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Editorial Introduction

This special issue of *Studia Logica* is devoted to recent developments in the field of the structures that incorporate algebraic methods to the study of non-classical logic, and can be of interest to both logicians and algebraists. It includes fifteen papers having Algebraic Logic as a common subject of investigation. Among the wide range of topics that Algebraic Logic covers, this issue contains articles on the following: lattices, meet-semilattices, pseudocomplemented distributive lattices, Heyting algebras, semi-Heyting algebras, Ockham and De Morgan algebras, MV algebras, ℓ -groups, monadic MV algebras, residuated lattices, BCK-algebras and Post algebras.

It will perhaps be useful to orient the reader by briefly describing the various contributions.

In her paper “Prime Filters, Normality and Irreducibility in Lattices,” Gabriela Hauser-Bordalo recalls some notions introduced and developed by António A. Monteiro, and shows how Monteiro’s ideas and results have influenced other mathematical areas.

The variety of Semi-Heyting algebras, introduced by Hanamantagouda P. Sankappanavar, is an algebraic abstraction of the variety Heyting algebras. Semi-Heyting algebras share some important properties with Heyting algebras. For example, they are pseudocomplemented distributive lattices and congruences on them are determined by filters. This special issue contains two papers on semi-Heyting algebras. In the paper “Semi-intuitionistic Logic,” Juan Manuel Cornejo introduces a new logic \mathcal{SI} , called semi-intuitionistic logic, such that the semi-Heyting algebras are the semantics for \mathcal{SI} . In the paper “Expansions of Semi-Heyting Algebras I: Discriminator Varieties,” Sankappanavar defines new equational classes of algebras, as expansions of semi-Heyting algebras.

Priestley duality generalizes the well-known Stone duality between Stone spaces and Boolean algebras and has developed into a powerful tool in the study of varieties and quasi-varieties that have an underlying distributive

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lattice structure. In the paper “Priestley Style Duality for Distributive Meet-semilattices,” Guram Bezhanishvili and Ramón Jansana generalize Priestley duality for distributive lattices to a duality for distributive meet-semilattices, while in “Subalgebras of Heyting and De Morgan Heyting Algebras,” Valeria Castaño and Marcela Muñoz Santis obtain characterizations of subalgebras of Heyting and De Morgan Heyting algebras by using a Priestley-style topological representation of the corresponding algebras.

In this issue there are three papers related to MV-algebras. In the paper “Boolean Skeletons of MV-algebras and ℓ -groups,” Roberto Cignoli shows that for each strong order unit u of an ℓ -group \mathbf{G} , the Boolean skeleton of the MV-algebra $\Gamma(\mathbf{G}, u)$ is isomorphic to the Boolean algebra of factor congruences of \mathbf{G} , where Γ is the Mundici’s functor. The paper “On Certain Quasivarieties of Quasi-MV Algebras,” by A. Ledda, T. Kowalski and F. Paoli, is devoted to studying the lattice of quasivarieties of a quasi-MV algebra. Finally, in the paper “Monadic MV-algebras are Equivalent to Monadic ℓ -groups with Strong Unit,” C. Cimadamore and J. P. Díaz Varela define monadic ℓ -groups and establish a natural equivalence between the category of monadic MV-algebras and the category of monadic ℓ -groups with strong unit.

Residuated lattices have been studied extensively in recent years mainly due to their connection with substructural logics. This issue features two articles on residuated lattices. In the paper “Compatible Operations on Residuated Lattices,” J. L. Castiglioni and H. J. San Martn obtain conditions for a function f to be compatible with respect to every congruence, while in “Free-decomposability in Varieties of Pseudocomplemented Residuated Lattices,” D. Castaño, J. P. Díaz Varela and A. Torrens prove that free pseudocomplemented residuated lattices are decomposable if and only if they satisfy the Stone identity $\neg x \vee \neg\neg x = 1$.

An Ockham algebra, a generalization of a Boolean algebra, is a bounded distributive lattice enriched with a dual endomorphism. In the paper “On Endomorphisms of Ockham Algebras with Pseudocomplementation,” T. S. Blyth and J. Fang study some classes of Ockham algebras endowed with an additional operation of pseudocomplementation that commutes with the negation. Particular classes of Ockham algebras are the well-known classes of De Morgan algebras, important for the study of the mathematical aspects of fuzzy logic. In “Classical Modal De Morgan Algebras,” Sergio A. Celani defines and studies the class of De Morgan algebras endowed with an interior operator that satisfies an additional condition.

BCK algebras are algebraic structures that describe fragments of the propositional calculus involving implication. In the paper “Quasivarieties and Congruence Permutability of Lukasiewicz Implication Algebras,” M. Campercholi, D. Castaño and J. P. Díaz Varela study Lukasiewicz implication algebras, a particular class of BCK algebras. They prove that every subquasivariety of Lukasiewicz implication algebras is in fact a variety.

In the paper “Algebraic Functions,” M. Campercholi and D. Vaggione present a collection of general tools for the study of algebraic functions and apply them to several varieties of algebraic structures. Finally, in the paper “Resolution of Algebraic Systems of Equations in the Variety of Cyclic Post Algebras,” J. P. Díaz Varela and B. F. López Martinolich show how to solve an algebraic system of equations over a cyclic Post algebra of order prime p . They use the equivalence between Post algebras of order prime p and finite fields, Gröbner bases and algorithms programmed in Maple.

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