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On the effect of noise and electronics bandwidth on a stochastic-resonance memory device

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Abstract. We recently showed that a ring of two bistable oscillators is capable of storing a single bit of information via stochastic resonance. Memory performance was characterized in terms of the probability of erroneous bit detection and was shown to be minimized for a range of noise intensities. Furthermore, memory persistence was also shown to exhibit a stochastic-resonance behavior. In this paper we investigate the influence on memory performance, in particular its resilience to noise, on both noise bandwidth and the limited time response of the bistable elements. We show that, for broad ranges of ST and noise bandwidths, the probability of erroneous bit retrieval is also minimized for an optimal noise intensity, exhibiting a deep well as a function of noise intensity. We are interested in the breadth of such a well as it points out to the robustness of the memory device under different working conditions. Moreover, we show that there exists a relation between the noise and ST bandwidths that favors wide wells. We believe that this relation may be of relevance as a design rule for practical memory devices sustained by noise.

Keywords: Stochastic Resonance, Memory Device
PACS: 05.45.-a

INTRODUCTION

The increasing capacity of modern computers has been driven by Moore's Law, which postulates that the number of transistors in an integrated circuit doubles roughly every two years. However, as noted in Ref. [1], noise immunity and power consumption do not follow Moore's law. On the contrary, higher transistor densities and power consumption require the use of smaller supply voltages. Together all these factors lead to tighter noise margins and higher error rates in computation [2, 3]. There have been several proposals to solve this problem, e.g., Refs. [1, 4, 5] take explicitly into account the fact that the results of a computation may be correct only with some probability, and Ref. [6] uses a set of orthogonal noise processes to represent logic values. Recently, it was shown how to implement basic logical operations (OR, AND, NOR, NAND) using nonlinear systems such that their performance improves in the presence of noise [7, 8, 9], a signature of stochastic resonance.

Stochastic resonance (SR) is usually associated with a nonlinear system where the noise helps, an otherwise weak signal, to induce transitions between stable equilibrium states [10, 11, 12]. The phenomenon of stochastic resonance has been studied in a large number of applications, ranging from biological and neurological systems [13, 14, 15] to information transmission sustained by noise [16, 17, 18, 19, 20, 21, 22] and information

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storage [23, 24, 25]. In Refs. [23, 24] a ring of identical oscillators was shown to be able to sustain a traveling wave with the aid of noise, long after the harmonic drive signal had been switched off. It is only natural to ask whether such a scheme can be used to store data, i.e., aperiodic signals, in noisy environments. In [26, 27, 28] we showed that such a ring of two bistable oscillators is capable of storing a single bit of information via stochastic resonance. Memory performance was characterized in terms of the probability of erroneous bit detection.

In particular, we showed that by the addition of small amounts of white Gaussian noise, the system outperformed the deterministic (noiseless) case. We also showed that information could be retrieved from any of the two oscillators, with the same probability of error, after an elapsed 'synchronization' time that decreases with increasing noise. Moreover, we found that there was a range of noise intensities that yields a minimum probability of error and, at the same time, a nearly minimum synchronization time. Memory persistence was also shown to exhibit a stochastic-resonance behavior. Finally, a 'discrete' model of the bistable oscillators, based on STs in a loop array, was built and shown to be capable of storing a single bit more efficiently for an optimum amount of noise.

In this paper we extend our investigations on the loop of two STs in two directions. On one hand we consider the bandwidth limitation of Schmitt triggers. On the other hand, we consider the influence of the correlation time of the added Gaussian noise. In particular, by means of numerical simulations, we analyze the influence on memory performance of both noise bandwidth and time response limitations of the bistable elements.

RESULTS

As a simple model for colored noise we assume an Ornstein-Uhlenbeck process $X(t)$ given by the following stochastic differential equation [29],

$$dX = -\omega_n X dt + \sigma_* dW_t,$$

where ω_n is the noise bandwidth and W_t is standard Brownian motion. It is simple to verify that the steady state variance of X is given by $\sigma^2 = \omega_n \sigma_*^2/2$. Schmitt triggers are modeled by the following differential equation [11]

$$\dot{v}_{out} = -\beta \left\{ v_{out} - \frac{V_{hi} - V_{lo}}{2} \tanh \left[A \left(\frac{v_{out} - \frac{V_{hi}+V_{lo}}{2}}{\frac{V_{hi}-V_{lo}}{2}} + \frac{v_{in} - \frac{V_{+}+V_{-}}{2}}{\frac{V_{+}-V_{-}}{2}} \right) \right] - \frac{V_{hi} + V_{lo}}{2} \right\},$$

where v_{in} and v_{out} are the input and output voltages of the ST circuit, V_- and V_+ are the nominal low and high thresholds, respectively; V_{lo} and V_{hi} are the nominal low and high output voltages, respectively; A is a parameter representing how close the response of the Schmitt trigger is to that of an ideal bistable device, and β is the ST relaxation rate, i.e., a parameter that models the Schmitt trigger's 'bandwidth'. In our simulations we set $V_- = -1$ V, $V_+ = +3V$, $V_{lo} = 0$ V and $V_{hi} = 2$ V, and $A = 10^5$.

Memory storage starts by driving the first ST with a supra-threshold square pulse of $+5$ V of 1 ms duration ('memory write'). Upon storage, and after an arbitrary elapsed

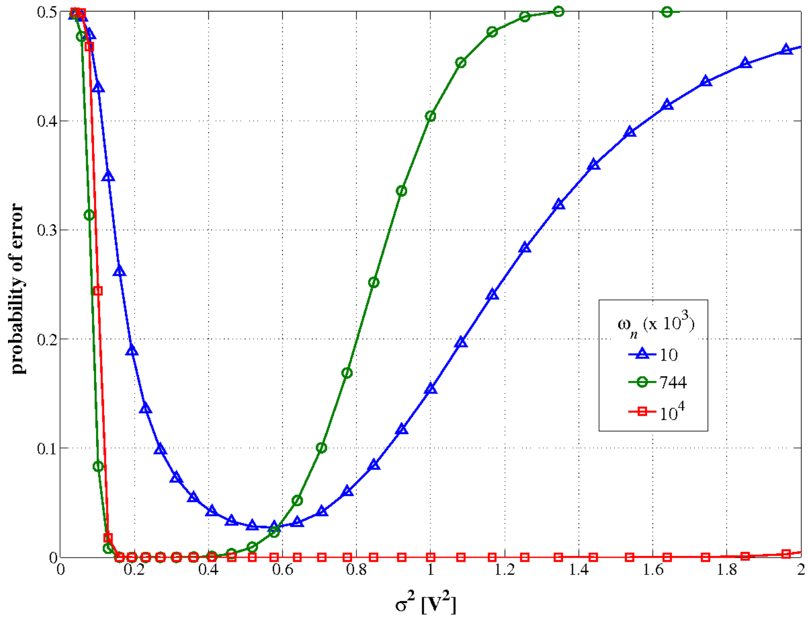

FIGURE 1. Probability of error vs. noise intensity for $\beta \approx 744$K 1/s and $\omega_n \approx 10\text{K}, 744\text{K}, 10^4\text{K}$ rad/s.

time, we interrogate the second ST ('memory read'). The retrieved voltage is then compared to a fixed threshold, and a decision is made on whether the device is storing a '0' or a '1' state. This operation is repeated for $\sim 10^5$ realizations of noise, and the probability of error is computed as the number of detected 0s divided by the total number of realizations.

Results on the dependence of device performance on noise bandwidth are shown in Fig. 1, where the ST response time was set to $\beta \approx 744 \times 10^3$ 1/s. First, the onset of stochastic resonance becomes apparent for there exists a noise range that minimizes the probability of error. Also, we observe that both the optimum noise intensity and the probability of error are noise-bandwidth dependent.

In Fig. 2 we show results on the dependence on the response time of the bistable devices for fixed $\omega_n \approx 744 \times 10^3$ rad/s. It can be readily verified that the 'slower' ST displays wider stochastic-resonance wells with optimum noise shifted towards higher intensities. This behavior can be explained as follows: when $\beta \ll \omega_n$ the device behaves as a low-pass filter for the noise present. This way, noise intensity is effectively reduced, thus decreasing the probability of error by haphazard noise events, and shifting the optimum point to a higher intensity. Conversely, if the device response time allows it to follow rapid noise fluctuations, performance is expected to degrade.

Next, we investigated the device resilience to added noise by looking at the noise dynamic range that guarantees an arbitrary low probability of error (in our case, 10^{-1}). Results are shown in Fig. 3 where the noise dynamic range, i.e. the width of the 'well' for a probability of error of 10^{-1}, is plotted against both device and noise bandwidths. As discussed above, we expect good device performance when $\beta \ll \omega_n$, and this becomes apparent at the figure's lower right. We also find poor performance when $\beta \gg \omega_n$. Interestingly, we find clear evidence of a regime that maximizes the noise dynamic

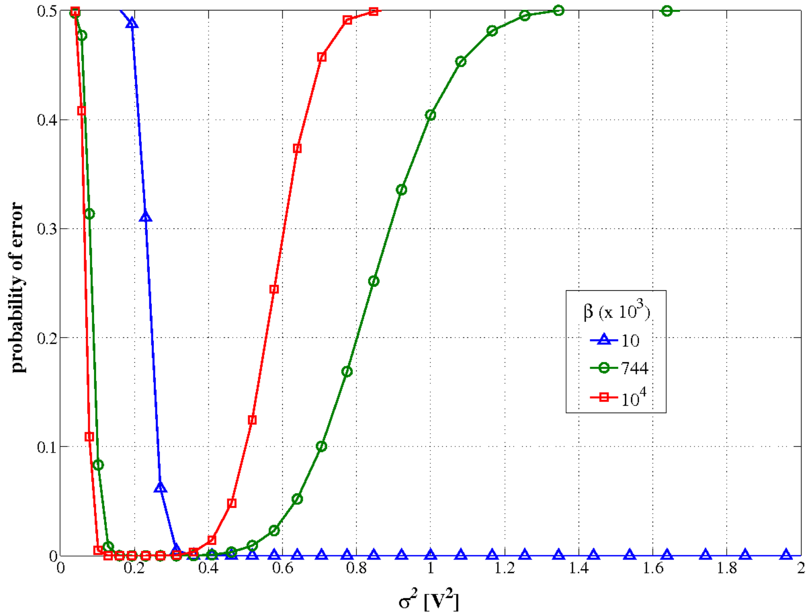

FIGURE 2. Probability of error vs. noise intensity for $\omega_n \approx 744$K 1/s and $\beta \approx 10\text{K}, 744\text{K}, 10^4\text{K}$ 1/s.

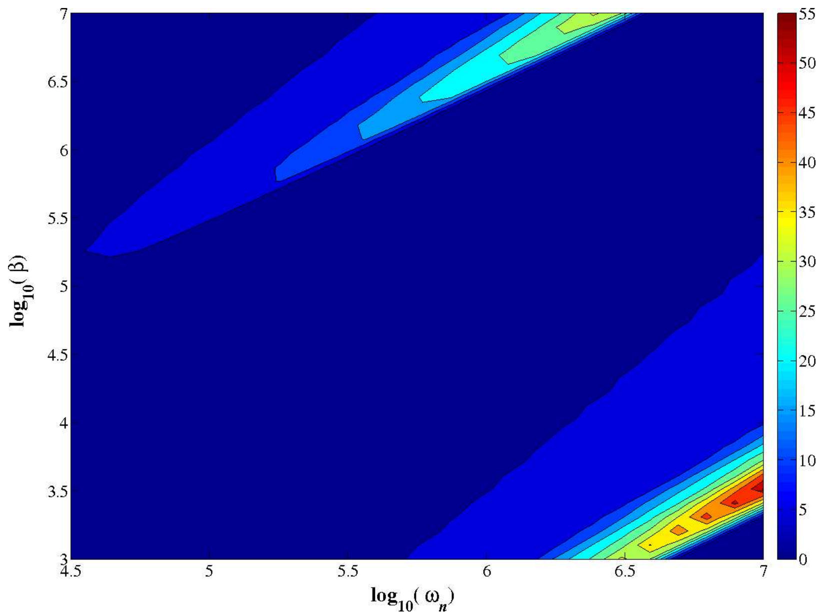

FIGURE 3. Contour plot, well width (at a probability of error $= 10^{-1}$) vs. ω_n, β.

range corresponding to the case where both noise and device bandwidths are of the same magnitude.

In summary, we presented results on the performance dependence of a stochastic-resonance memory device based on Schmitt triggers in a loop configuration. Parameters such as noise bandwidth and time response of the bistable elements were shown to affect device performance. We found that the device is more resilient to the action of noise, i.e., by delivering a low probability of detection errors, when the noise and Schmitt trigger bandwidths are of the same magnitude. We believe this result may be of relevance as a design rule for the practical implementation of memory devices sustained by noise.

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