observations in the India-Asia collision zone [*England and Houseman*, 1986]; the calculations of *Cohen and Morgan* [1986] are also based on this model, which assumes a vertically averaged power law rheology like that of the present paper. *Vilotte et al.* [1982, 1986] considered a closely related model, also with a vertically averaged rheology (slightly more general as it includes a yield stress parameter); they also calculated the continental deformation due to an indenter, using a wider variety of boundary conditions than the other authors. The main difference between these and the present model is that here it is assumed that the velocity depends on the vertical coordinate, while all the thin sheet models ignore this dependence, so that viscous stresses can only arise due to the horizontal changes of the velocity; this approximation may be appropriate if only a small part L of the continental border is invested by the indenter as in the India-Asia collision, then L is the natural scale of the velocity gradients, but some doubts can arise when most of its length is involved (so that the scale L drops out of the problem). Another difference is that these authors perform numerical experiments, no attempt being made to derive approximate analytical scaling laws, as we do here. The bulldozer-driven prism model of *Davis et al.* [1983] implies a depth-dependent velocity as in the present paper, but as it aims primarily to describe the mechanics of fold-and-thrust belts and accretionary wedges Coulomb rheology is assumed, and an integral description of the dynamics of the whole growing range, including the inland slope, is not attempted.

Spreading and Collapse of an Isostatically Balanced Range

Let us first argue why the depth dependence of the velocity during the process of deformation may be an important factor in determining the topography of a range. To this purpose we shall begin by ascertaining if the lithospheric rocks are sufficiently strong to prevent collapse of a mountain range in isostatic balance. In the following we

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shall be concerned only with average properties on a regional scale rather than the detailed structure of the range (such as local topography, faults, etc.).

The regionally averaged profile of the crust of a range is shown schematically in Figure 1 assuming an Airy model for isostatic compensation; ρ denotes the density of the crust and $\rho+\rho'$ that of the supporting layer (mantle lithosphere); h is the height of the range and h' the depth of the crustal root (or equivalently, the depth of the trough in the mantle lithosphere). Isostasy requires that

$$\rho' h' = \rho h \quad (1)$$

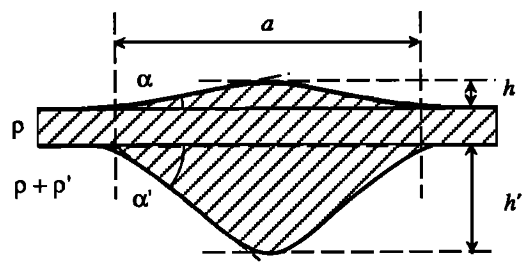

Fig. 1. Schematic cross section of a range in isostatic balance.

Let a denote the width of the range, so that $\alpha \approx 2h/a$ represents the typical value of its slope and $\alpha' \approx 2h'/a$ that of its root. A profile like Figure 1 is not in hydrostatic equilibrium since both the slopes of the range and that of its root will tend to collapse and spread laterally for nonzero α, α' unless the strength (deviatoric stress capacity) of the lithosphere acts to balance the forces due to gravity and hydrostatic pressure. By dimensional analysis one can estimate typical values of the shearing stress tending to produce the collapse of the slope of the range as

$$\tau \approx \rho g h \alpha \approx 2\rho g h^2/\alpha \quad (2)$$

(the local value of the stress increases linearly with depth, so that it depends on which point we are considering, but as the incipient failure involves a portion of the crust whose depth is of the order of h, the stresses involved will have a characteristic value given by (2)). In (2), g denotes the acceleration of gravity. Since failure will occur if τ exceeds the strength τ_y of the crust, the stability condition will be

$$\vartheta \geq 2\rho g h/\tau_y \quad (3)$$

where $\vartheta = a/h$ denotes the "aspect ratio" of the range. A similar argument can be applied to the root; the shearing stress that tends to produce its collapse will be typically

$$\tau' \approx \rho' g h' \alpha' \approx 2\rho' g h'^2/a \quad (4)$$

and to achieve stability τ' must not exceed the strength τ'_y of the medium (τ_y and τ'_y will be different, in general). Using equations (1) and (4) one obtains the condition:

$$\vartheta \geq 2\rho g h\gamma/\tau_y \qquad \gamma = \rho\tau_y/\rho'\tau'_y \quad (5)$$

Here it must be observed that it is not sufficient to overcome the strength of the crustal rocks to produce the collapse of the root, since it is necessary at the same time that a slumping of the mantle lithosphere (over which rests the crust) accompanies the sliding of crustal material: the slopes of the mantle lithosphere trough that contains and supports the root must also fail. In other words, a change in the shape of the root requires displacements in both media. Clearly, the stability of the shape of the interface between two media is determined by the stronger one. Then, the appropriate τ'_y in (5) is that of the stronger of both layers, which according to present knowledge is the mantle lithosphere (see, for example, *Ranalli and Murphy* [1987]). However, for the purpose of the present analysis it is irrelevant which layer is the stronger, provided one uses the appropriate τ'_y.

Which condition, (3) or (5), is the most restrictive will depend on the value of γ. If the belt formed by a thrust along the continental border, it seems reasonable that its slopes would be as steep as allowed by conditions (3) and (5), independent of the details of the buildup mechanics. This is what happens, for instance, when a bulldozer slowly pushes a mound of earth (in this connection see *Davis et al.* [1983]). It is easy to see that young belts are consistent with rather small values of τ_y and/or τ'_y, of the order of 0.1 kbar. For example, typical values for the Andes (h = 4.5 km, ϑ = 60-70) yield crustal strengths of τ_y = 35-40 bar, or τ'_y = 160-190 bar. These values are much smaller than observed strengths, which are in the kilobar range (see, for example, *Brace and Kohlstedt* [1980]). This discrepancy is expected since the behavior of rocks at time scales corresponding to the growth of a range must be very different from that corresponding to fast processes such as faulting. However, the extrapolation of experimentally determined properties of rocks to conditions that presumably prevail at the depth of mountain roots and to strain rates typical of orogeny [e.g., *Hobbs et al.*, 1975] indicates that the strength τ'_y required to explain observed profiles is roughly 1 order of magnitude too large.

Therefore it appears that static equilibrium at time scales typical of orogeny is implausible since too much strength is required in the roots, and plastic failure must occur there. There is ample evidence that plastic deformation is a key process in orogeny. For example, *Davis et al.* [1983] observe that Coulomb wedge theory is not valid toward the interior of very wide mountain ranges and accretionary wedges, when the thickness of the crust exceeds the depth of the brittle-plastic transition. Apparently, plastic flow rather than static strength is the main factor that determines the large-scale profile of mountain ranges.

This conclusion is consistent with the fact that large-scale profiles of ranges are generally in equilibrium with respect to fast processes such as faulting: the conditions (3) and (5) are fulfilled if τ_y and τ'_y correspond to short time scale behavior. This has no relevance to large-scale features such as the profile of the range. These are determined by the behavior on very long time scales.

Because lithospheric viscosity decreases with depth, flow will occur mainly in the vicinity of roots, which will lead to lateral spreading. As the root broadens and shallows due to lateral flow, the topography of the range will undergo a corresponding isostatic adjustment. Near the surface this readjustment may occur either by way of plastic deformation or by faulting.

Let us consider now that the roots are spreading laterally due to viscous flow and that viscous stresses balance the stresses due to buoyancy, as given by equation (4). The problem here is to estimate the viscous stresses, since it is necessary to evaluate the velocity gradients, which in turn require a knowledge of the flow pattern. Of course, we have no direct evidence of the type of flow that occurs near the roots of a belt. In lack of better knowledge two extreme hypotheses can be made: (1) the flow is characterized by strong vertical gradients of the velocity, i. e., $\partial u/\partial z$ is large (u is the velocity, and z, x denote the vertical and horizontal coordinate, respectively); (2) the velocity depends weakly on z, so that its gradients are essentially horizontal. In the light of the preceding discussion it seems reasonable to favor the first possibility, as it appears that the flow should consist of an almost horizontal outward motion of the crustal material of the root, accompanied by a counterflow of the mantle lithosphere tending to fill the root trough as it is being vacated (in addition there must be deformations of the upper part of the crust to preserve isostasy); this pattern is consistent with the buoyancy forces that drive the motion. It is also reasonable to expect that the flow will be confined within a layer whose depth is of the order of the thickness of the structure (i.e., of the order of h') since the buoyancy stresses are localized there (the rest of the lithosphere is in hydrostatic equilibrium). We are then led to expect that the motion will exhibit a considerable velocity shear in nearly horizontal planes, and that $\partial u/\partial z \gg \partial u/\partial x$, since the scale lengths of the gradients are, respectively, the vertical and horizontal scales of the structure h' and a, and $h' \ll a$. Thus, we shall assume that the flow is strongly two-dimensional, and that the main contribution to the viscous stresses arises from $\partial u/\partial z$, and not from $\partial u/\partial x$, that can be neglected in a first approximation. This assumption differentiates the present model from the thin sheet models, already discussed in the Introduction, which correspond to the assumption (2). In fact, these models postulate $\partial u/\partial z \equiv 0$, so that flows whose velocity varies with depth are excluded. The test of the validity of the assumptions (1) or (2) will ultimately be the consistency of the predictions of our model with the observed large scale topography of of mountain belts, and other data concerning the style of deformation, as shall be discussed later on. For completeness we shall derive in the last Section the results corresponding to the assumption that $\partial u/\partial z \approx 0$, i.e., that the main contribution to the viscous stresses originates from $\partial u/\partial x$, as in the case of the thin sheet models; it will be seen that it leads to predictions that are not very different from those derived under the assumption $\partial u/\partial z \gg \partial u/\partial x$ for the rheologies usually assumed for the lithosphere. By considering the two extreme cases, all possible intermediate patterns of flow are bracketed by our analysis (in the spirit of the present approximations, the only conceivable scale lengths for the velocity gradients are the height and the width of the range), and the fact that regardless of which is chosen one derives very similar predictions indicates that the gross features of the processes are not very sensitive to the details of the flow pattern.

To estimate the velocity gradients we observe that $u \approx v/2$ where $v = da/dt$ is the rate of increase of a, so that $\partial u/\partial z \approx v/2h'$. Let us first assume, for greater simplicity, a Newtonian rheology. If μ denotes the viscosity, the viscous stress is given by

$$\tau'' \approx \mu v/2h' \tag{6}$$

Here, as said above, the crustal flow associated with the broadening of the root must be accompanied by a corresponding flow of the mantle lithosphere tending to fill the root trough. The rate of change of a will be determined by the most viscous of the layers involved, and this gives the value of μ that must be used in (6).

Equating τ'' to τ' (equation (4)) one obtains the spreading rate

$$v = (4g\rho/\mu)(\rho/\rho')^2 h^3/a \tag{7}$$

The characteristic time of the collapse of the range resulting from root spreading can be estimated as

$$t^* = a/v \tag{8}$$

i.e., t^* is the time required to double the width of the range, and accordingly, to reduce its height to one half. For example, if $\mu = 10^{22}$ P, then v = 2.1-2.5 cm/yr and t^* = 11-15 m.y. for the Andes. The corresponding strain rate is roughly 10^{-14} s^{-1}, and the viscous stress is approximately 160-180 bar.

As the roots spread and become more shallow (and therefore colder) the viscosity must increase, so that the collapse of the range will not proceed indefinitely.

It could be argued that a Newtonian rheology is not acceptable as a model of the lithosphere. A more realistic assumption would be to chose a vertically averaged power law rheology of the type [*Byrd*, 1976

$$\tau_{ij} = BE^{(1/n-1)}\varepsilon_{ij} \tag{9}$$

(usually referred to as the Norton-Hoff law) where τ_{ij} are the vertically averaged components of the deviatoric stress tensor, ε_{ij} are the strain rates, and

$$E = (\varepsilon_{ij}\varepsilon_{ij})^{1/2} \tag{10}$$

In (9) B is a constant that depends on the vertically integrated structure of the lithosphere (more precisely, the layer, or layers, that make the major contribution to its strength for the deformation one is considering). The justification for disregarding the rheological stratification of the lithosphere, and using a constitutive relation like (9), has been discussed in detail by *England* [1983], *Sonder and England* [1986], and *Houseman and England* [1986] in the context of thin sheet models of lithospheric deformation. Their arguments, which for brevity we shall not repeat in detail here, are also appropriate to the purpose of the present analysis, as we are not attempting to describe the local and small-scale details of the phenomenon. We are contented here to derive scaling laws and other relations between the characteristic values of large-scale physical variables, in order to describe the gross features of the process. These are not sensitive to local properties but only to average ones.

Equation (9) implies assuming a stress-dependent effective viscosity

$$\mu = BE^{(1/n-1)} \tag{11}$$

The exponent n depends on the combination of deformation mechanisms across the (relevant layers of the) lithosphere; $n = 3$ would be appropriate if the rheology is dominated by the power-law creep of olivine; higher values of n are a better approximation if high stress plasticity is prevalent or when friction on faults is a significant factor. A Newtonian fluid corresponds to $n = 1$, and $n \rightarrow \infty$ simulates a perfectly plastic behavior. It is generally accepted that a power law constitutive equation is adequate for practically all crustal and upper mantle rocks [*Kirby*, 1983; *Ranalli and Murphy*, 1987; see also *Goetze*, 1978]; vertically averaged rheologies similar to that assumed here have been used by *Bird and Piper* [1980], *England* [1983], *Houseman and England* [1986], *Cohen and Morgan* [1986], *Vilotte et al.* [1982, 1986], as well as others in studies of lithospheric deformations. See also *Karner et al.* [1983], *Sonder and England* [1986], and *Paterson* [1987] for further discussions on this subject.

The spreading of the root can be estimated on the assumption of a power law rheology by a slight modification of the preceding arguments. In place of (6) one has now

$$\tau'' = B(v/2h')^{1/n} \tag{12}$$

where as said before, B is a constant that can be obtained from tabulated properties of lithospheric materials (see, for example, *Ranalli and Murphy* [1987]) by means of some convenient vertical averaging procedure (which we shall not further discuss) over the relevant lithospheric layers. Then (7) is changed to

$$v = 2(h\rho/\rho')^{n+1}(2\rho gh/Ba)^n \tag{13}$$

With the same data as above, if one assumes $n = 3$ (olivine [*Goetze*, 1978]), and $B = 10^{13}$ (cgs), which are reasonable values for the lithospheric materials at the depth of the root [see *Ranalli and Murphy*, 1987], one obtains $t^* \approx 10$ m.y., that is of the same order of magnitude of the result derived above.

Notice that in any case t^* is of the order of the orogenic time scale, and this indicates that spreading flow is an important phenomenon in the course of the growth of a range. This flow involves a (almost horizontal) sliding of crustal material out of the deepest parts of the root and toward its sides and a corresponding slumping of the lithospheric mantle into the root trough, with vertical gradients of the velocity (in both instances) of the order of v/h'. Then we conclude that this dependence of the velocity with depth should be taken into account when describing the growth of mountain belts.

CRUSTAL SHORTENING AND THE DYNAMICS OF THE GROWTH OF A RANGE

Consider the situation of simultaneous crustal shortening and plastic flow at the root of the range. Let the rate of shortening be v_s, presumably caused by a thrust along the border such as may result from subduction. For simplicity we assume that the border is straight and infinitely long, that the horizontal forces that produce the shortening are normal to it, and that no mass is added to the crust. We shall also assume that the width of the continental plate is infinite (In practice this means that the results of the analysis can be applied only as long as the width of the range is considerably less than that of the continent). If d is the thickness of undeformed crust, then the volume of the range plus that of its root must be proportional to dl, where l denotes the amount of shortening that has taken place. The geometry of the problem is sketched in Figure 2, where for simplicity we have assumed a triangular profile for the range; the justification of this idealization will be discussed later. If t is the time elapsed since the beginning of the process and v_s is constant, $l = v_s t$; then

$$a(h+h') = 2dv_s t \quad (14)$$

Note that the addition of mass (for example due to magmatism) could be taken into account, if required, by introducing an appropriate coefficient in equation (14).

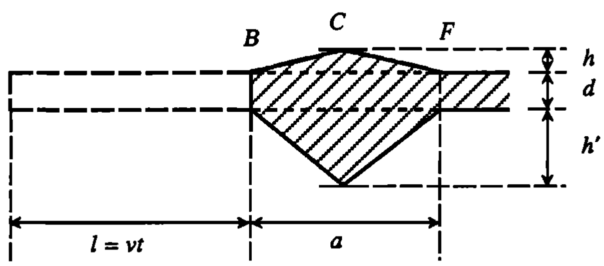

Fig. 2. Sketch showing simplified geometry of crust deformation due to shortening.

If flow at the root is taking place simultaneously, then stresses given by (4) are balanced by viscous stresses given by (6) or, more generally, by (12), and the range will tend to spread with a characteristic velocity v given by (7), or (13) (actually, the situation will be more complicate than that discussed in the previous Section, as the subduction complex participates in the range buildup process, and its presence must certainly affect the flow pattern; it is however reasonable to assume that the order of magnitude of the spreading velocity can still be estimated by (7) or (13)). Notice that now v will not be equal to da/dt.

Lastly, we assume that isostatic compensation is occurring during the process, so that condition (1) holds for all t. This is reasonable because, as stated above, the characteristic time of isostatic recovery is much shorter than the time scale of range buildup [*Vilotte et al.*, 1986].

The evolution of the range will be the result of the combined effects of the shortening, the spreading flow, and of isostasy. We shall now present arguments to show that a state of dynamic balance in which $v \approx v_s$ is maintained during the process. First, we observe that as the plate is shortened, the volume of the range must increase according to (14). This can in principle be accommodated either by varying a, or h' (and then h by isostasy), or both. But v increases as h' increases, and decreases with increasing a. Clearly, v cannot exceed v_s, otherwise the crustal material that is spreading out from the root would be left behind by the border (this would mean, in fact, that the effective rate of shortening should be v, and not v_s as assumed by hypothesis); then any increase of h' must be accompanied by an appropriate increase of a, and should not be too fast, in order to keep $v \leq v_s$. On the other hand, if h' has been increasing too slowly in the past, v will be accordingly small; suppose, for instance, that $v < v_s$: in this case, since the range is spreading very slowly, the shortening due to the motion of the border will actually effect a reduction of the width of the range, i.e. a will diminish with time. But in this situation, conservation of mass will lead to a fast increase of h', and then of v, and this increase will continue until $v \approx v_s$. We conclude that the process is autoregulated, in such a way that a state of dynamic balance is maintained, in which $v \approx v_s$ as stated above. In what follows we shall put $v_s = v$ to take into account this autoregulation.

We shall now derive the scaling laws that describe the evolution of a range, taking into account shortening, root spreading, and isostasy.

Let us consider first the case of a Newtonian rheology. Then

$$v\mu/h' = 4\rho' g h'^2/a \quad (15)$$

Combining (14), (15), and (1) yields $h(t)$ and $a(t)$:

$$h = d(t/T_s)^{1/2}(T_f/t)^{1/4} \quad (16)$$

$$a = 2d(t/T_s)^{1/2}(t/T_f)^{1/4} \quad (17)$$

where

$$T_s = d(1+\lambda)/v\lambda \qquad T_f = \lambda(1+\lambda)/2Ad \quad (18)$$

$$\lambda = \rho'/\rho \qquad A = \rho g/\mu \quad (19)$$

These formulae show that the evolution of the range is determined by two characteristic times: T_s associated with crustal shortening, and T_f associated with viscous flow. Due to the combination of these processes the evolution of the profile is self similar, with a vertical scale that increases with the 1/4 power of time while the horizontal scale increases as the 3/4 power of time. From (16) and (17) one obtains the evolution of the aspect ratio:

$$\vartheta = 2(t/T_f)^{1/2} \quad (20)$$

As a range grows in height its average slope becomes gentler; i.e., it spreads while its height increases and crust shortens, This dynamic equilibrium between growth and spreading explains the puzzle we mentioned in the introduction.

Figure 3 is a sketch of the evolution of the profile of a mountain belt, as given by equations (16) and (17). As the range front (indicated by F in the figure) advances inland, the surface of the crust between F and the crest C rises, whereas crust left behind by the crest (i.e., those points between B and C) subsides. The kinematics of the profile are reminiscent of a wave: point P rises as the front reaches that point (that can be visualized as a locus on the surface). This motion continues until P is riding the crest of the profile; afterwards P slowly subsides. These features agree qualitatively with the style of deformation for a young range such as the Andes in which thrust faulting occurs to the east of the crest of the belt, instances of normal faulting occur to the west, and the age of tectonic deformation decreases steadily eastward [*Suarez et al.*, 1983].

It can be observed from (18) and (20) that ϑ depends only on T_f and t and is independent of the rate of crustal shortening. Then if λ, μ, d, and the age of the range are known, the average profile can be predicted. Conversely, the geographical data (height and width) and the

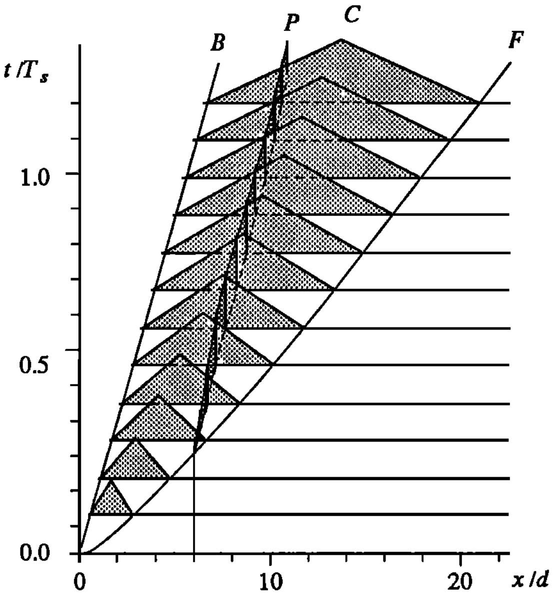

Fig. 3. Sketch of the time evolution of the visible profile of a mountain belt; x denotes the horizontal coordinate, measured from the initial position of the plate border, which changes in time due to the shortening of the plate so that the distance of B from the vertical axis gives the amount of shortening that has taken place; F denotes the position of the front of the deformation wave. A point on the surface such as P is at rest until reached by the front of the deformation wave (a little before $t = 0.3\ T_s$), then is uplifted until it rides the crest (near $t = 0.6\ T_s$); after that it slowly subsides. Uplift occurs at all points of the deformation wave between the front and the crest, while subsidence takes place between the crest and the border. The vertical exaggeration is 10x; $T_s = 1.46\text{x}10^3\ T_f$

age of the range provide constraints on the rheological properties at the depth of the root. Consider, for example, the Central Andes where $\lambda = 0.217$, $d = 33$ km, $h = 4.5$ km, and $\vartheta = 60\text{-}70$. If the profile of the belt is due to the present orogeny which started some 15 m.y. ago (e.g., *Audebaud et al.* [1973]) then m = 2.9-3.0x10^{22} P; the rate of crustal shortening is $v = 0.76$ cm/yr.

A second example is the Tibetan plateau where λ and d are as before, and h is 4.5 km, the average height of the top of the plateau (we shall discuss below why the present scalings, which were derived with the assumption that the range is sharply peaked, may be also applied to the case of a plateau). The top of the plateau is more appropriate for h than the height of the Himalaya since this range seems to be supported by flexure of the Indian plate (see, for example, *Lyon-Caen and Molnar* [1983] and *Karner and Watts* [1983]), not accounted for by the present model of isostatic support. Then $\vartheta = 220\text{-}230$. If orogeny started 55 m.y. ago, then $\mu = 0.9\text{-}1.0\text{x}10^{22}$ P and $v = 0.7$ cm/yr.

In both instances the values of μ are of the same order of magnitude and fall within the range of typical values for the lowest portions of the crust and the upper mantle as obtained from isostatic rebound data (see, for example, *Vening-Meinesz* [1937] and *Nakiboglu and Lambeck* [1983]) and from studies of creep (see the review article of *Weertman and Weertman* [1975]). Of course, we must be aware that the present comparisons suffer from various limitations. In the first place, the present topography only allows a comparison with the profiles at single point of the time axis, while it would be desirable to compare the actual evolution of the ranges and the predictions, for different times. Second, erosion has not been taken into account; this process will modify the mass balance equation, introducing additional nonlinear terms that could perhaps spoil the self similarity of the profile evolution. However, the effect may not be very large for fast growing ranges.

Consider now a power law rheology. Then (15) must be replaced by

$$B(v/2h')^{1/n} = 2\rho' g h'^2/a \tag{21}$$

and one obtains in place of (16), (17), and (20)

$$h = d(t/T_s)^{(n+1)/(3n+1)}(T_f/t)^{1/(3n+1)} \tag{22}$$

$$a = 2d(t/T_s)^{2n/(3n+1)}(t/T_f)^{1/(3n+1)} \tag{23}$$

$$\vartheta = 2(t/T_s)^{(n-1)/(3n+1)}(t/T_f)^{2/(3n+1)} \tag{24}$$

in which T_s is given by (18) as before, but the definition of T_f is changed into

$$T_f = (\lambda+1)(\lambda B/\rho g d)^n \tag{25}$$

Now the height of the range increases with the $n/(3n+1)$ power of time, while the width varies with the $(2n+1)/(3n+1)$ power, and the aspect ratio changes according to the $(n+1)/(3n+1)$ power. While the exponents of the scaling laws depend on the rheology, it is clear that this dependence is weak (as n ranges from 1 to ∞ the exponents vary, respectively, in rather narrow intervals: 1/4-1/3, 3/4-2/3, and 1/2-1/3) and does not change qualitatively the behavior we discussed above: the evolution of the profile is also self similar; it can be observed that the result that the aspect ratio increases with the age of the range is independent of the rheology, and the same is true of the main characteristics of the style of deformation, that are represented in Figure (3); in particular, uplift occurs in front of the crest, and subsidence occurs behind. We also observe that for $n \neq 1$, the aspect ratio depends only extremely weakly on v (for $n = 3$, $\vartheta \sim v^{1/5}$). The data of the examples can of course be fitted for any n by appropriate choices of B. Taking $n = 3$, which is a value frequently accepted as adequate for the lithosphere, the corresponding B fall within the range of typical values [see *Ranalli and Murphy*, 1987].

Finally, notice that the evolution of the range can also be obtained by assuming that stresses given by (4) that tend to collapse the range are balanced by rock strength, i.e., the range evolves through a succession of stable states of equilibrium instead of continuous evolution. With this hypothesis, equation (15) should be replaced by

$$h^2 = a\gamma\tau_y/2\rho g \tag{26}$$

so that equations (16)-(17) become

$$h = 2(\rho g d/\gamma\tau_y)^{-1/3}(t/T_s)^{1/3} \tag{27}$$

$$a = 2d(\rho g d/\gamma\tau_y)^{1/3}(t/T_s)^{2/3} \tag{28}$$

The formula for the aspect ratio is changed to

$$\vartheta = 2(\rho g d/\gamma\tau_y)^{2/3}(t/T_s)^{1/3} \tag{29}$$

The evolution given by equations (27)-(29) is qualitatively similar to that corresponding to equations (16), (17), and (20), or to (22)-(25). The exponents of the power laws correspond to the limit $n \to \infty$ in (22)-(25). The data of the examples can be fitted by appropriate choices of τ_y and τ'_y, but the values thus obtained are not related to behavior of rocks during short time scales; furthermore, the resulting τ'_y is larger than expected for prevailing temperatures at root depth and at typical orogenic strain rates as discussed earlier.

The present analysis is clearly limited to young, fast growing belts since erosion was not taken into account. Also, the scaling laws are more appropriate for advanced stages of growth of a range (i.e., when a considerable size has already been attained), when the height is varying slowly (see Figure 3), and the same happens to the depth of the root. Then the effective viscosity will also vary slowly, so that the assumption of it being constant is more justified.

Now we shall discuss the assumption of a triangular profile for the belt. In the first place, we want to stress that this hypothesis is not essential at all: we could have assumed any other reasonable shape, as long as it can be characterized by a "width," a, and a "height," h, and still the arguments leading to the scaling laws could be carried through just the same (the only difference would be that the continuity equation (14) would include a constant dimensionless geometrical factor of the order of unity), so that the same self similar scalings would be obtained within numerical factors that can be ignored in the spirit of the present dimensional analysis. The main reason for chosing the triangular shape is simplicity: we wish to avoid cluttering the formulae with geometrical factors that will obscure the basic facts we want to bring out, without gaining any real advantage, because we can not calculate these factors unless we know the exact theoretical profile. In addition, a roughly symmetrical triangular profile seems to be a reasonable first approximation for ranges associated to subduction along a continental border, as the Andes, on the basis of observational data. In Figure 4 a series of topographic profiles from across the Andes of Peru are represented; these profiles span an area 880 km long and 500 km wide as indicated in the inserted map; each profile represents the average relief of a strip 80 km wide, at right angles to the axis of the cordillera (that is nearly straight in the portion considered), and has been smoothed on a scale length of 40 km, transversely to the range, to cancel out small-scale (i.e., $< d$) topographic details. It can be appreciated that the profiles are nearly triangular and that their average width a is very well defined; the height h is also reasonably well defined, although not as sharply as a. We must be wary, however, and not take this evidence as proving that the correct theoretical shape of a range is triangular; actually, the observed averaged topography tends to be flat near the top (a fact that must be expected on theoretical grounds); obviously this feature is not correctly described by a triangle. On the other hand, many theoretical shapes that are characterized by vertical and horizontal scales h, a, can be imagined, that fit the observed data equally well, or even better than the triangle, and that have in addition a rounded top. To give just an example, the profiles of Figure 4 can be fitted very well by a segment of an ellipse whose major half axes are given by $3h$ and $2a/3$ (approximately). When this segment of an ellipse is scaled self similarly to a greater age (50-60 m.y., say) according to (16), (17) (or (22), (23)), the top flattens and the profile looks very plateau-like. For this reason we feel justified in using the present scalings for a plateau such as Tibet.

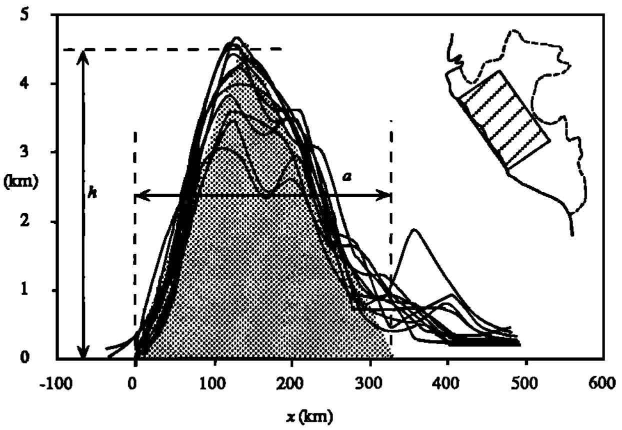

Fig. 4. Smoothed topographic profiles of the Andes of Peru. Each profile represents the relief of an 80-km-wide strip, at right angle to the axis of the cordillera, and has been smoothed on a scale length of 40 km transversely to the range to average small-scale topographic details. The 11 profiles span an area of 880 x 500 km as indicated in the inserted map. The profile assumed in the text is represented by the shadowed triangle.

It must be emphasized that we have given a very simplified treatment of the present model, which does not allow a theoretical prediction of the profile; at the present stage of development of our theory the shape must be assigned on an empirical basis. Nevertheless, an important theoretical result follows from our dimensional analysis, namely that we can predict that the evolution of the profile of the range is self similar, even if we are not able to compute its exact shape; this is a direct consequence of our assumptions concerning the governing parameters of the phenomenon.

To conclude this section, we shall complement the preceding statement by presenting some arguments to show (1) that profiles like those of Figure 4 are consistent with the physics of the present model, if appropriate boundary conditions are included, and (2) why the present model predicts a localized range, with a characteristic height and width.

To predict the observed shape, the theory must include the following features: an horizontal push to produce shortening, buoyancy, and in addition, (1) a mechanism to support the foot of the range on the side of the continental border (to the left of the crest in the figures), (2) a mechanism to localize the deformation near the continental border (i.e., to restrain its spreading inland from the crest, so that the range does not become arbitrarily wide). Basal shear traction due to the subducting plate, as in the model of *Davis et al.* [1983] is the obvious candidate as mechanism (1); clearly, except in narrow portion near the border (where the Coulomb wedge theory of *Davis et al.* [1983] is applicable) a power law viscous rheology such as (9) could be a reasonable choice; by this mechanism as shortening takes place, the crust will thicken in a wedgelike shape, thus buttressing the root of the range on the side of the continental border. An intuitive image of the process can be given as follows: this wedge acts like the blade of a fictitious bulldozer, pushing inland the rest of the root and causing the deformation to spread into the continent with a downward slope. The top of the range marks the position of the blade of this contrived bulldozer. Thus, each slope buttresses the other, and their feet are restrained from spreading by viscous stresses. To describe this situation, the theory must provide an adequate flow pattern, which probably should include vertical velocity shear; In this respect thin sheet models such as that of *England and McKenzie* [1982] may not be adequate since they lack this feature. On the other hand the present model includes it from the beginning; in fact, it is the balance between buoyancy and the viscous forces due to the vertical velocity shear as shortening is occurring that determines the slope of the range on the side of the continental border. The same viscous forces also provide naturally the required mechanism (2) that localizes the deformation and determines the slope on the other (inland) side.

It should be clear that while we cannot yet calculate the exact profile of the belt, the physics of the present model very definitely predict that the shortening will be concentrated in a localized range. This can be traced to the assumption that as expressed by equation (21) the viscous stresses (given by equation (12)) must balance the buoyancy stresses (given by equation (4)) that produce the spreading of the root. The autoregulation mechanism discussed at the beginning of this section ensures that this balance is maintained throughout the process of growth of the range. But (21) establishes a constraint linking a to h', and by the isostasy assumption (1), to h, that can be written (omitting constant factors) as

$$a \sim h^{2+1/n} \nu^{1/n} \tag{30}$$

The relationship (30) is, of course, implicit in equations (22) and

(23). It tells us that for any rate of shortening the width a cannot be chosen freely, since it is uniquely determined by the height of the range. Since on the other hand the volume of the range ($\sim ah$) is determined by the amount of shortening that has taken place, both a and h are uniquely determined at any given time (as expressed by equations (22) and (23)). It is then impossible to balance stresses for other values of a. In other words, viscosity can balance the buoyancy stresses if, and only if, the deformation is localized in a well-defined width. Any other assumption regarding the deformation (such as a diffuse thickening across the entire continent) must be ruled out: it cannot be reconciled with the present model and leads to inconsistencies. For example, a uniform thickening of the plate implies that the horizontal velocity varies linearly with x, which in turn means that the viscous stress is zero; it is clearly impossible to generate from the initial conditions of the problem such a velocity field, without passing first through an phase in which the horizontal forces arising from buoyancy (that appear because the slope is nonzero) that act to set in motion portions of the medium farther and farther away from the border (where the perturbation started) are balanced by viscosity; this balance determines the velocity of the motion at each point. This initial phase is precisely that in which we are interested. In this context, a uniform thickening from the very start of the process could be accepted only if it is assumed that nonviscous (for instance, elastic) forces, which are not considered in the present model, govern the phenomenon. But observational evidence rules out this possibility. Actually the argument may be turned around: the fact that a localized range (and not a uniform thickening of the continent) is observed indicates that viscous forces are a key factor. Summarizing, the localization of the deformation, the characteristic width a, and the height h, descend from the basic assumptions of the model: once nonviscous forces are excluded, autoregulation and its consequences follow by necessity.

SCALING FOR FLOWS OF THE THIN SHEET MODEL TYPE

In thin sheet models it is assumed that the velocity field does not depend on the vertical coordinate. As we are considering an infinitely long straight range, a horizontal velocity gradient can occur only transversely to the range, and can be estimated as $v/2a$ (instead of v/h' as we assumed in the preceding section). We shall consider a power law rheology; then (21) must be replaced by

$$B(v/2a)^{1/n} = 2\rho' g h'^2/a \quad (31)$$

Using (1), (14), and (31) one obtains the following scalings

$$h = d(4\lambda)^{-1/(3n-1)}(t/T_s)^{n/(3n-1)}(T_f/t)^{1/(3n-1)} \quad (32)$$

$$a = 2d(4\lambda)^{1/(3n-1)}(t/T_s)^{(2n-1)/(3n-1)}(t/T_f)^{1/(3n-1)} \quad (33)$$

$$\vartheta = 2(4l)^{2/(3n-1)}(t/T_s)^{(n-1)/(3n-1)}(t/T_f)^{2/(3n-1)} \quad (34)$$

It can be observed that (apart from a numerical factor that depends on the density contrast factor λ) the ratio T_s/T_f is equal to the n-th power of the Argand number, which was introduced by *England and McKenzie* [1982] in the context of the thin sheet model.

For Newtonian rheology ($n = 1$), it can be seen that one obtains the peculiar result that the height of an infinitely long range is independent of time; its value is given by

$$h = \sqrt{(\mu\lambda v/4\rho g)} \quad (35)$$

so that it is proportional to the square root of the shortening rate (for example, if $\mu = 3\times10^{22}$ P, $\lambda = 0.217$, $v = 1$ cm/yr, one obtains $h \approx 1.4$ km). Except for this special case, the scalings agree qualitatively with the behavior already discussed in the preceding section, and tend to coincide with (22)-(24) for large n. In particular for $n = 3$, which as said before, is a value generally accepted for the lithosphere, it can be observed that the scalings (32)-(34) already differ very little from the previous results (22)-(24) which were derived assuming a very different flow pattern. This somewhat surprising result implies that the gross visible features of the process of growth of a mountain belt are rather insensitive to the details of the velocity field within the lithosphere.

It must be stressed that the scaling (31) follows from the thin sheet model equations by assuming an indenter of infinite length ($L \to \infty$), so that $\partial/\partial y \equiv 0$ (y denotes the horizontal coordinate along the continental border). The calculations of *Houseman and England* [1986], *Cohen and Morgan* [1986] and others were made with the assumption of an indenter of finite length; then L is also a scale length for the velocity gradients; as a consequence, the horizontal velocity gradients on vertical planes transverse to the range, which must be also taken into account, give rise to additional stresses so that (31) no longer expresses the actual stress balance. Owing to the effect of the transverse stresses one obtains $dh/dt \neq 0$ in these calculations, even for Newtonian rheology. This is not in contradiction with our result, which is valid only for $L \to \infty$.

DISCUSSION AND CONCLUSIONS

Physical arguments that account for crustal shortening, isostasy, and a vertically averaged power law rheology permit scaling laws to be derived that describe the growth of a range and the shape of its profile. The formation of a localized range is predicted. The scalings of the width and height of the range are only weakly dependent on the rheology. As long as the crust is undergoing horizontal compression and shortening along its border, the range will grow both in height and width, and the average slope will decrease with time. Surface points away from the border are set in motion as the deformation wave propagates inland, being first uplifted until they ride the crest of the range and then subsided. Only order of magnitude estimates and qualitative predictions are possible due to the extreme simplifications involved in the present analysis. Within these limitations, the present results agree with the data of young structures such as the Andes (see, for example, *Suarez et al.* [1983]) and the Tibetan plateau [*Molnar and Tapponnier*, 1978]. On this basis we tentatively conclude that flow at the depth of roots is, together with isostasy and crustal shortening, the basic process that governs large-scale evolution and characteristics of young mountain belts and plateaus.

A more detailed description of the dynamics of the growth of mountain belts based on the present ideas may be obtained by solving the hydrodynamic equations that describe the buoyancy-driven creeping flow at the roots. If the simplifying assumption of constant viscosity is retained, the problem may be solved analytically by appropriate generalizations of the lubrication approximation (see, for example, *Huppert* [1982, 1986]). More realistic assumptions could be tackled numerically.

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