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The symmetric Radon–Nikodým property for tensor norms ☆

Daniel Carando [a,b], Daniel Galicer [a,b,*]

[a] Departamento de Matemática – Pab I, Facultad de Cs. Exactas y Naturales, Universidad de Buenos Aires, (1428) Buenos Aires, Argentina
[b] CONICET, Argentina

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ABSTRACT

We introduce the symmetric Radon–Nikodým property (sRN property) for finitely generated s-tensor norms β of order n and prove a Lewis type theorem for s-tensor norms with this property. As a consequence, if β is a projective s-tensor norm with the sRN property, then for every Asplund space E, the canonical mapping $\widetilde{\bigotimes}_{\beta}^{n,s} E' \to (\widetilde{\bigotimes}_{\beta'}^{n,s} E)'$ is a metric surjection. This can be rephrased as the isometric isomorphism $\mathcal{Q}^{min}(E) = \mathcal{Q}(E)$ for some polynomial ideal $\mathcal{Q}$. We also relate the sRN property of an s-tensor norm with the Asplund or Radon–Nikodým properties of different tensor products. As an application, results concerning the ideal of n-homogeneous extendible polynomials are obtained, as well as a new proof of the well-known isometric isomorphism between nuclear and integral polynomials on Asplund spaces. An analogous study is carried out for full tensor products.

0. Introduction

A result of Boyd and Ryan [5] and also of Carando and Dimant [9] implies that, for an Asplund space E, the space $\mathcal{P}_I^n(E)$ of integral polynomials is isometric to the space $\mathcal{P}_N^n(E)$ of nuclear polynomials (the isomorphism between these spaces was previously obtained by Alencar in [1,2]). In other words, if E is Asplund, the space of integral polynomials on E coincides isometrically with its minimal hull $(\mathcal{P}_I^n)^{min}(E) = \mathcal{P}_N^n(E)$. This fact was used, for example, in [5,6,17] to study geometric properties of spaces of polynomials and tensor products (e.g., extreme and exposed points of their unit balls), and in [7,8] to characterize isometries between spaces of polynomials and centralizers of symmetric tensor products. When the above mentioned isometry is stated as the isometric coincidence between a maximal ideal and its minimal hull, it resembles the Lewis theorem for operator ideals and (2-fold) tensor norms (see [24] and [15, 33.3]). The Radon–Nikodým property for tensor norms is a key ingredient for Lewis theorem.

The aim of this article is to find conditions under which the equality $\mathcal{Q}(E) = \mathcal{Q}^{min}(E)$ holds isometrically for a maximal polynomial ideal $\mathcal{Q}$. In terms of symmetric tensor products, we want conditions on an s-tensor norms ensuring the isometry $\widetilde{\bigotimes}_{\beta}^{n,s} E' = (\widetilde{\bigotimes}_{\beta'}^{n,s} E)'$. To this end, we introduce the symmetric Radon–Nikodým property for s-tensor norms and show our main result, a Lewis-type theorem (Theorem 2.2): if an s-tensor norm has the symmetric Radon–Nikodým property (sRN property), then the canonical mapping $\widetilde{\bigotimes}_{\backslash\beta/}^{n,s} E' \to (\widetilde{\bigotimes}_{/\beta'\backslash}^{n,s} E)'$ is a metric surjection for every Asplund space E (see the notation below). As a consequence, if $\mathcal{Q}$ is the maximal ideal (of n-homogeneous polynomials) associated to a projective s-tensor norm β with the sRN property, then $\mathcal{Q}^{min}(E) = \mathcal{Q}(E)$ isometrically.

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* Corresponding author at: Departamento de Matemática – Pab I, Facultad de Cs. Exactas y Naturales, Universidad de Buenos Aires, (1428) Buenos Aires, Argentina.

E-mail addresses: dcarando@dm.uba.ar (D. Carando), dgalicer@dm.uba.ar (D. Galicer).

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As an application of this result, we reprove the isometric isomorphism between integral and nuclear polynomials on Asplund spaces (note that the result proved in [5,9] is stronger). We also show that the ideal of extendible polynomials coincide with its minimal hull for Asplund spaces, and obtain as a corollary that the space of extendible polynomials on E has a monomial basis whenever E' has a basis.

We present examples of s-tensor norms associated to well-known polynomial ideals which have the sRN property. We also relate the sRN property of an s-tensor norm with the Asplund property. More precisely, we show that, for β is projective with the sRN, then β' preserves the Asplund property, in the sense that $\widetilde{\bigotimes}^{n,s}_{\beta'} E$ is Asplund whenever E is. As an application, we show that the space of extendible polynomials on E has the Radon–Nikodým property if and only if E is Asplund. One might be tempted to infer that a projective β with the sRN property preserves the Radon–Nikodým property, but this is not the case, as can be concluded from a result by Bourgain and Pisier [4]. However, we show that this is true with additional assumptions on the space E.

In order to prove our main theorem, we must show an analogous result for full tensor norms, which we feel can be of independent interest. It should be noted that, although we somehow follow some ideas of Lewis theorem's proof in [15, 33.3], that proof is based on some factorizations of linear operators and not on properties of bilinear forms. Therefore, the weaker nature of the sRN property introduced in this work together with our multilinear/polynomial framework makes our proof more complicated. As a consequence, we decided to postpone the proof of the main result to Section 4. The article is then organized as follows: In Section 1 we recall same basic definitions and facts about the theory of full and symmetric tensor norms and set some notation. In Section 2 we define the sRN property, state our main theorem and prove the related results described above. We also exhibit some examples of tensor norms having the sRN property. In Section 3 we consider the sRN property in full tensor products and show the Lewis-type result for this spaces (Theorem 3.5). In Section 4 we give the proof of Theorem 2.2 and conclude the article with some questions.

We refer to [15] for the theory of tensor norms and operator ideals, and also for all undefined notation used throughout the article. We also refer to [18–21] for symmetric tensor products and polynomial ideals.

1. Preliminaries

In this section we present the definitions and the general results that we use throughout the article.

A surjective mapping $T: E \to F$ is called *a metric surjection* if

$$\left\| Q(x) \right\|_F = \inf\left\{ \|y\|_E :\ Q(y) = x \right\},$$

for all $x \in E$. As usual, a mapping $I: E \to F$ is called *isometry* if $\|Ix\|_F = \|x\|_E$ for all $x \in E$. We will use the notation $\overset{1}{\twoheadrightarrow}$ and $\overset{1}{\hookrightarrow}$ to indicate a metric surjection or an isometry, respectively. We also write $E \overset{1}{=} F$ if E and F are isometrically isomorphic Banach spaces (i.e. there exists a surjective isometry $I: E \to F$). For a Banach space E with unit ball B_E, we call the mapping $Q_E: \ell_1(B_E) \overset{1}{\twoheadrightarrow} E$ given by $(a_x)_x \mapsto \sum a_x x$ the *canonical quotient mapping*.

For a natural number n, *a full tensor norm α of order n* assigns to every n-tuple of Banach spaces $(E_1, \dots, E_n)$ a norm $\alpha(\cdot; \bigotimes_{i=1}^n E_i)$ on the n-fold (full) tensor product $\bigotimes_{i=1}^n E_i$ such that

(1) $\varepsilon \leqslant \alpha \leqslant \pi$ on $\bigotimes_{i=1}^n E_i$.
(2) $\| \bigotimes_{i=1}^n T_i : (\bigotimes_{i=1}^n E_i, \alpha) \to (\bigotimes_{i=1}^n F_i, \alpha)\| = \|T_1\| \dots \|T_n\|$ for each set of operators $T_i \in \mathcal{L}(E_i, F_i)$, $i = 1, \dots, n$.

We say that α is *finitely generated* if for all $E_i \in BAN$ (the class of all Banach spaces) and z in $\bigotimes_{i=1}^n E_i$

$$\alpha\left(z, \bigotimes_{i=1}^n E_i\right) := \inf\left\{ \alpha\left(z, \bigotimes_{i=1}^n M_n\right):\ z \in \bigotimes_{i=1}^n M_i \right\},$$

the infimum being taken over all n-tuples $M_1, \dots, M_n$ of finite-dimensional subspaces of $E_1, \dots, E_n$ respectively whose tensor product contains z.

The name "*full tensor norms*" stresses the fact that they are defined on the full tensor product, to distinguish them from the s-tensor norms, that are defined on symmetric tensor products.

We say that β is an *s-tensor norm of order n* if β assigns to each Banach space E a norm $\beta(\cdot; \bigotimes^{n,s} E)$ on the n-fold symmetric tensor product $\bigotimes^{n,s} E$ such that

(1) $\varepsilon_s \leqslant \beta \leqslant \pi_s$ on $\bigotimes^{n,s} E$.
(2) $\| \bigotimes^{n,s} T : (\bigotimes^{n,s} E, \beta) \to (\bigotimes^{n,s} F, \beta)\| = \|T\|^n$ for each operator $T \in \mathcal{L}(E, F)$.

β is called *finitely generated* if for all $E \in BAN$ and $z \in \bigotimes^{n,s} E$

$$\beta\left(z, \bigotimes^{n,s} E\right) = \inf\left\{ \beta\left(z, \bigotimes^{n,s} M\right):\ M \in FIN(E),\ z \in \bigotimes^{n,s} M \right\}.$$

In this article we will only work with finitely generated tensor norms. Therefore, we will assume that all tensor norms are always finitely generated.

If α is a full tensor norm of order n, then the *dual tensor norm* α' is defined on *FIN* (the class of finite-dimensional Banach spaces) by

$$\Big(\bigotimes_{i=1}^n M_i, \alpha'\Big) \overset{1}{:=} \Big[\Big(\bigotimes_{i=1}^n M_i', \alpha\Big)\Big]'$$

and on BAN by

$$\alpha'\Big(z, \bigotimes_{i=1}^n E_i\Big) := \inf\Big\{\alpha'\Big(z, \bigotimes_{i=1}^n M_n\Big)\colon z \in \bigotimes_{i=1}^n M_i\Big\},$$

the infimum being taken over all n-tuples $M_1, \dots, M_n$ of finite-dimensional subspaces of $E_1, \dots, E_n$ respectively whose tensor product contains z.

Analogously, for β an s-tensor norm of order n, its *dual tensor norm* β' is defined on *FIN* by

$$\Big(\bigotimes^{n,s} M, \beta'\Big) \overset{1}{:=} \Big[\Big(\bigotimes^{n,s} M', \beta\Big)\Big]'$$

and extended to *BAN* as above.

The projective and injective associates (or hulls) of α will be denoted, by extrapolation of the 2-fold case, as $\backslash\alpha/$ and $/\alpha\backslash$ respectively (we refer to [15, 20.1] for the definitions of projective and injective 2-fold tensor norms). *The projective associate of* α will be the (unique) smallest projective tensor norm greater than α. Following [15, Theorem 20.6] we can see that $\backslash\alpha/$ satisfies

$$\Big(\bigotimes_{i=1}^n \ell_1(E_i), \alpha\Big) \overset{1}{\twoheadrightarrow} \Big(\bigotimes_{i=1}^n E_i, \backslash\alpha/\Big).$$

The injective associate of α will be the (unique) greatest injective tensor norm smaller than α. As in [15, Theorem 20.7] we have

$$\Big(\bigotimes_{i=1}^n E_i, /\alpha\backslash\Big) \overset{1}{\hookrightarrow} \Big(\bigotimes_{i=1}^n \ell_\infty(B_{E_i'}), \alpha\Big).$$

It is rather easy to check that an n-linear form A belongs to $(\bigotimes_{i=1}^n E_i, \backslash\alpha/)'$ if and only if $A \circ (Q_{E_1}, \dots, Q_{E_n}) \in (\bigotimes_{i=1}^n \ell_1(B_{E_i}), \alpha)'$. Moreover, we have

$$\|A\|_{(\bigotimes_{i=1}^n E_i, \backslash\alpha/)'} = \big\|A \circ (Q_{E_1}, \dots, Q_{E_n})\big\|_{(\bigotimes_{i=1}^n E_i, \alpha)'}.$$

On the other hand, an n-linear form A is in $(\bigotimes_{i=1}^n E_i, /\alpha\backslash)'$ if and only if it has an extension to $\ell_\infty(B_{E_1'}) \times \dots \times \ell_\infty(B_{E_n'})$ which belongs to $(\bigotimes_{i=1}^n \ell_\infty(B_{E_i'}), \alpha)'$. Moreover, the norm of A in $(\bigotimes_{i=1}^n E_i, /\alpha\backslash)'$ is the infimum of the norms in $(\bigotimes_{i=1}^n \ell_\infty(B_{E_i'}), \alpha)'$ of all such extensions.

It is clear that a tensor norm α is injective if and only if $\alpha = /\alpha\backslash$. Also, α is projective if and only if $\alpha = \backslash\alpha/$.

Note that in our notation, the symbols "\" and "/" by themselves lose their original meanings, as well as the left and right sides of α.

The projective and injective associates for a s-tensor norm β can be defined in a similar way, and they satisfy

$$\Big(\bigotimes^{n,s} \ell_1(E), \beta\Big) \overset{1}{\twoheadrightarrow} \Big(\bigotimes^{n,s} E, \backslash\beta/\Big),$$
$$\Big(\bigotimes_{i=1}^n E, /\beta\backslash\Big) \overset{1}{\hookrightarrow} \Big(\bigotimes^{n,s} \ell_\infty(B_{E'}), \beta\Big).$$

Again, the s-tensor norm β is injective or projective if and only if $\beta = /\beta\backslash$ or $\beta = \backslash\beta/$ respectively.

The description of the n-homogeneous polynomial Q belonging to $(\bigotimes^{n,s} E, \backslash\beta/)'$ or to $(\bigotimes^{n,s} E, /\beta\backslash)'$ is analogous to that for multilinear forms.

It is not hard to check, following the ideas of [15, Proposition 20.10], the following duality relations for a full tensor norms α or an s-tensor norm β:

$$\big(/\alpha\backslash\big)' = \backslash\alpha'/, \qquad \big(\backslash\alpha/\big)' = /\alpha'\backslash, \qquad \big(/\beta\backslash\big)' = \backslash\beta'/, \qquad \big(\backslash\beta/\big)' = /\beta'\backslash.$$

Just as in [15, Corollary 20.8], if $E_1, \dots, E_n$ are $\mathcal{L}_{1,\lambda}$ spaces for every $\lambda > 1$ then α and $\backslash\alpha/$ are equal on $\bigotimes_{i=1}^n E_i$. On the other hand, if $E_1, \dots, E_n$ are $\mathcal{L}_{\infty,\lambda}$ spaces for every $\lambda > 1$ then α and $/\alpha\backslash$ coincide in $\bigotimes_{i=1}^n E_i$. A similar result holds for s-tensor norms: if E is an $\mathcal{L}_{1,\lambda}$ space for every $\lambda > 1$, then β and $\backslash\beta/$ coincide on $\bigotimes^{n,s} E$. On the other hand, if E is an $\mathcal{L}_{\infty,\lambda}$ space for every $\lambda > 1$, then β and $/\beta\backslash$ coincide in $\bigotimes^{n,s} E$.

Let us recall some definitions from the theory of Banach ideals of multilinear forms. *A Banach ideal of continuous scalar valued n-linear forms* is a pair $(\mathcal{U}, \|\cdot\|_{\mathcal{U}})$ such that:

(i) $\mathcal{U}(E_1,\dots,E_n)=\mathcal{A}\cap\mathcal{L}(E_1,\dots,E_n)$ is a linear subspace of $\mathcal{L}(E_1,\dots,E_n)$, the space of all continuous multilinear forms on $E_1\times\cdots\times E_n$, and $\|\cdot\|_{\mathcal{U}}$ is a norm which makes the pair $(\mathcal{U},\|\cdot\|_{\mathcal{U}})$ a Banach space.
(ii) If $T_i\in\mathcal{L}(F_i,E_i)$ $(i=1,\dots,n)$, $A\in\mathcal{U}(E_1,\dots,E_n)$ then $A\circ(T_1\times\cdots\times T_n)\in\mathcal{U}(F_1,\dots,F_n)$ and

$$\big\|A\circ(T_1\times\cdots\times T_n)\big\|_{\mathcal{U}(F_1,\dots,F_n)}\leqslant\|A\|_{\mathcal{U}(E_1,\dots,E_n)}\|T_1\|\dots\|T_n\|.$$

(iii) $(z_1,\dots,z_n)\mapsto z_1,\dots,z_n$ belongs to $\mathcal{U}(\mathbb{K},\dots,\mathbb{K})$ and has norm 1.

Let $(\mathcal{U},\|\cdot\|_{\mathcal{U}})$ be the Banach ideal of continuous scalar valued n-linear forms and, for $A\in\mathcal{U}(E_1,\dots,E_n)$, define

$$\|A\|_{\mathcal{U}^{max}(E)}:=\sup\big\{\|A|_{M_1\times\cdots\times M_n}\|_{\mathcal{U}(M_1,\dots,M_n)}\colon\ M_i\in FIN(E_i)\big\}\in[0,\infty].$$

The maximal kernel of $\mathcal{U}$ is the ideal given by $\mathcal{U}^{max}:=\{A\in\mathcal{U}^n\colon\ \|A\|_{\mathcal{U}^{max}}<+\infty\}$. It is a Banach ideal with the norm $\|\cdot\|_{\mathcal{U}^{max}}$. An ideal $\mathcal{U}$ is said to be *maximal* if $\mathcal{U}\overset{1}{=}\mathcal{U}^{max}$.

The minimal kernel of $\mathcal{U}$ is defined as the composition ideal $\mathcal{U}^{min}:=\mathcal{U}\circ(\overline{\mathfrak{F}}\times\cdots\times\overline{\mathfrak{F}})$, where $\overline{\mathfrak{F}}$ stands for the ideal of approximable operators. In other words, a multilinear form A belongs to $\mathcal{U}^{min}(E_1,\dots,E_n)$ if it admits a factorization

$$\begin{tikzcd} E_1\times\cdots\times E_n \arrow[r, "A"] \arrow[d, "T_1\times\cdots\times T_n"'] & \mathbb{K} \\ F_1\times\cdots\times F_n \arrow[ur, "B"] & \end{tikzcd} \tag{1}$$

where $F_1,\dots,F_n$ are Banach spaces, $T_i:E_i\to F_i$ $(i=1,\dots,n)$ are approximable operator and B is in $\mathcal{U}(F_1,\dots,F_n)$. The minimal norm is given by

$$\|A\|_{\mathcal{U}^{min}}:=\inf\big\{\|B\|_{\mathcal{U}(F_1,\dots,F_n)}\|T_1\|\dots\|T_n\|\big\},$$

where the infimum runs over all possible factorizations as in (2). An ideal $\mathcal{U}$ is said to be *minimal* if $\mathcal{U}\overset{1}{=}\mathcal{U}^{min}$.

A Banach ideal of continuous scalar valued n-homogeneous polynomials is a pair $(\mathcal{Q},\|\cdot\|_{\mathcal{Q}})$ such that:

(i) $\mathcal{Q}(E)=\mathcal{Q}\cap\mathcal{P}^n(E)$ is a linear subspace of $\mathcal{P}^n(E)$ and $\|\cdot\|_{\mathcal{Q}}$ is a norm which makes the pair $(\mathcal{Q},\|\cdot\|_{\mathcal{Q}})$ a Banach space.
(ii) If $T\in\mathcal{L}(E_1,E)$, $P\in\mathcal{Q}(E)$ then $P\circ T\in\mathcal{Q}(E_1)$ and

$$\|P\circ T\|_{\mathcal{Q}(E_1)}\leqslant\|P\|_{\mathcal{Q}(E)}\|T\|^n.$$

(iii) $z\mapsto z^n$ belongs to $\mathcal{Q}(\mathbb{K})$ and has norm 1.

Let $(\mathcal{Q},\|\cdot\|_{\mathcal{Q}})$ be the Banach ideal of continuous scalar valued n-homogeneous polynomials and, for $P\in\mathcal{P}^n(E)$, define $\|P\|_{\mathcal{Q}^{max}(E)}:=\sup\{\|P|_M\|_{\mathcal{Q}(M)}\colon\ M\in FIN(E)\}\in[0,\infty]$.

The maximal kernel of $\mathcal{Q}$ is the ideal given by $\mathcal{Q}^{max}:=\{P\in\mathcal{P}^n\colon\ \|P\|_{\mathcal{Q}^{max}}<+\infty\}$. Endowed with the norm $\|\cdot\|_{\mathcal{Q}^{max}}$ it is a Banach ideal. An ideal $\mathcal{Q}$ is called *maximal* if $\mathcal{Q}\overset{1}{=}\mathcal{Q}^{max}$.

The minimal kernel of $\mathcal{Q}$ is defined as the composition ideal $\mathcal{Q}^{min}:=\mathcal{Q}\circ\overline{\mathfrak{F}}$, where $\overline{\mathfrak{F}}$ stands for the ideal of approximable operators. In other words, a polynomial P belongs to $\mathcal{Q}^{min}(E)$ if it admits a factorization

$$\begin{tikzcd} E \arrow[rr, "P"] \arrow[dr, "T"'] & & \mathbb{K} \\ & F \arrow[ur, "Q"'] & \end{tikzcd} \tag{2}$$

where F is a Banach space, $T:E\to F$ is an approximable operator and Q is in $\mathcal{Q}(F)$. The minimal norm is given by $\|P\|_{\mathcal{Q}^{min}}:=\inf\{\|Q\|_{\mathcal{Q}(F)}\|T\|^n\}$, where the infimum runs over all possible factorizations as in (2). An ideal $\mathcal{Q}$ is said to be *minimal* if $\mathcal{Q}\overset{1}{=}\mathcal{Q}^{min}$.

For properties about maximal and minimal ideals of homogeneous polynomials and examples see [19,21] and the references therein.

If $\mathcal{U}$ is an ideal of n-linear forms, its *associated full tensor norm* α is the unique tensor norm satisfying

$$\mathcal{U}(M_1,\dots,M_n)\overset{1}{=}\Big(\bigotimes_{i=1}^n M_i,\alpha\Big),$$

for all finite-dimensional spaces M_i $(i=1,\dots,n)$. As in the representation theorem [15, 17.1], if $\mathcal{U}$ is maximal then we have

$$\mathcal{U}(E_1,\dots,E_n)\overset{1}{=}\Big(\bigotimes_{i=1}^n E_i,\alpha'\Big)',$$

for all Banach spaces E_i.

If $\mathcal{Q}$ is a polynomial ideal, its *associated s-tensor norm* β is the unique tensor norm satisfying

$$\mathcal{Q}(M) \overset{1}{=} \bigotimes\nolimits_{\beta}^{n,s} M,$$

for every finite-dimensional space M. The polynomial representation theorem asserts that, if $\mathcal{Q}$ is maximal, then we have

$$\mathcal{Q}(E) \overset{1}{=} \Big(\bigotimes\nolimits_{\beta'}^{n,s} E\Big)',$$

for every Banach space E [21, 3.2].

If $\mathcal{U}$ is a maximal ideal of n-linear forms associated to the tensor norm α, we will denote by $\backslash\mathcal{U}/$ the unique maximal ideal associated to $\backslash\alpha/$ (i.e. $\backslash\mathcal{U}/(E_1,\dots,E_n) = (\bigotimes_{i=1}^n E_i, /\alpha'\backslash)'$). Analogously, for $\mathcal{Q}$ a maximal ideal of n-homogeneous polynomials associated to the s-tensor norm β we will denote by $\backslash\mathcal{Q}/$ the maximal ideal associated to $\backslash\beta/$.

Let $\mathcal{Q}$ be a maximal polynomial ideal associated to the s-tensor norm β. There is a natural quotient mapping from $\widetilde{\bigotimes}_{\beta}^{n,s} E' \overset{1}{\twoheadrightarrow} \mathcal{Q}^{min}(E)$ defined on $\bigotimes^{n,s} E'$ by the following rule: $\sum_{j=1}^r \otimes^n x_j' \mapsto \sum_{j=1}^r \underline{\bigotimes}^n x_j'$, where $(\sum_{j=1}^r \underline{\bigotimes}^n x_j')(x) := \sum_{j=1}^r x_j'(x)^n$ [19, Theorem 4.2]. We will denote by J_β the composition mapping

$$\widetilde{\bigotimes}_{\beta}^{n,s} E' \overset{1}{\twoheadrightarrow} \mathcal{Q}^{min}(E) \hookrightarrow \mathcal{Q}(E) \overset{1}{=} \Big(\widetilde{\bigotimes}_{\beta'}^{n,s} E\Big)'.$$

Sometimes J_β will be referred to as the *natural mapping* from $\widetilde{\bigotimes}_{\beta}^{n,s} E'$ to $(\widetilde{\bigotimes}_{\beta'}^{n,s} E)'$.

2. The symmetric Radon–Nikodým property

It is well know that the Radon–Nikodým property permitted to understand the full duality of a tensor norm π and ε, describing conditions under which $E' \widetilde{\otimes}_\pi F' = (E \widetilde{\otimes}_\varepsilon F)'$ holds. Lewis in [24] obtained many results of the form $E' \widetilde{\otimes}_\alpha F' = (E \widetilde{\otimes}_{\alpha'} F)'$ or, in other words, results about $\mathcal{U}^{min}(E, F') = \mathcal{U}(E, F')$ (if $\mathcal{U}$ is the maximal operator ideal associated with α).

For $\mathcal{Q}$ a maximal ideal of n-homogeneous polynomials, we want to find conditions under which the next equality holds

$$\mathcal{Q}^{min}(E) = \mathcal{Q}(E). \tag{3}$$

A related question is the following: if β is the s-tensor norm of order n associated to $\mathcal{Q}$, when does the natural mapping

$$J_\beta : \widetilde{\bigotimes}_{\beta}^{n,s} E' \overset{1}{\twoheadrightarrow} \mathcal{Q}^{min}(E) \hookrightarrow \mathcal{Q}(E) \overset{1}{=} \Big(\widetilde{\bigotimes}_{\beta'}^{n,s} E\Big)', \tag{4}$$

become a metric surjection? Note that, in this case, we get the equality (3). To give an answer to this question we will need the next definition. In a sense, it is a symmetric version of the one which appears in [15, 33.2].

Definition 2.1. A finitely generated s-tensor norm β of order n has the symmetric Radon–Nikodým property (sRN property) if

$$\widetilde{\bigotimes}_{\beta}^{n,s} \ell_1 \overset{1}{=} \Big(\widetilde{\bigotimes}_{\beta'}^{n,s} c_0\Big)'.$$

Here equality means that canonical mapping $J_\beta : \widetilde{\bigotimes}_{\beta}^{n,s} \ell_1 \to (\widetilde{\bigotimes}_{\beta'}^{n,s} c_0)'$ (as in (4) with $E = c_0$) is an isometric isomorphism.

Since ℓ_1 and c_0 are, respectively, $\mathcal{L}_{1,\lambda}$ and $\mathcal{L}_{\infty,\lambda}$ spaces for every $\lambda > 1$, β and $\backslash\beta/$ coincide in $\bigotimes^{n,s}\ell_1$ and $(\backslash\beta/)' = /\beta'\backslash$ coincides with β' on $\bigotimes^{n,s} c_0$. As a consequence, from the very definition we have that an s-tensor norm β has the sRN property if and only if its projective hull $\backslash\beta/$ does.

Also, ℓ_1 has the metric approximation property and, by [19, Corollary 5.2 and Proposition 7.5], the mapping J_β is always an isometry. Therefore, to prove that β has the sRN property we only have to check that J_β is surjective. Note that, for $\mathcal{Q}$ the maximal n-homogeneous polynomial ideal associated to β, our previous definition is equivalent to

$$\mathcal{Q}^{min}(c_0) = \mathcal{Q}(c_0), \tag{5}$$

and the isometry is automatic.

Our interest in the sRN property is motivated by the following Lewis-type theorem:

Theorem 2.2. *Let β be an s-tensor norm with the sRN property and E be an Asplund space. Then we have*

$$\widetilde{\bigotimes}_{\backslash\beta/}^{n,s} E' \overset{1}{\twoheadrightarrow} \Big(\widetilde{\bigotimes}_{/\beta'\backslash}^{n,s} E\Big)',$$

i.e., the natural mapping $J_{\backslash\beta/}$ is a metric surjection.

As a consequence, if $\mathcal{Q}$ is the maximal ideal of n-homogeneous polynomials associated with β, then

$$\big(\backslash\mathcal{Q}/\big)^{min}(E) = \backslash\mathcal{Q}/(E) \quad \textit{isometrically}.$$

One may wonder if the projective hull of the tensor norm β is really necessary in Theorem 2.2. Let us see that, in general, it cannot be avoided. Take any injective s-tensor norm β and let $\mathcal{Q}$ be the associated maximal polynomial ideal. If T is the dual of the original Tsirelson space (which is reflexive and therefore Asplund), then we can see that $\mathcal{Q}(T) \neq \mathcal{Q}^{min}(T)$. Indeed, we consider for each m, the polynomial on ℓ_2 given by $P_m(x) = \sum_{j=1}^m x_j^n$. Since β is injective, we have

$$\begin{aligned}\|P_m\|_{\mathcal{Q}(\ell_2)} &= \beta\left(\sum_{j=1}^m e'_j \otimes \cdots \otimes e'_j, \bigotimes\nolimits^{n,s} \ell_2\right) \\ &\leqslant /\pi\backslash\left(\sum_{j=1}^m e'_j \otimes \cdots \otimes e'_j, \bigotimes\nolimits^{n,s} \ell_2\right) \\ &\leqslant K\varepsilon\left(\sum_{j=1}^m e'_j \otimes \cdots \otimes e'_j, \bigotimes\nolimits^{n,s} \ell_2\right) \leqslant K,\end{aligned}$$

where the second inequality (and the constant K) are taken from [13, Lemma 2.6], and the third inequality is immediate. So we have shown that $\|P_m\|_{\mathcal{Q}(\ell_2)}$ is uniformly bounded. Since T does not contain $(\ell_2^m)_m$ nor $(\ell_\infty^m)_m$ uniformly complemented (see [14, pp. 33 and 66]), we can conclude that $\mathcal{Q}(T)$ cannot be separable by [13, Proposition 3.9]. As a consequence, $\mathcal{Q}(T)$ cannot coincide with $\mathcal{Q}^{min}(T)$.

In order to prove Theorem 2.2, an analogous result for full tensor products (and multilinear forms) will be necessary. As a consequence, we postpone the proof of Theorem 2.2 to Section 4.

Let us then present different tensor norms with the sRN. We begin with two basic examples. The following identities are simple and well known:

$$\left(\widetilde{\bigotimes}{}^{n,s}_{\pi'_s} c_0\right)' = \left(\widetilde{\bigotimes}{}^{n,s}_{\varepsilon_s} c_0\right)' = \widetilde{\bigotimes}{}^{n,s}_{\pi_s} \ell_1$$

and

$$\left(\widetilde{\bigotimes}{}^{n,s}_{\varepsilon'_s} c_0\right)' = \left(\widetilde{\bigotimes}{}^{n,s}_{\pi_s} c_0\right)' = \widetilde{\bigotimes}{}^{n,s}_{\varepsilon_s} \ell_1$$

(they easily follow from the analogous identities for full tensor products, since the symmetrization operator is a continuous projection). Therefore, we have:

Example 2.3. The tensor norms π_s and ε_s have the sRN property.

It should be noted the (2-fold) tensor norm ε does not have the classical Radon–Nikodým property [15, 33.2]. Therefore, the sRN property defined here for s-tensor norms and in Section 3 for full tensor norms is less restrictive than the Radon–Nikodým property for tensor norms.

In [1,2], Alencar showed that if E is Asplund, then integral and nuclear polynomials on E coincide, with equivalent norms. Later, Boyd and Ryan [5] and, independently, Carando and Dimant [9], showed that this coincidence is isometric (with a slightly more general assumption: that $\widetilde{\bigotimes}{}^{n,s}_{\varepsilon_s} E$ does not contain a copy of ℓ_1). Note that the isometry between nuclear and integral polynomials on Asplund spaces is an immediate consequence of Theorem 2.2 for $\beta = \pi_s$:

Corollary 2.4. *If E is Asplund, then $\mathcal{P}_I^n(E) = \mathcal{P}_N^n(E)$ isometrically.*

If we apply Theorem 2.2 and [19, Corollary 5.2] to $\beta = \varepsilon_s$, we obtain for E' with the bounded approximation property

$$\mathcal{P}_e^n(E) = \left(\mathcal{P}_e^n\right)^{min}(E) = \widetilde{\bigotimes}{}^{n,s}_{\backslash\varepsilon_s/} E' \quad \text{isometrically},$$

where $\mathcal{P}_e^n$ stands for the ideal of extendible polynomials (see [27] and the references therein for properties and definitions). Combining this with the main result in [22] we have:

Corollary 2.5. *Let E be a Banach space such that E' has a basis. Then, the monomials associated to this basis is a Schauder basis for the space of extendible polynomials $\mathcal{P}_e^n(E)$.*

We now give other examples of s-tensor norms associated to well-known maximal polynomial ideals having the sRN property.

The ideal of r-factorable polynomials. For $n \leqslant r \leqslant \infty$, a polynomial $P \in \mathcal{P}^n(E)$ is called **r-factorable** [20] if there is a positive measure space (Ω, μ), an operator $T \in \mathcal{L}(E, L_r(\mu))$ and $Q \in \mathcal{P}^n(L_r(\mu))$ with $P = Q \circ T$. The space of all such polynomials will be denoted by $\mathcal{L}_r^n(E)$. With

$$\|P\|_{\mathcal{L}_r^n(E)} = \inf\bigl\{\|Q\|\|T\|^n:\ P : E \xrightarrow{T} L_r(\mu) \xrightarrow{Q} \mathbb{K}\bigr\}.$$

Example 2.6. Let ρ_n^r be the s-tensor norm associated to $\mathcal{L}_r^n$ ($r \geqslant n \geqslant 2$). Then, ρ_n^r has the sRN property.

Proof. We can assume that $r < \infty$ since $\mathcal{L}_\infty^n(c_0) = \mathcal{P}_e^n(c_0)$ [20, Proposition 3.4]. For $P \in \mathcal{L}_r^n(c_0)$ there is a measure space (Ω, μ), an operator $T \in \mathcal{L}(c_0, L_r(\mu))$ and a polynomial $Q \in \mathcal{P}^n(L_r(\mu))$ with $P = Q \circ T$. Since $L_r(\mu)$ is reflexive, as a direct consequence of the Schauder theorem and the Schur property of ℓ_1, the operator T is approximable. On the other hand Q is trivially in $\mathcal{L}_r^n(L_r(\mu))$. Hence P belongs to $(\mathcal{L}_r^n)^{min}(c_0)$. $\square$

The ideal of positively r-factorable polynomials. An n-homogeneous polynomial $Q : F \to \mathbb{K}$ on a Banach lattice F is called positive, if $\check{Q} : F \to \mathbb{K}$ is positive, i.e., $\check{Q}(f_1, \dots, f_n) \geqslant 0$ for $f_1, \dots, f_n \geqslant 0$. For $n \leqslant r \leqslant \infty$, a polynomial $P \in \mathcal{P}^n(E)$ is called **positively r-factorable** [20] if there is a positive measure space (Ω, μ), an operator $T \in \mathcal{L}(E, L_r(\mu))$ and $Q \in \mathcal{P}^n(L_r(\mu))$ positive with $P = Q \circ T$. The space of all such polynomials will be denoted by $\mathcal{J}_r^n(E)$. With

$$\|P\|_{\mathcal{L}_r^n(E)} = \inf\bigl\{\|Q\|\|T\|^n:\ P : E \xrightarrow{T} L_r(\mu) \xrightarrow{Q} \mathbb{K}\bigr\}.$$

Using the ideas of the previous proof we have:

Example 2.7. Let δ_n^r be the s-tensor norm associated to $\mathcal{J}_r^n$ ($2 \leqslant n \leqslant r < \infty$). Then, δ_n^r has the sRN property.

The ideal of r-dominated polynomials. For $x_1, \dots, x_m \in E$, we define

$$w_r\bigl((x_i)_{i=1}^m\bigr) = \sup_{x' \in B_{E'}} \left(\sum_i \bigl|\langle x', x_i\rangle\bigr|^r\right)^{1/r}.$$

A polynomial $P \in P^n(E)$ is r**-dominated** (for $r \geqslant n$) if there exists $C > 0$ such that for every finite sequence $(x^i)_{i=1}^m \subset E$ the following holds

$$\left(\sum_{i=1}^m \bigl|P(x_i)\bigr|^{\frac{r}{n}}\right)^{\frac{n}{r}} \leqslant C w_r\bigl((x_i)_{i=1}^m\bigr)^n.$$

We will denote the space of all such polynomials by $\mathcal{D}_r^n(E)$. The least of such constants C is called the *r-dominated norm* and denoted by $\|P\|_{\mathcal{D}_r^n(E)}$.

In [10, Section 4], an n-fold full tensor norm $\alpha_{r'}^n$ was introduced, so that the ideal of dominated multilinear forms is dual to $\alpha_{r'}^n$. If we use the same notation for the analogous s-tensor norm, we have that $(\alpha_{r'}^n)'$ is the s-tensor norm associated to $\mathcal{D}_r$.

Example 2.8. The s-tensor norm $(\alpha_{r'}^n)'$ has the sRN property.

Proof. By [26] we know that $\mathcal{D}_r^n = \mathcal{P}^n \circ \Pi_r$, where Π_r is the ideal of r-summing operators (see [19] for notation). Thus, for $P \in \mathcal{D}_r^n(c_0)$ we have the factorization $P = Q \circ T$ where $T : c_0 \to G$ is an r-summing operator and $Q : G \to \mathbb{K}$ an n-homogeneous continuous polynomial. We may assume without lost of generality that $G = F'$ for a Banach space F (think on the Aron–Berner extension). By [15, Proposition 33.5] the tensor norm $(\alpha_{r',1})'$ has the Radon–Nikodým property. Using this, and the identity $(\alpha^t)' = (\alpha')^t$ (which holds for every tensor norm of order two α) we easily get

$$\begin{aligned}\Pi_r(c_0, G) &= \Pi_r\bigl(c_0, F'\bigr) = (c_0 \otimes_{\alpha_{1,r'}} F)' = (F \otimes_{\alpha_{r',1}} c_0)' \\ &= F' \otimes_{(\alpha_{r',1})'} \ell_1 = \ell_1 \otimes_{(\alpha_{1,r'})'} F' = \ell_1 \otimes_{(\alpha_{1,r'})'} G.\end{aligned}$$

Therefore, we have proved that $\Pi_r(c_0, G) = (\Pi_r)^{min}(c_0, G)$. Now is easy that $\mathcal{D}_r^n(c_0) = (\mathcal{D}_r^n)^{min}(c_0)$. $\square$

A natural and important question about a tensor norm is if it preserves some Banach space property. The following result shows that the sRN property is closely related to the preservation of the Asplund property under tensor products:

Theorem 2.9. *Let E be Banach space and β a projective s-tensor norm with sRN property. The tensor product $\widetilde{\bigotimes}_{\beta'}^{n,s} E$ is Asplund if and only if E is Asplund.*

Proof. Necessity is clear. For the converse, let S be a separable subspace of $\widetilde{\bigotimes}^{n,s}_{\beta'} E$ and let us see that it has a separable dual. We can take $(x_k)_{k\in\mathbb{N}}$ a sequence of vectors in E such that S is contained in $F = \overline{[\bigotimes^n x_k\colon k \in \mathbb{N}]}$. Since β' is injective, we have the isometric inclusion $S \overset{1}{\hookrightarrow} \widetilde{\bigotimes}^{n,s}_{\beta'} F$. Now, F' is separable (since E is Asplund) and therefore, by Theorem 2.2, the mapping

$$\widetilde{\bigotimes}^{n,s}_{\beta'} F' \to \Big(\widetilde{\bigotimes}^{n,s}_{\beta'} F\Big)'$$

is surjective. So, $(\widetilde{\bigotimes}^{n,s}_{\beta'} F)'$ is a separable Banach space and hence is also S' (since we have a surjective mapping $(\widetilde{\bigotimes}^{n,s}_{\beta'} F)' \twoheadrightarrow S'$). □

The following is an application of the previous theorem to $\beta = \backslash\varepsilon_s/$:

Corollary 2.10. *For a Banach space E and $n \in \mathbb{N}$, $\mathcal{P}^n_e(E)$ has the Radon–Nikodým property if and only if E is Asplund.*

Looking at Theorem 2.9 a natural question arises: if β is a projective s-tensor norm with the sRN property, does $\widetilde{\bigotimes}^{n,s}_{\beta} E$ have the Radon–Nikodým property whenever E has the Radon–Nikodým property? Burgain and Pisier [4, Corollary 2.4] presented a Banach space E with the Radon–Nikodým property such that $E \otimes_\pi E$ contains c_0 and, consequently, does not have the Radon–Nikodým property. This construction gives a negative answer to our question since the copy of c_0 in $E \otimes_\pi E$ is actually contained in the symmetric tensor product of E and π_s (which has the sRN property) is equivalent to the restriction of π to the symmetric tensor product.

However, $\widetilde{\bigotimes}^{n,s}_{\beta} E$ inherits the Radon–Nikodým property of E if, in addition, E is a dual space with the bounded approximation property (this should be compared to [16], where an analogous result for the 2-fold projective tensor norm π is shown):

Corollary 2.11. *Let β be a projective s-tensor norm with the sRN property and E a dual Banach space with the bounded approximation property. Then, $\widetilde{\bigotimes}^{n,s}_{\beta} E$ has the Radon–Nikodým property if and only if E does.*

Proof. Let F be a predual of E and suppose E has the Radon–Nikodým property. Since F is Asplund, by Theorem 2.9 so is $\widetilde{\bigotimes}^{n,s}_{\beta'} F$. On the other hand, by Theorem 2.2 we have a metric surjection $\widetilde{\bigotimes}^{n,s}_{\beta} E \overset{1}{\twoheadrightarrow} (\widetilde{\bigotimes}^{n,s}_{\beta'} F)'$. Since $E = F'$ has the bounded approximation property, by [18, Corollary 5.2], the mapping is injective. Thus, $\widetilde{\bigotimes}^{n,s}_{\beta} E \overset{1}{=} (\widetilde{\bigotimes}^{n,s}_{\beta'} F)'$. Therefore, $\widetilde{\bigotimes}^{n,s}_{\beta} E$ is the dual of an Asplund Banach space and has the Radon–Nikodým property.

Since E is complemented in $\widetilde{\bigotimes}^{n,s}_{\beta} E$, the converse follows. □

Any Banach space E with a boundedly complete Schauder basis $\{e_k\}_k$ is a dual space with the Radon–Nikodým property and the bounded approximation property. Indeed, E turns out to be the dual of the subspace F of E' spanned by the dual basic sequence $\{e'_k\}_k$ (which is, by the way, a shrinking basis of F). Then we have

$$\widetilde{\bigotimes}^{n,s}_{\beta} E \overset{1}{=} \Big(\widetilde{\bigotimes}^{n,s}_{\beta'} F\Big)'. \tag{6}$$

The monomials associated to $\{e_k\}_k$ and to $\{e'_k\}_k$ with the appropriate ordering (see [22]) are Schauder basis of, respectively, $\widetilde{\bigotimes}^{n,s}_{\beta} E$ and $\widetilde{\bigotimes}^{n,s}_{\beta'} F$. By the equality (6), monomials form a boundedly complete Schauder basis of $\widetilde{\bigotimes}^{n,s}_{\beta} E$ and a shrinking Schauder basis of $\widetilde{\bigotimes}^{n,s}_{\beta'} F$.

On the other hand, if we start with a Banach space E with a shrinking Schauder basis and take F as its dual, we are in the analogous situation with the roles of E and F interchanged. So we have:

Corollary 2.12. *Let β be a projective s-tensor norm with the sRN property.*

(a) *If E has a boundedly complete Schauder basis, then so does $\widetilde{\bigotimes}^{n,s}_{\beta} E$.*
(b) *If E has a shrinking Schauder basis, then so does $\widetilde{\bigotimes}^{n,s}_{\beta'} E$.*

The corresponding statement for the 2-fold full tensor norm π was shown by Holub in [23].

3. The sRN property for full tensor norms

In order to prove Theorem 2.2 we must first show an analogous result for full tensor products (see Theorem 3.5 below). So let us first introduce the sRN property for full tensor products in the obvious way:

Definition 3.1. A finitely generated full tensor norm of order n α has the symmetric Radon–Nikodým property (sRN property) if

$$\Big(\widetilde{\bigotimes}_{i=1}^{n}\ell_1,\alpha\Big)=\Big(\widetilde{\bigotimes}_{i=1}^{n}c_0,\alpha'\Big)'.$$

As in [15, Lemma 33.3] we have the following symmetric result for ideals of multilinear form.

Proposition 3.2. *Let α be a finitely generated full tensor norm of order n with the sRN property. Then,*

$$\Big(\widetilde{\bigotimes}_{i=1}^{n}\ell_1(J_i),\alpha\Big)=\Big(\widetilde{\bigotimes}_{i=1}^{n}c_0(J_i),\alpha'\Big)'$$

holds isometrically for all index sets $J_1,\dots,J_n$.

Proof. Fix $J_1,\dots,J_n$ index sets, and let us define $\mathcal{U}(c_0(J_1),\dots,c_0(J_n))=(\widetilde{\bigotimes}_{i=1}^{n}c_0(J_i),\alpha')'$. We must show $\mathcal{U}(c_0(J_1),\dots,c_0(J_n))=\mathcal{U}^{min}(c_0(J_1),\dots,c_0(J_n))$ with equal norms. For $T\in\mathcal{U}(c_0(J_1),\dots,c_0(J_n))$, let us see that the set $L=\{(j_1,\dots,j_n)\colon T(e_{j_1},\dots,e_{j_n})\neq 0\}$ is countable. If not, there exist $(j_1^k,\dots,j_n^k)_{k\in\mathbb{N}}$ different indexes such that

$$\big|T(e_{j_1^k},\dots,e_{j_n^k})\big|>\varepsilon.$$

Without loss of generality we can assume that the sequence of first coordinates j_1^k has all its elements pairwise different. Passing to subsequences we can also assume that $e_{j_i^k}$ are weakly convergent, moreover, $e_{j_1^k}\overset{w}{\to}0$. This contradicts the Littlewood–Bogdanowicz–Pełczyński property of c_0 [3,25].

Let $\Omega_k: J_1\times\cdots\times J_n\to J_k$ given by $\Omega_k(j_1,\dots,j_n)=j_k$. And let L_k be the set $\Omega_k(L)\subset J_k$. Consider, ξ_k the mapping $c_0(J_k)\to c_0(L_k)$ given by

$$(a_j)_{j\in J_k}\mapsto(a_j)_{j\in L_k}.$$

And the inclusion $\imath_k: c_0(L_k)\to c_0(J_k)$ defined by

$$(a_j)_{j\in L_k}\mapsto(b_j)_{j\in J_k},$$

where b_j is a_j if $j\in L_k$ and zero otherwise. Then, we can factor

$$\begin{tikzcd}
c_0(J_1)\times\cdots\times c_0(J_1) \arrow[r, "T"] \arrow[d, "\xi_1\times\cdots\times\xi_n"'] & \mathbb{K} \\
c_0(L_1)\times\cdots\times c_0(L_n) \arrow[ur, "\overline{T}"'] &
\end{tikzcd}$$

where $\overline{T}:=T\circ(\imath_1\times\cdots\times\imath_n)$. Since α has the sRN property we know that $\mathcal{U}(c_0(L_1),\dots,c_0(L_n))=\mathcal{U}^{min}(c_0(L_1),\dots,c_0(L_n))$ with equal norms. Therefore $\overline{T}$ is in $\mathcal{U}^{min}(c_0(L_1),\dots,c_0(L_n))$ with

$$\|\overline{T}\|_{\mathcal{U}^{min}}=\|\overline{T}\|_{\mathcal{U}}\leqslant\|T\|_{\mathcal{U}}.$$

Thus $\overline{T}$ belongs to $\mathcal{U}^{min}(c_0(L_1),\dots,c_0(L_n))$ which implies that T also is $\mathcal{U}^{min}(c_0(J_1),\dots,c_0(J_n))$. Moreover,

$$\|T\|_{\mathcal{U}^{min}}\leqslant\|\overline{T}\|_{\mathcal{U}^{min}}\|\xi_1\times\cdots\times\xi_n\|\leqslant\|T\|_{\mathcal{U}}.\qquad\square$$

Let $E_1,\dots,E_n$ be Banach spaces. For every $k=1,\dots,n$, denote by $I_k: E_k\to\ell_\infty(B_{E_k'})$ the inclusion mapping. Also, let EXT_k denote the canonical extension of a multilinear form to the bidual in the k-th coordinate (i.e. the multilinear version of the canonical extension $\varphi^\wedge$ and ${}^\wedge\varphi$ of a bilinear form φ, as in [15, 1.9]). We can describe this extension by the following way:

$$EXT_k(T)(x_1,\dots,Z_k,\dots,x_n)=\lim_{x_{k,\alpha}\overset{w^*}{\to}z_k}T(x_1,\dots,x_{k,\alpha},\dots,x_n),$$

for all $x_j\in E_j$, $z_k\in E_k''$, where $x_{k,\alpha}\overset{w^*}{\to}z_k$ stands for any bounded net on E_k weak-star convergent to z_k. It is important to mention that a multilinear version of the *Extension Lemma* [15, 13.2] holds, with the same proof. In other words, extending a multilinear form to the bidual of any of the Banach spaces where it is defined preserves the norm as a linear functional on the tensor product, for any finitely generated tensor norm.

For $\varphi: E_1\times\cdots\times E_n\to\mathbb{K}$ we denote by φ^n the associated $(n-1)$-linear mapping $\varphi^n: E_1\times\cdots\times E_{n-1}\to E_n'$. Now, if $T: E_n'\to F'$ is a linear operator, then the $(n-1)$-linear mapping $\rho: E_1\times\cdots\times E_{n-1}\times F'\to\mathbb{K}$ given by $T\circ\varphi^n$ induces an n-linear form on $E_1\times\cdots\times E_{n-1}\times F$. It is not hard to check that

$$\rho(e_1,\dots,e_{n-1},f) = (EXT_n)\varphi\big(e_1,\dots,e_{n-1},T' J_F(f)\big),$$

where $J_F : F \to F''$ is the canonical inclusion mapping.

For every $k = 1,\dots,n$ we define an operator

$$\Psi_k : \Big(\Big(\widetilde{\bigotimes}_{j=1}^{k-1} E_j\Big) \widetilde{\otimes}\, c_0(B_{E_k'}) \,\widetilde{\otimes} \Big(\widetilde{\bigotimes}_{j=k+1}^{n} E_j\Big), /\alpha'\backslash\Big)' \to \Big(\widetilde{\bigotimes}_{i=1}^{n} E_i, /\alpha'\backslash\Big)',$$

by the composition $((\widetilde{\bigotimes}_{j=1}^{k-1} Id_{E_k}) \,\widetilde{\otimes}\, I_k \,\widetilde{\otimes}\, (\widetilde{\bigotimes}_{j=k+1}^{n} Id_{E_k}))' \circ EXT_k$.

The following remark is easy to check:

Remark 3.3. Let $E_1,\dots,E_n$ be Banach spaces. For every k the following diagram conmutes:

$$\begin{tikzcd}
((\widetilde{\bigotimes}_{j=1}^{k-1} E_j') \,\widetilde{\otimes}\, \ell_1(B_{E_k'}) \,\widetilde{\otimes}\, (\widetilde{\bigotimes}_{j=k+1}^{n} E_j'), \backslash\alpha/) \arrow[r] \arrow[d, "(\widetilde{\bigotimes}_{j=1}^{k-1} Id_{E_j'}) \widetilde{\otimes} Q_k \widetilde{\otimes} (\widetilde{\bigotimes}_{j=k+1}^{n} Id_{E_j'})"] & ((\widetilde{\bigotimes}_{j=1}^{k-1} E_j) \,\widetilde{\otimes}\, c_0(B_{E_k'}) \,\widetilde{\otimes}\, (\widetilde{\bigotimes}_{j=k+1}^{n} E_j), /\alpha'\backslash)' \arrow[d, "\Psi_k"] \\
((\widetilde{\bigotimes}_{j=1}^{k-1} E_j') \,\widetilde{\otimes}\, E_k' \,\widetilde{\otimes}\, (\widetilde{\bigotimes}_{j=k+1}^{n} E_j'), \backslash\alpha/) \arrow[r] & (\widetilde{\bigotimes}_{i=1}^{n} E_i, /\alpha'\backslash)',
\end{tikzcd}$$

where $Q_k : \ell_1(B_{E_k'}) \overset{1}{\twoheadrightarrow} E_k'$ is the canonical quotient mapping.

Now an important proposition:

Proposition 3.4. *Let $E_1,\dots,E_n$ be Banach spaces. If E_k is Asplund then Ψ_k is a metric surjection.*

Proof. We will prove it assuming that $k = n$ (the other cases are analogous). Notice that Ψ_n has norm less or equal to one (since EXT_n is an isometry).

Fix $\varphi \in (\widetilde{\bigotimes}_{i=1}^{n} E_i, /\alpha'\backslash)'$ and $\varepsilon > 0$ and let $\widetilde{\varphi} \in ((\widetilde{\bigotimes}_{i=1}^{n-1} E_i) \,\widetilde{\otimes}\, \ell_\infty(B_{E_n'}), /\alpha'\backslash)'$ a Hahn–Banach extension of φ. Since E_n' has the Radon–Nikodým property, by the Lewis–Stegall theorem the adjoint of the canonical inclusion $I_n : E_n \to \ell_\infty(B_{E_n'})$ factors through $\ell_1(B_{E_n'})$ via

$$\begin{tikzcd}
\ell_\infty(B_{E_n'})' \arrow[rr, "I_n'"] \arrow[dr, "A"'] & & E_n' \\
& \ell_1(B_{E_n'}) \arrow[ur, "Q_n"'] &
\end{tikzcd} \tag{7}$$

where Q_n is the canonical quotient mapping and $\|A\| \leqslant (1+\varepsilon)$. Let $\rho : E_1 \times \dots \times E_{n-1} \times c_0(B_{E_n'}) \to \mathbb{K}$ given by the formula $\rho(x_1,\dots,x_{n-1},a) = (EXT_n)\widetilde{\varphi}(x_1,\dots,x_{n-1},A' J_{c_0(B_{E_n'})}(a))$ (ρ is the n-linear form on $E_1 \times \dots \times E_{n-1} \times c_0(B_{E_n'})$ associated to $A \circ (\widetilde{\varphi})^n$). Using the ideal property and the fact that the extension to the bidual is an isometry $\rho \in ((\widetilde{\bigotimes}_{i=1}^{n-1} E_i) \,\widetilde{\otimes}\, c_0(B_{E_n'}), /\alpha'\backslash)'$ and $\|\rho\| \leqslant \|\varphi\|(1+\varepsilon)$.

If we show that $\Psi_n(\rho) = \varphi$ we are done. It is an easy exercise to prove that $I_n'(\widetilde{\varphi})^n = \varphi^n$. It is also easy to see that $I_n(x)(a) = Q_n(a)(x)$ for $x \in E_n$ and $a \in \ell_1(B_{E_n'})$.

Now, $\Psi_n(\rho) = (\widetilde{\bigotimes}_{i=1}^{n-1} Id_{E_i} \,\widetilde{\otimes}\, I_n)' \circ (EXT_n)(\rho)$. Then,

$$\begin{aligned}
\Psi_n(\rho)(x_1,\dots,x_n) &= (I_n x_n)\big[\rho(x_1,\dots,x_{n-1},\cdot)\big] \\
&= (I_n x_n) A(\widetilde{\varphi})^n(x_1,\dots,x_{n-1}) \\
&= Q_n A(\widetilde{\varphi})^n(x_1,\dots,x_{n-1})(x_n) \\
&= I_n'(\widetilde{\varphi})^n(x_1,\dots,x_{n-1})(x_n) \\
&= \varphi^n(x_1,\dots,x_{n-1})(x_n) \\
&= \varphi(x_1,\dots,x_n). \qquad \square
\end{aligned}$$

The following result is the version of Theorem 2.2 for full tensor products. It should be noted that it holds for tensor products of different spaces.

Theorem 3.5. *Let α be a tensor norm with the sRN property and $E_1,\dots,E_n$ be Asplund spaces. Then*

$$\Big(\widetilde{\bigotimes}_{i=1}^{n} E_i', \backslash\alpha/\Big) \overset{1}{\twoheadrightarrow} \Big(\widetilde{\bigotimes}_{i=1}^{n} E_i, /\alpha'\backslash\Big)'.$$

In particular if $(\mathfrak{A}, \mathbf{A})$ is the maximal ideal (of multilinear forms) associated with α, then

$$\big(\backslash\mathfrak{A}/\big)^{min}(E_1, \ldots, E_n) = \backslash\mathfrak{A}/(E_1, \ldots, E_n).$$

Proof. Using Remark 3.3 we know that the following diagram commutes in each square.

$$\begin{tikzcd}
(\widetilde{\bigotimes}_{i=1}^n \ell_1(B_{E_i'}), \backslash\alpha/) \arrow[rr, "R_0"] \arrow[dd, "\widetilde{\otimes}_{i=1}^{n-1} Id_{\ell_1(B_{E_i'})} \widetilde{\otimes} P_n"] & & (\widetilde{\bigotimes}_{i=1}^n c_0(B_{E_i'}), /\alpha'\backslash)' \arrow[d, "EXT_n"] \\
& & ((\widetilde{\bigotimes}_{i=1}^{n-1} c_0(B_{E_i'})) \widetilde{\otimes} \ell_\infty(B_{E_n'}), /\alpha'\backslash)' \arrow[d, "((\widetilde{\bigotimes}_{i=1}^{n-1} Id_{c_0(B_{E_i'})}) \widetilde{\otimes} I_n)'"] \\
((\widetilde{\bigotimes}_{i=1}^{n-1} \ell_1(B_{E_i'})) \widetilde{\otimes} E_n', \backslash\alpha/) \arrow[rr, "R_1"] \arrow[dd, "(\widetilde{\bigotimes}_{i=1}^{n-2} Id_{\ell_1(B_{E_i'})}) \widetilde{\otimes} P_{n-1} \widetilde{\otimes} Id_{E_n'}"] & & ((\widetilde{\bigotimes}_{i=1}^{n-1} c_0(B_{E_i'})) \widetilde{\otimes} E_n, /\alpha'\backslash)' \arrow[d, "EXT_{n-1}"] \\
& & ((\widetilde{\bigotimes}_{i=1}^{n-2} c_0(B_{E_i'})) \widetilde{\otimes} \ell_\infty(B_{E_{n-1}'}) \widetilde{\otimes} E_n, /\alpha'\backslash)' \arrow[d, "((\widetilde{\bigotimes}_{i=1}^{n-1} Id_{c_0(B_{E_i'})}) \widetilde{\otimes} I_{n-1} \widetilde{\otimes} Id_{E_n'})'"] \\
((\widetilde{\bigotimes}_{i=1}^{n-2} \ell_1(B_{E_i'})) \widetilde{\otimes} E_{n-1}' \widetilde{\otimes} E_n', \backslash\alpha/) \arrow[rr, "R_2"] \arrow[dd] & & ((\widetilde{\bigotimes}_{i=1}^{n-2} c_0(B_{E_i'})) \widetilde{\otimes} E_{n-1} \widetilde{\otimes} E_n, /\alpha'\backslash)' \arrow[dd] \\
& \cdots & \\
\cdots \arrow[dd] & \cdots & \cdots \arrow[dd] \\
& \cdots & \\
(\ell_1(B_{E_1'}) \widetilde{\otimes} (\widetilde{\bigotimes}_{i=2}^n E_i'), \backslash\alpha/) \arrow[rr, "R_{n-1}"] \arrow[dd, "P_1 \widetilde{\otimes} (\widetilde{\bigotimes}_{i=2}^n Id_{E_i'})"] & & (c_0(B_{E_1'}) \widetilde{\otimes} (\widetilde{\bigotimes}_{i=2}^n E_i), /\alpha'\backslash)' \arrow[d, "EXT_1"] \\
& & (\ell_\infty(B_{E_1'}) \widetilde{\otimes} (\widetilde{\bigotimes}_{i=2}^n E_i), /\alpha'\backslash)' \arrow[d, "(I_1 \widetilde{\otimes} (\widetilde{\bigotimes}_{i=2}^n Id_{E_i'}))'"] \\
(\widetilde{\bigotimes}_{i=1}^n E_i', \backslash\alpha/) \arrow[rr, "R_n"] & & (\widetilde{\bigotimes}_{i=1}^n E_i, /\alpha'\backslash)'
\end{tikzcd}$$

Let us take a look at the first commutative square. Since α has the sRN property, R_0 is a metric surjection by Proposition 3.2. Moreover, by Proposition 3.4 the mapping $((\widetilde{\bigotimes}_{i=1}^{n-1} Id_{c_0(B_{E_i'})}) \widetilde{\otimes} I_n)' \circ EXT_n$ is also a metric surjection. As a consequence of these two facts we get that R_1 is a metric surjection. The same argument can be applied to the second commutative square, now that we know that R_1 is metric surjection. Thus, R_2 is also a metric surjection. Reasoning like this, it follows that $R_n : (\widetilde{\bigotimes}_{i=1}^n E_i', \backslash\alpha/) \to (\widetilde{\bigotimes}_{i=1}^n E_i, /\alpha'\backslash)'$ is a metric surjection. □

We will call $\Psi : (\widetilde{\bigotimes}_{i=1}^n c_0(B_{E_i}'))' \to (\widetilde{\bigotimes}_{i=1}^n E_i')'$ the composition of the downward mappings in the right side of the last diagram. The following proposition shows how to describe the mapping Ψ more easily (it will be useful to prove the polynomial version of the last theorem).

Proposition 3.6. *The mapping $\Psi : (\widetilde{\bigotimes}_{i=1}^n c_0(B_{E_i}'), /\alpha'\backslash)' \to (\widetilde{\bigotimes}_{i=1}^n E_i, /\alpha'\backslash)'$ is the composition mapping*

$$\Big(\widetilde{\bigotimes}_{i=1}^n c_0\big(B_{E_i}'\big), /\alpha'\backslash\Big)' \xrightarrow{EXT} \Big(\widetilde{\bigotimes}_{i=1}^n \ell_\infty\big(B_{E_i}'\big), /\alpha'\backslash\Big)' \xrightarrow{(\widetilde{\otimes}_{i=1}^n I_i)'} \Big(\widetilde{\bigotimes}_{i=1}^n E_i, /\alpha'\backslash\Big)',$$

where EXT stands for the iterated extension to the bidual given by $(EXT_n) \circ \cdots \circ (EXT_1)$ (we extend from the left to the right).

Proof. For the readers' sake we will give a proof for the case $n = 2$. Let $\rho \in (c_0(B_{E_1'}) \widetilde{\otimes} c_0(B_{E_2'}), /\alpha'\backslash)'$, then

$$\begin{aligned}
\Psi(\rho)(e_1, e_2) &= (id_{E_1} \widetilde{\otimes} I_2)'(EXT_2)(I_1 \widetilde{\otimes} Id_{c_0(B_{E_2'})})'(EXT_1)(\rho)(e_1, e_2) \\
&= (EXT_2)(I_1 \widetilde{\otimes} Id_{c_0(B_{E_2'})})'(EXT_1)(\rho)\big(e_1, I_2(e_2)\big) \\
&= I_2(e_2)\big((I_1 \widetilde{\otimes} Id_{c_0(B_{E_2'})})'(EXT_1)(\rho)(e_1, \cdot)\big) \\
&= I_2(e_2)\big(a \mapsto I_1(e_1)\rho(\cdot, a)\big) \\
&= I_2(e_2)\big((EXT_1)(\rho)\big(I_1(e_1), \cdot\big)\big) \\
&= (EXT)(\rho)\big(I_1(e_1), I_2(e_2)\big) \\
&= (I_1 \widetilde{\otimes} I_2)'(EXT)(\rho)(e_1, e_2). \qquad \square
\end{aligned}$$

Now, this proposition shows that the diagram

$$\begin{tikzcd}
(\widetilde{\bigotimes}_{i=1}^n \ell_1(B_{E_i'}), \backslash\alpha/) \arrow[r, two heads] \arrow[d, "P"] & (\widetilde{\bigotimes}_{i=1}^n c_0(B_{E_i'}), /\alpha'\backslash)' \arrow[d, "\Psi"] \\
(\widetilde{\bigotimes}_{i=1}^n E_i', \backslash\alpha/) \arrow[r] & (\widetilde{\bigotimes}_{i=1}^n E_i, /\alpha'\backslash)'
\end{tikzcd}$$

conmutes and, by the proof of Theorem 3.5, for $E_1, \ldots, E_n$ Asplund spaces, the mapping Ψ is a metric surjection.

The next remark will be very useful. It can be proved following carefully the proof of Proposition 3.4 and using Proposition 3.6.

Remark 3.7. Let E be an Asplund space and $A : \ell_\infty(B_{E'})' \to \ell_1(B_{E'})$ be the operator obtained by the Lewis–Stegall theorem with $\|A\| \leqslant 1 + \varepsilon$ as in diagram (7). Given $\varphi \in (\widetilde{\bigotimes}_{i=1}^n E, /\alpha'\backslash)'$, if we take a Hahn–Banach extension $\widetilde{\varphi} \in (\widetilde{\bigotimes}_{i=1}^n \ell_\infty(B_{E'}), /\alpha'\backslash)'$, then the linear functional $\rho \in (\widetilde{\bigotimes}_{i=1}^n c_0(B_{E'}), /\alpha'\backslash)'$ given by

$$\rho(a_1, \ldots, a_n) := (EXT)(\widetilde{\varphi})\big(A' J(a_1), \ldots, A' J(a_n)\big), \tag{8}$$

satisfies $\Psi(\rho) = \varphi$ and $\|\rho\| \leqslant \|\varphi\|(1 + \varepsilon)^n$.

We end this section with the statement of the non-symmetric versions of Theorem 2.9, Corollary 2.11 and Corollary 2.12, which readily follow:

Theorem 3.8. *Let $E_1, \ldots, E_n$ be Banach spaces and α a full tensor norm with sRN. The tensor product $(E_1 \widetilde{\otimes} \cdots \widetilde{\otimes} E_n, /\alpha'\backslash)$ is Asplund if and only if E_i is Asplund for $i = 1, \ldots, n$.*

Corollary 3.9. *Let α be a projective full tensor norm with the sRN property and $E_1, \ldots, E_n$ dual Banach spaces with the bounded approximation property. Then, $(E_1 \widetilde{\otimes} \cdots \widetilde{\otimes} E_n, \alpha)$ has the Radon–Nikodým property if and only if each E_i does.*

Corollary 3.10. *Let α be a projective full tensor norm with the sRN property and $E_1, \ldots, E_n$ be Banach spaces.*

(a) *If each E_i has a boundedly complete Schauder basis, then so does $(E_1 \widetilde{\otimes} \cdots \widetilde{\otimes} E_n, \alpha)$.*
(b) *If each E_i has a shrinking Schauder basis, then so does $(E_1 \widetilde{\otimes} \cdots \widetilde{\otimes} E_n, \alpha')$.*

4. The proof of Theorem 2.2 and some questions

To prove Theorem 3.5 we used a multilinear version of the *Extension Lemma* whose proof follows identical to the one in [15, 6.7]. For polynomials a similar result is needed:

Proposition 4.1. *(See [11, Corollary 3.4].) Let β a finitely generated s-tensor norm. For each $P \in (\widetilde{\bigotimes}_\beta^{n,s} E)'$ its Aron–Berner extension $AB(P)$ of P belongs to $(\widetilde{\bigotimes}_\beta^{n,s} E'')'$ and*

$$\|P\|_{(\widetilde{\bigotimes}_\beta^{n,s} E)'} = \big\|AB(P)\big\|_{(\widetilde{\bigotimes}_\beta^{n,s} E'')'}.$$

This was obtained as a consequence of the isometry of the iterated extension to ultrapowers for maximal polynomial ideals. However, this can also be proved without the ultrapower techniques: just use the principle of local reflexivity instead of local determination of ultrapowers and proceed as in [11].

Proof of Theorem 2.2. As in the multilinear case, the next diagram commutes:

$$\begin{tikzcd}
\widetilde{\bigotimes}^{n,s}_{\backslash\beta/}\ell_1(B_{E'}) \arrow[r, two heads] \arrow[d, two heads, "\widetilde{\otimes}^{n,s}P"] & (\widetilde{\bigotimes}^{n,s}_{/\beta'\backslash}c_0(B_{E'}))' \arrow[d, "\Psi"] \\
\widetilde{\bigotimes}^{n,s}_{\backslash\beta/}E' \arrow[r] & (\widetilde{\bigotimes}^{n,s}_{/\beta'\backslash}E)'
\end{tikzcd}$$

where Ψ is the composition mapping

$$\Big(\widetilde{\bigotimes}^{n,s}_{/\beta'\backslash}c_0(B_{E'})\Big)' \xrightarrow{AB} \Big(\widetilde{\bigotimes}^{n,s}_{/\beta'\backslash}\ell_\infty(B_{E'})\Big)' \xrightarrow{(\widetilde{\otimes}^{n,s}I)'} \Big(\widetilde{\bigotimes}^{n,s}_{/\beta'\backslash}E\Big)'.$$

Fix $P \in (\widetilde{\bigotimes}^{n,s}_{/\beta'\backslash}E)'$. Denote by $\overline{P} \in (\widetilde{\bigotimes}^{n,s}_{/\beta'\backslash}\ell_\infty(B_{E'}))'$ a Hahn–Banach extension of P and by A an operator obtained from the Lewis–Stegall theorem such that $\|A\| \leqslant 1+\varepsilon$ (see diagram (7)). Since the Aron–Berner is an isometry for maximal ideals (Proposition 4.1), as in Remark 3.7, the linear functional $L \in (\widetilde{\bigotimes}^{n,s}_{/\beta'\backslash}c_0(B_{E'}))'$ given by $L(a) := (AB)(\overline{P})(A' J_{c_0(B_{E'})}a)$ satisfies that $\Psi(L) = P$ and $\|L\|_{(\widetilde{\otimes}^{n,s}_{/\beta'\backslash}c_0(B_{E'}))'} \leqslant \|P\|_{(\widetilde{\otimes}^{n,s}_{/\beta'\backslash}E)'}(1+\varepsilon)^n$. Thus, Ψ is a metric surjection and, by the diagram, we easily get that $\widetilde{\bigotimes}^{n,s}_{\backslash\beta/}E' \to (\widetilde{\bigotimes}^{n,s}_{/\beta'\backslash}E)'$ is also a metric surjection. $\square$

We conclude the article with a couple of questions:

Since we do not know of any example of an s-tensor without the sRN property, we ask: *Does every s-tensor norm have the sRN property*?

A more precise, but not necessarily easier, question is the following: *does* $/\pi_s\backslash$ *have the sRN property*? In the case of a positive answer we would conclude that every natural s-tensor norm (see [12]) have this property.

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