Testing for three-body quark forces in L=1 excited baryons

Dan Pirjol and Carlos Schat

Citation: AIP Conference Proceedings **1296**, 159 (2010); doi: 10.1063/1.3523161
View online: http://dx.doi.org/10.1063/1.3523161
View Table of Contents:
http://scitation.aip.org/content/aip/proceeding/aipcp/1296?ver=pdfcov
Published by the AIP Publishing

Testing for three-body quark forces in $L = 1$ excited baryons

Dan Pirjol* and Carlos Schat[1]†

* *National Institute for Physics and Nuclear Engineering, Department of Particle Physics, 077125 Bucharest, Romania*
†*Department of Physics and Astronomy, Ohio University, Athens, Ohio 45701, USA*
Departamento de Física, FCEyN, Universidad de Buenos Aires, Ciudad Universitaria, Pab.1, (1428) Buenos Aires, Argentina

Abstract. We discuss the matching of the quark model to the effective mass operator of the $1/N_c$ expansion using the permutation group S_N. As an illustration of the general procedure we perform the matching of the Isgur-Karl model for the spectrum of the negative parity $L = 1$ excited baryons. Assuming the most general two-body quark Hamiltonian, we derive two correlations among the masses and mixing angles of these states which should hold in any quark model. These correlations constrain the mixing angles and can be used to test for the presence of three-body quark forces.

Keywords: Quark model, 1/N expansion of QCD, excited baryons
PACS: 12.39.-x, 12.38.-t, 14.20-c

INTRODUCTION

Quark models provide a simple and intuitive picture of the physics of ground state baryons and their excitations [1, 2]. An alternative description is provided by the $1/N_c$ expansion, which is a systematic approach to the study of baryon properties [3]. This program can be realized in terms of a quark operator expansion, which gives rise to a physical picture similar to the one of the phenomenological quark models, but is closer connected to QCD. In this context quark models gain additional significance.

The $1/N_c$ expansion has been applied both to the ground state and excited nucleons [4, 5, 6, 7, 8, 9, 10]. In the system of negative parity $L = 1$ excited baryons this approach has yielded a number of interesting insights for the spin-flavor structure of the quark interaction.

In a recent paper [11] we showed how to match an arbitrary quark model Hamiltonian onto the operators of the $1/N_c$ expansion, thus making the connection between these two physical pictures. This method makes use of the transformation of the states and operators under $S_N^{\mathrm{sp-fl}}$, the permutation group of N objects acting on the spin-flavor degrees of the quarks. This is similar to the method discussed in Ref. [12] for $N_c = 3$ in terms of S_3^{orb}, the permutation group of three objects acting on the orbital degrees of freedom.

The main result of [11] can be summarized as follows: consider a two-body quark

[1] Speaker, *XI Hadron Physics*, March 21-26, 2010, São Paulo, Brazil

Hamiltonian $V_{qq} = \sum_{i<j} O_{ij} R_{ij}$, where O_{ij} acts on the spin-flavor quark degrees of freedom and R_{ij} acts on the orbital degrees of freedom. Then the hadronic matrix elements of the quark Hamiltonian on a baryon state $|B\rangle$ contains only the projections O_α of O_{ij} onto a few irreducible representations of $S_N^{\text{sp-fl}}$ and can be factorized as $\langle B|V_{qq}|B\rangle = \sum_\alpha C_\alpha \langle O_\alpha \rangle$. The coefficients C_α are related to reduced matrix elements of the orbital operators R_{ij}, and are given by overlap integrals of the quark model wave functions. The matching procedure has been discussed in detail for the Isgur-Karl (IK) model in Ref. [13] providing a simple example of this general formalism.

In Ref. [14] we used the general S_N approach to study the predictions of the quark model with the most general two-body quark interactions, and to obtain information about the spin-flavor structure of the quark interactions from the observed spectrum of the $L = 1$ negative parity baryons. This talk summarizes the main ideas and emphasizes their relevance as a possible test for three-body forces in excited baryons.

THE MASS OPERATOR OF THE ISGUR-KARL MODEL

The Isgur-Karl model is defined by the quark Hamiltonian

$$\mathscr{H}_{IK} = \mathscr{H}_0 + \mathscr{H}_{\text{hyp}}\,, \tag{1}$$

where $\mathscr{H}_0$ contains the confining potential and kinetic terms of the quark fields, and is symmetric under spin and isospin. The hyperfine interaction $\mathscr{H}_{\text{hyp}}$ is given by

$$\mathscr{H}_{\text{hyp}} = A \sum_{i<j} \left[\frac{8\pi}{3} \vec{s}_i \cdot \vec{s}_j \delta^{(3)}(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} (3\vec{s}_i \cdot \hat{r}_{ij}\, \vec{s}_j \cdot \hat{r}_{ij} - \vec{s}_i \cdot \vec{s}_j) \right], \tag{2}$$

where A determines the strength of the interaction, and $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ is the distance between quarks i, j. The first term is a local spin-spin interaction, and the second describes a tensor interaction between two dipoles. This interaction Hamiltonian is an approximation to the gluon-exchange interaction, neglecting the spin-orbit terms[2].

In the original formulation of the IK model [2] the confining forces are harmonic. We will derive in the following the form of the mass operator without making any assumption on the shape of the confining quark forces. We refer to this more general version of the model as IK-V(r).

The $L = 1$ quark model states include the following SU(3) multiplets: two spin-1/2 octets $8_{\frac{1}{2}}, 8'_{\frac{1}{2}}$, two spin-3/2 octets $8_{\frac{3}{2}}, 8'_{\frac{3}{2}}$, one spin-5/2 octet $8'_{\frac{5}{2}}$, two decuplets $10_{\frac{1}{2}}, 10_{\frac{3}{2}}$ and two singlets $1_{\frac{1}{2}}, 1_{\frac{3}{2}}$. States with the same quantum numbers mix. For the $J = 1/2$ states we define the relevant mixing angle θ_{N1} in the nonstrange sector as

$$N(1535) = \cos\theta_{N1} N_{1/2} + \sin\theta_{N1} N'_{1/2}\,, \tag{3}$$

$$N(1650) = -\sin\theta_{N1} N_{1/2} + \cos\theta_{N1} N'_{1/2} \tag{4}$$

---

[2] In Ref.[2] A is taken as $A = \frac{2\alpha_S}{3m^2}$.

and similar equations for the $J = 3/2$ states, which define a second mixing angle θ_{N3}.

We find [13] that the most general mass operator in the IK-V(r) model depends only on three unknown orbital overlap integrals, plus an additive constant c_0 related to the matrix element of $\mathscr{H}_0$, and can be written as

$$\hat{M} = c_0 + aS_c^2 + bL_2^{ab}\{S_c^a, S_c^b\} + cL_2^{ab}\{s_1^a, S_c^b\}, \tag{5}$$

where the spin-flavor operators are understood to act on the state $|\Phi(SI)\rangle$ constructed as a tensor product of the core of quarks 2,3 and the 'excited' quark 1, as given in [6, 11]. The coefficients are given by

$$a = \frac{1}{2}\langle R_S\rangle, \; b = \frac{1}{12}\langle Q_S\rangle - \frac{1}{6}\langle Q_{MS}\rangle, \; c = \frac{1}{6}\langle Q_S\rangle + \frac{1}{6}\langle Q_{MS}\rangle. \tag{6}$$

The reduced matrix elements R_S, Q_S, Q_{MS} for the orbital part of the interaction contain the unknown spatial dependence and are defined in Refs. [6, 13]. Evaluating the matrix elements using Tables II, III in Ref. [6] we find the following explicit result for the mass matrix

$$M_{1/2} = \begin{pmatrix} c_0 + a & -\frac{5}{3}b + \frac{5}{6}c \\ -\frac{5}{3}b + \frac{5}{6}c & c_0 + 2a + \frac{5}{3}(b+c) \end{pmatrix}, \tag{7}$$

$$M_{3/2} = \begin{pmatrix} c_0 + a & \frac{\sqrt{10}}{6}b - \frac{\sqrt{10}}{12}c \\ \frac{\sqrt{10}}{6}b - \frac{\sqrt{10}}{12}c & c_0 + 2a - \frac{4}{3}(b+c) \end{pmatrix}, \tag{8}$$

$$M_{5/2} = c_0 + 2a + \frac{1}{3}(b+c), \tag{9}$$

$$\Delta_{1/2} = \Delta_{3/2} = c_0 + 2a. \tag{10}$$

Computing the reduced matrix elements with the interaction given by Eq. (2), one finds that the reduced matrix elements in the IK model with harmonic oscillator wave functions are all related and can be expressed in terms of the single parameter δ as

$$\langle Q_{MS}\rangle = \langle Q_S\rangle = -\frac{3}{5}\delta \qquad ; \qquad \langle R_S\rangle = \delta. \tag{11}$$

This gives a relation among the coefficients a, b, c of the mass matrix Eq. (5)

$$a = \frac{1}{2}\delta, \qquad b = \frac{1}{20}\delta, \qquad c = -\frac{1}{5}\delta. \tag{12}$$

We recover the well known result that in the harmonic oscillator model, the entire spectroscopy of the $L = 1$ baryons is fixed by one single constant $\delta = M_\Delta - M_N \sim 300$ MeV, along with an overall additive constant c_0, and the model becomes very predictive.

In Fig. 1 we show the result of a best fit of a, b, c in the IK-V(r) model together with the predictions of the IK model. The IK-V(r) spectrum is the best fit possible for a potential model with the spin-flavor interaction given in Eq. (2).

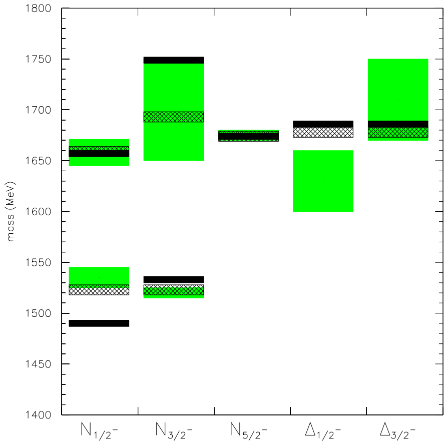

FIGURE 1. Masses predicted by the IK model (black bars), by the IK-V(r) model (hatched bars) and the experimental masses (green boxes) from Ref. [15].

THE MOST GENERAL TWO-BODY QUARK HAMILTONIAN

The most general two-body quark interaction Hamiltonian in the constituent quark model can be written in generic form as $V_{qq} = \sum_{i<j} V_{qq}(ij)$ with

$$V_{qq}(ij) \;=\; \sum_k f_{0,k}(\vec{r}_{ij}) O_{S,k}(ij) + f^a_{1,k}(\vec{r}_{ij}) O^a_{V,k}(ij) + f^{ab}_{2,k}(\vec{r}_{ij}) O^{ab}_{T,k}(ij)\,, \tag{13}$$

where $a,b = 1,2,3$ denote spatial indices, O_S, O^a_V, O^{ab}_T act on spin-flavor, and $f_k(\vec{r}_{ij})$ are orbital functions. Their detailed form is unimportant for our considerations. The scalar, spin-orbit and tensor parts of the interaction yield factors of $\mathbf{1}, L^i, L^{ij}_2 = \frac{1}{2}\{L^i, L^j\} - \frac{1}{3}\delta^{ij}L(L+1)$, which are coupled to the spin-flavor part of the interaction as shown in Table I of Ref. [14]

Following Refs. [11, 14] one finds that the most general form of the effective mass operator in the presence of these two-body quark interactions is a linear combination of 10 nontrivial spin-flavor operators

$$\begin{aligned} &O_1 = T^2\,,\; O_2 = \vec{S}_c^2\,,\; O_3 = \vec{s}_1\cdot\vec{S}_c\,,\; O_4 = \vec{L}\cdot\vec{S}_c\,,\; O_5 = \vec{L}\cdot\vec{s}_1\,,\; O_6 = L^i t^a_1 G^{ia}_c\,,\\ &O_7 = L^i g^{ia}_1 T^a_c\,,\; O_8 = L^{ij}_2\{S^i_c, S^j_c\}\,,\; O_9 = L^{ij}_2 s^i_1 S^j_c\,,\; O_{10} = L^{ij}_2 g^{ia}_1 G^{ja}_c \end{aligned} \tag{14}$$

and the unit operator.

It turns out that the 11 coefficients C_{0-10} contribute to the mass operator of the negative parity N^* states only in 9 independent combinations: $C_0, C_1 - C_3/2, C_2 + C_3, C_4, C_5, C_6, C_7, C_8 + C_{10}/4, C_9 - 2C_{10}/3$. This implies the existence of two universal

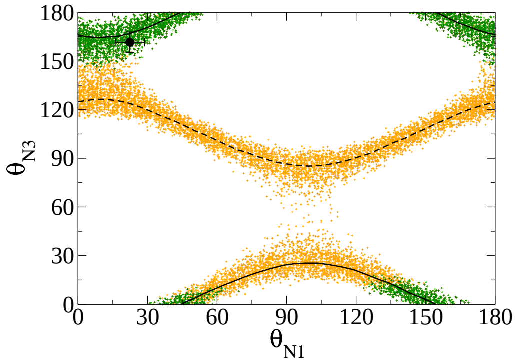

FIGURE 2. Correlation in the $(\theta_{N1}, \theta_{N3})$ plane from the quark model with the most general two-body quark interactions.

relations among the masses of the 9 multiplets plus the two mixing angles, which must hold in any quark model containing only two-body quark interactions.

The first universal relation involves only the nonstrange hadrons, and requires only isospin symmetry. It can be expressed as a correlation among the two mixing angles θ_{N1} and θ_{N3} (see Fig. 2)

$$\frac{1}{2}(N(1535)+N(1650))+\frac{1}{2}(N(1535)-N(1650))(3\cos 2\theta_{N1}+\sin 2\theta_{N1}) \qquad (15)$$
$$-\frac{7}{5}(N(1520)+N(1700))+(N(1520)-N(1700))\left[-\frac{3}{5}\cos 2\theta_{N3}+\sqrt{\frac{5}{2}}\sin 2\theta_{N3}\right]$$
$$=-2\Delta_{1/2}+2\Delta_{3/2}-\frac{9}{5}N_{5/2}\,.$$

This correlation holds also model independently in the $1/N_c$ expansion, up to corrections of order $1/N_c^2$, since for non-strange states the mass operator to order $O(1/N_c)$ [6, 7] is generated by the operators in Eq. (14). An example of an operator which violates this correlation is $L^i g^{ja}\{S_c^j, G_c^{ia}\}$, which can be introduced by three-body quark forces. In Fig.2 we show the two solutions correlating the two mixing angles, where the solid line and the dashed line are obtained replacing the central values of the baryon masses in Eq. (15). These lines are expanded into bands given by the scatter plot when the experimental errors in the masses are taken into account. The second universal relation expresses the spin-weighted SU(3) singlet mass $\overline{\Lambda}=\frac{1}{6}(2\Lambda_{1/2}+4\Lambda_{3/2})$ in terms of the nonstrange hadronic parameters and can be found in Ref. [14]. The green area in Fig. 2 shows the allowed region for $(\theta_{N1}, \theta_{N3})$ compatible with both relations and singles out the solid line as the preferred solution.

On the same plot we show also, as a black dot with error bars, the values of the mixing angles obtained in Ref. [16] from an analysis of the $N^* \to N\pi$ strong decays and in Ref. [17] from photoproduction amplitudes.

These angles are in good agreement with the correlation Eq. (15), and provide no evidence for the presence of spin-flavor dependent three-body quark interactions, within errors. It would be interesting to narrow down the errors on masses and mixing angles, and also compare with the upcoming results of lattice calculations for these excited states, to see if violations of the correlation given by Eq. (15) become apparent.

CONCLUSIONS

We presented the Isgur-Karl model mass operator in a form that makes the connection with the $1/N_c$ operator expansion clear. This simple and explicit calculation (for details see Ref. [13]) should serve as an illustration of the general matching procedure discussed in Ref. [11] using the permutation group.

We used the more general matching procedure [14] to saturate the contribution of all possible two-body forces to the masses of the negative parity $L = 1$ excited baryons, without making any assumptions about the orbital hadronic wave functions. We derived two universal correlations among masses and mixing angles, which will be broken by the presence of three-body forces, and could be used to set bounds on their strength given a more precise determination of all the masses and mixing angles for the negative parity $L = 1$ baryons.

ACKNOWLEDGMENTS

The work of C.S. was supported by CONICET and partially supported by the U. S. Department of Energy, Office of Nuclear Physics under contract No. DE-FG02-93ER40756 with Ohio University.

REFERENCES

1. A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. D **12**, 147 (1975).
2. N. Isgur and G. Karl, Phys. Lett. B **72**, 109 (1977).
3. R. F. Dashen, E. Jenkins and A. V. Manohar, Phys. Rev. D **49**, 4713 (1994) [Erratum-ibid. D **51**, 2489 (1995)]; R. F. Dashen, E. Jenkins and A. V. Manohar, Phys. Rev. D **51**, 3697 (1995).
4. J. L. Goity, Phys. Lett. B **414**, 140 (1997).
5. D. Pirjol and T. M. Yan, Phys. Rev. D **57**, 1449 (1998); Phys. Rev. D **57**, 5434 (1998).
6. C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed, Phys. Lett. B **438**, 327 (1998); Phys. Rev. D **59**, 114008 (1999).
7. C. L. Schat, J. L. Goity and N. N. Scoccola, Phys. Rev. Lett. **88**, 102002 (2002); J. L. Goity, C. L. Schat and N. N. Scoccola, Phys. Rev. D **66**, 114014 (2002).
8. J. L. Goity, C. Schat and N. N. Scoccola, Phys. Lett. B **564**, 83 (2003).
9. D. Pirjol and C. Schat, Phys. Rev. D **67**, 096009 (2003); AIP Conf. Proc. **698**, 548 (2004).
10. N. Matagne and F. Stancu, Phys. Rev. D **71**, 014010 (2005); Phys. Lett. B **631**, 7 (2005); Phys. Rev. D **74**, 034014 (2006).
11. D. Pirjol and C. Schat, Phys. Rev. D **78**, 034026 (2008).
12. H. Collins and H. Georgi, Phys. Rev. D **59**, 094010 (1999).
13. L. Galeta, D. Pirjol and C. Schat, Phys. Rev. D **80**, 116004 (2009).
14. D. Pirjol and C. Schat, Phys. Rev. Lett. **102**, 152002 (2009).
15. C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).
16. J. L. Goity, C. Schat and N. Scoccola, Phys. Rev. D **71**, 034016 (2005).
17. N. N. Scoccola, J. L. Goity and N. Matagne, Phys. Lett. B **663**, 222 (2008).