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Integral representation of holomorphic functions on Banach spaces

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Abstract

In this paper we discuss the problem of integral representation of analytic functions over a complex Banach space E. We obtain, for a wide class of functions, integral representations of the form

$$f(x) = \int_{E'} e^{\gamma(x)} \overline{f_1(\gamma)}\, dW(\gamma) \quad \text{and} \quad f(x) = \int_{E'} \frac{1}{1 - \frac{\gamma(x)}{\|\gamma\|}} \overline{f_2(\gamma)}\, dW(\gamma),$$

where W is an abstract Wiener measure on E' and f_1, f_2 are transformations of f involving the covariance operator of W.

Keywords: Integral representation; Cauchy integral formula; Gaussian measures

Introduction

The Cauchy integral formula has no true analogue in infinite-dimensional holomorphy. The usual generalisation, though quite useful, is essentially the one-dimensional formula

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in each direction: given x and $|\lambda| < r$,

$$f(\lambda x) = \frac{1}{2\pi i} \int_{|\omega|=r} \frac{f(\omega x)}{\omega - \lambda} \, d\omega.$$

In this paper, we discuss the problem of generalising the Cauchy integral formula to infinite-dimensional Banach spaces, and derive two related formulas valid for a wide class of functions on such spaces.

Integral expressions valid for some homogeneous polynomials and some holomorphic functions on a Banach space E have been proposed, all of which involve integration over the dual space E' rather than E. An integral k-homogeneous polynomial over E [4], for example, is

$$P(x) = \int_{B_{E'}} \gamma(x)^k \, d\mu(\gamma).$$

We begin our discussion by considering the Cauchy integral formula on C, and possible generalisations to larger spaces. The Cauchy integral formula

$$f(z) = \frac{1}{2\pi i} \int_{S^1} \frac{f(\omega)}{\omega - z} \, d\omega, \tag{1}$$

may be written, after a change in variables, as

$$f(z) = \int_{S^1} \frac{1}{1 - z\bar{\omega}} f(\omega) \, dP(\omega), \tag{2}$$

where P is normalised Lebesgue measure on the circle. One may also write

$$f(z) = \int_{C} \frac{1}{1 - z\frac{\bar{\omega}}{|\bar{\omega}|}} f\left(\frac{\omega}{|\omega|}\right) dG(\omega), \tag{3}$$

where G is a Gaussian measure on C.

In the following heuristic introduction, we consider the problem of generalising these formulas to n or infinite-dimensional complex spaces. We refer to [5] for infinite-dimensional holomorphy.

In n-dimensional space the first formula leads to the Cauchy formula over the polydisc, where integration is over the distinguished boundary. Only rarely (i.e., in ℓ^∞) can the infinite-dimensional polydisc be found in a Banach space; also, division by $\omega - z$ seems meaningless, so we do not pursue this idea.

The second formula involves integration on the sphere and makes some sense if one is willing to take $\omega \in E'$. This and the consideration of integral polynomials is what gave rise to the definition of integral holomorphic function $f : B_E^\circ \to C$ in [3],

$$f(x) = \int_{B_{E'}} \frac{1}{1 - \gamma(x)} \, d\mu(\gamma).$$

In this expression, integration is over the unit ball of E' (the w^*-closure of the unit sphere) and μ is a regular Borel measure on $(B_{E'}, w^*)$. The measure μ is said to represent the function f, but there are many such representing μ's, and little has been said in the way of expressing these measures in terms of the function f, for example as $d\mu(\gamma) = \tilde{f}(\gamma)\, dM(\gamma)$, where $\tilde{f}$ is some transformation of f and M a universal measure (i.e., the same measure for all f). Consider the problem in C^n. For the second formula to hold one sees after expanding the functions $\frac{1}{1-\langle z,\omega\rangle}$ and f in Taylor series, that we would need (integrating on the $(2n-1)$-dimensional sphere S)

$$\int_S \omega^\beta \bar{\omega}^\alpha \, dP(\omega) = \delta_{\alpha\beta} \frac{\alpha!}{|\alpha|!}.$$

However, one obtains

$$\int_S \omega^\beta \bar{\omega}^\alpha \, dP(\omega) = \delta_{\alpha\beta} \frac{\alpha!}{|\alpha|!} \binom{|\alpha| + n - 1}{n - 1}^{-1}.$$

This can be compensated for by multiplying each k-homogeneous term in the integral by the combinatorial number $\binom{k+n-1}{n-1}$. When assigned to $\frac{1}{1-\langle z,\omega\rangle}$, the Szegö kernel for the sphere S appears. It could also be assigned to f obtaining a transform f_n of f for which the formula

$$f(z) = \int_S \frac{1}{1 - \langle z, \omega\rangle} f_n(\omega)\, dP(\omega)$$

holds. However, both the Szegö kernels and the f_n's tend to infinity as the dimension n increases. Thus, this approach also seems inappropriate for infinite-dimensional holomorphy.

Consider now the third formula and proceed as above to generalise it to C^n. One is led to calculate

$$\int_{C^n} \omega^\beta \bar{\omega}^\alpha \, dG(\omega) = \delta_{\alpha\beta} \frac{\alpha!}{|\alpha|!} |\alpha|!.$$

Here the extant factor is $|\alpha|!$. This can be compensated for by multiplying each k-homogeneous term in the integral by $1/k!$, independently of the dimension n. When assigned to $\frac{1}{1-\langle z,\omega\rangle}$, one obtains the formula

$$f(z) = \int_{C^n} e^{\langle z,\omega\rangle} f(\omega)\, dG(\omega). \tag{A}$$

Alternatively, it can be assigned to f, obtaining a transform $f^\diamond$ of f which is entire if f has radius of convergence 1. Normalising in $\frac{1}{1-\langle z,\omega\rangle}$ to avoid poles, one obtains

$$f(z) = \int_{C^n} \frac{1}{1 - \left\langle z, \frac{\omega}{\|\omega\|}\right\rangle} f^\diamond\big(\|\omega\|\omega\big)\, dG(\omega). \tag{B}$$

These last two integral formulas, (A) for entire functions and (B) for functions holomorphic on the unit ball, are the ones that we will prove in C^n and then generalise to infinite-dimensional Banach spaces.

In Section 1 we recall and adapt to our needs some of the elements of the theory of abstract Wiener spaces. Formulas (A) and (B) are proved for C^n and for an infinite-dimensional Banach space in Section 2. The functions to which they apply are presented rather abstractly in this section. Finally, we discuss in Section 3 more tractable classes of functions for which these results hold.

1. The measures

We need to consider Gaussian measures on Banach spaces and will use the theory of abstract Wiener spaces (see [8,10]). Since our spaces will be complex, we need to review some of the interplay between the real and complex structures. This will also serve to fix notation.

Let H be a separable complex Hilbert space and denote its inner product by $\langle\,,\rangle$. H of course also has a real Hilbert space structure, which we will denote by H_R. Its inner product is $\langle\,,\rangle_R = \mathrm{Re}\langle\,,\rangle$. Note that x and ix are orthogonal in H_R. Also, fix (e_n), an orthonormal basis of H. Then (e_n, ie_n) is an orthonormal basis of H_R. The duals of both spaces are (real) isometric, via the mapping $H'_R \to H'$, $\phi \mapsto \tilde{\phi}$ where $\tilde{\phi}(x) = \phi(x) - i\phi(ix)$. As always, H' can be identified with H via $I : H' \to H$ such that for $x \in H$ and $\phi \in H'$, $\phi(x) = \langle x, I(\phi)\rangle$. The isomorphism I is *conjugate linear*. Our need for analyticity will lead us to correct the lack of linearity of I with involutions in H and H'. If $x = \sum x_n e_n$ is an element of H, we denote $x^* = \sum \overline{x_n} e_n$. Note that $\langle x^*, y\rangle = \overline{\langle x, y^*\rangle}$. Similarly, if $\phi \in H'$, define ϕ^* so that $I(\phi^*) = I(\phi)^*$, and note that $\phi(x^*) = \overline{\phi^*(x)}$. These involutions depend on the basis chosen in H. The same involution is obtained—however—if a basis (f_n) is used, for which $f_n^* = f_n$.

A complex-valued Gaussian random variable (with mean m and variance σ^2) is one whose density function $f : C \to R$ is

$$f(w) = \frac{1}{\pi\sigma^2} e^{-\frac{|w-m|^2}{\sigma^2}}.$$

Its real and imaginary parts are independent real-valued Gaussian random variables with mean $\mathrm{Re}\, m$ and $\mathrm{Im}\, m$ and variance $\frac{\sigma^2}{2}$.

If P is a finite-rank orthogonal projector in H, a cylinder set in H is a set of the form

$$A = \{x \in H:\ Px \in B\},$$

where B is a Borel subset of PH. The collection of such sets is a field, but not a σ-field. We will denote by Γ the Gaussian cylinder measure defined on cylinder sets:

$$\Gamma(A) = \frac{1}{\pi^n} \int_B e^{-|w|^2}\, dw,$$

where n is the complex dimension of PH, and the integral is with respect to Lebesgue measure. Γ is not a true measure (it is not σ-additive). However, integrals of cylinder functions ($F : H \to C$ of the form $F = h \circ P$) may be defined by setting

$$\int_A F\,d\Gamma = \int_B h\,dG,$$

where G is standard n-dimensional Gaussian measure. Note also that the involution $*$ is Γ-preserving.

Elements $\phi \in H'$ are complex-valued Gaussian random variables with mean 0 and variance $\langle \phi, \phi \rangle$. Note that elements in $\alpha \in H'_R$ are real-valued random variables with mean 0 and variance $\frac{1}{2}\langle \alpha, \alpha \rangle$.

Consider on H a norm $\|.\|$ with the following property: given any $\varepsilon > 0$ there is a finite-rank orthogonal projector P_ε such that for all $P \perp P_\varepsilon$,

$$\Gamma\left\{x \in H\colon\ \|Px\| > \varepsilon\right\} < \varepsilon.$$

Such a norm is called *measurable* [8]. If S is a Hilbert–Schmidt operator on H, $\|.\|_S = \langle S(.), S(.)\rangle^{\frac{1}{2}}$ is an example of a measurable norm. Upon completing $(H, \|.\|)$ one obtains a Banach space X. $\iota : H \hookrightarrow X$ is called an abstract Wiener space. The inclusion ι is continuous and dense. Given $x'_1, \ldots, x'_n \in X'$ and a Borel set $B \subset C^n$, one defines, if $C_X = \{x \in X\colon\ (x'_1(x), \ldots, x'_n(x)) \in B\}$, and $C_H = C_X \cap H$,

$$\tilde{\Gamma}(C_X) = \Gamma(C_H).$$

$\tilde{\Gamma}$ is σ-additive, and extends to a measure W (called Wiener measure) on the Borel σ-algebra $\mathcal{B}$ of X. This generalises the situation $C^1 \hookrightarrow C[0, 1]$ giving rise to the 'original' Wiener measure on $C[0, 1]$. We shall use the following important theorems. It is easily checked by following the proofs in [8] for example, where the real versions of these theorems are given, that the complex versions hold.

Theorem 1.1 (Gross [7]). *If X is a separable Banach space, there is a Hilbert space H such that $\iota : H \hookrightarrow X$ is an abstract Wiener space. Furthermore, there is a smaller abstract Wiener space $H \hookrightarrow X_0 \hookrightarrow X$ and an increasing sequence of finite-rank orthogonal projectors (p_n) converging to the identity in H; these extend to P_n on X_0 where they converge to the identity as well. Also, $W(X_0) = 1$.*

Theorem 1.2 (Fernique [6]). *There is an $\varepsilon > 0$ such that*

$$\int_X e^{\varepsilon\|x\|^2}\,dW(x) < \infty.$$

If $\iota' : X' \to H'$ is the transpose of the inclusion ι, we define $\iota^* = * \circ \iota'$. In the following commutative diagram:

$$\begin{array}{ccc} H & \xrightarrow{\ \iota\ } & X \\ \uparrow{\scriptstyle I} & & \uparrow{\scriptstyle A} \\ H' & \xleftarrow[\ \iota^*\ ]{} & X' \end{array}$$

the arrow on the right, $A = \iota \circ I \circ \iota^*$ is called the *covariance operator* of W. ι^* is dense and one-to-one and we can choose $(z_n) \subset X'$ such that $\iota^*(z_n) = e'_n$, the orthonormal basis of H' dual to (e_n). Note that (z_n) is a sequence of independent Gaussian complex random variables with mean 0 and variance 1 on the probability space $(X, \mathcal{B}, W)$. We will denote by T the (densely-defined and unbounded) inverse of the operator A.

2. The formulas

We now prove finite-dimensional versions of formulas (A) and (B) of the Introduction. G will denote standard Gaussian measure on C^n. The projections to the ith coordinates are normally distributed complex-valued random variables with mean zero and variance 1. These therefore have density functions

$$\frac{1}{\pi} e^{-|z|^2}.$$

$\alpha = (\alpha_1, \dots, \alpha_n)$ and $\beta = (\beta_1, \dots, \beta_n)$ will be multi-indices. We will employ the usual notations: $|\alpha| = \alpha_1 + \cdots + \alpha_n$; $\alpha! = \alpha_1! \cdots \alpha_n!$; $\omega^\alpha = \omega_1^{\alpha_1} \cdots \omega_n^{\alpha_n}$. In the following lemmas, N denotes any norm $N : C^n \to [0, \infty)$.

Lemma 2.1. *Let G be standard Gaussian measure on C^n. Then*

$$\int_{C^n} \omega^\alpha \bar{\omega}^\beta \, dG(\omega) = \int_{C^n} \omega^\alpha \bar{\omega}^\beta N(\omega)^{|\alpha|-|\beta|} \, dG(\omega) = \delta_{\alpha\beta} \alpha!.$$

Proof. When $\alpha \neq \beta$ the integrals are zero by considerations of symmetry. If $\alpha = \beta$,

$$\begin{aligned}\int_{C^n} \omega^\alpha \bar{\omega}^\beta \, dG(\omega) &= \int_{C^n} \omega^\alpha \bar{\omega}^\beta N(\omega)^{|\alpha|-|\beta|} \, dG(\omega) = \int_{C^n} |\omega_1|^{2\alpha_1} \cdots |\omega_n|^{2\alpha_n} \, dG(\omega) \\ &= \prod_{i=1}^n \int_C |\omega_i|^{2\alpha_i} \, dG_i(\omega_i) = \prod_{i=1}^n \frac{1}{\pi} \int_C |\omega_i|^{2\alpha_i} e^{-|\omega_i|^2} \, d\omega_i \\ &= \prod_{i=1}^n \int_0^\infty \rho^{2\alpha_i} e^{-\rho^2} 2\rho \, d\rho = \prod_{i=1}^n \int_0^\infty u^{\alpha_i} e^{-u} \, du = \prod_{i=1}^n \alpha_i! = \alpha!. \qquad \square\end{aligned}$$

Lemma 2.2. *Let $h : C^n \to C$ be an entire function in $L^p(G)$ $(p > 1)$. Then for every $z \in C^n$,*

$$h(z) = \int_{C^n} e^{\langle z, \omega \rangle} h(\omega) \, dG(\omega).$$

Proof. Since $e^{\langle z, \omega \rangle}$ is in $L^q(G)$ for all $q < \infty$, the integral exists by Hölder. Write

$$h(\omega) = \sum_{k \geqslant 0} \sum_{|\alpha|=k} a_\alpha \omega^\alpha \quad \text{and} \quad e^{\langle z, \omega \rangle} = \sum_{j \geqslant 0} \frac{1}{j!} \left(\sum_{i=1}^n z_i \overline{\omega_i} \right)^j = \sum_{j \geqslant 0} \sum_{|\beta|=j} \frac{1}{\beta!} z^\beta \bar{\omega}^\beta,$$

and note that these series are uniformly convergent in any ball. Now

$$\begin{aligned}\int_{C^n} e^{\langle z,\omega\rangle} h(\omega)\,dG(\omega) &= \int_{C^n} \sum_{j\geqslant 0}\sum_{|\beta|=j} \frac{1}{\beta!} z^\beta \overline{\omega}^\beta \sum_{k\geqslant 0}\sum_{|\alpha|=k} a_\alpha \omega^\alpha\,dG(\omega)\\ &= \sum_{j\geqslant 0}\sum_{|\beta|=j}\sum_{k\geqslant 0}\sum_{|\alpha|=k} a_\alpha z^\beta \frac{1}{\beta!}\int_{C^n} \omega^\alpha \bar{\omega}^\beta\,dG(\omega)\\ &= \sum_{j\geqslant 0}\sum_{|\beta|=j}\sum_{k\geqslant 0}\sum_{|\alpha|=k} a_\alpha z^\beta \frac{1}{\beta!}\delta_{\alpha\beta}\alpha!\\ &= \sum_{k\geqslant 0}\sum_{|\alpha|=k} a_\alpha z^\alpha = h(z). \qquad \square\end{aligned}$$

Lemma 2.3. *Let B be the open unit ball of C^n and $h : B \to C$ analytic. If $h(\omega) = \sum_{k\geqslant 0}\sum_{|\alpha|=k} a_\alpha \omega^\alpha$, define $h^\diamond : C^n \to C$ by $h^\diamond(\omega) = \sum_{k\geqslant 0}\frac{1}{k!}\sum_{|\alpha|=k} a_\alpha\omega^\alpha$. Then if $h^\diamond(N(\omega)\omega) \in L^1(G)$, for any z such that* $\sup_\omega |\langle z, \frac{\omega}{N(\omega)}\rangle| < 1$, *we have*

$$h(z) = \int_{C^n} \frac{1}{1-\langle z, \frac{\omega}{N(\omega)}\rangle} h^\diamond\big(N(\omega)\omega\big)\,dG(\omega).$$

Proof. Note that since h has radius of convergence one, $h^\diamond$ is entire. Also,

$$\left|\left\langle z, \frac{\omega}{N(\omega)}\right\rangle\right| \leqslant c < 1 \quad\text{so}\quad \left|\frac{1}{1-\langle z, \frac{\omega}{N(\omega)}\rangle}\right| \leqslant \frac{1}{1-c},$$

and the integral exists. Define I_r to be the indicator function of the set $\{\omega\colon |\omega_i| \geqslant \frac{1}{r}$ for all $i\}$ and write

$$\frac{I_r(\omega)}{1-\langle z, \frac{\omega}{N(\omega)}\rangle} = I_r(\omega)\sum_{j\geqslant 0}\left(\sum_{i=1}^n \frac{z_i\overline{\omega_i}}{N(\omega)}\right)^j = I_r(\omega)\sum_{j\geqslant 0}\sum_{|\beta|=j}\frac{j!}{\beta!}\frac{z^\beta\bar{\omega}^\beta}{N(\omega)^j}.$$

Then the integral is the limit as $r \mapsto \infty$ of

$$\begin{aligned}&\int_{C^n} \frac{I_r(\omega)}{1-\langle z, \frac{\omega}{N(\omega)}\rangle} h^\diamond\big(N(\omega)\omega\big)\,dG(\omega)\\ &\quad= \int_{C^n} I_r(\omega)\sum_{j\geqslant 0}\sum_{|\beta|=j}\frac{j!}{\beta!}\frac{1}{N(\omega)^j} z^\beta\bar{\omega}^\beta \sum_{k\geqslant 0}\frac{1}{k!}\sum_{|\alpha|=k} a_\alpha N(\omega)^k\omega^\alpha\,dG(\omega)\\ &\quad= \sum_{j\geqslant 0}\sum_{|\beta|=j}\sum_{k\geqslant 0}\sum_{|\alpha|=k} a_\alpha z^\beta \frac{j!}{k!\beta!}\int_{C^n} I_r(\omega)\omega^\alpha\bar{\omega}^\beta N(\omega)^{|\alpha|-|\beta|}\,dG(\omega),\end{aligned}$$

which, as in Lemma 2.1, converges to

$$\sum_{j\geqslant 0}\sum_{|\beta|=j}\sum_{k\geqslant 0}\sum_{|\alpha|=k} a_\alpha z^\beta\frac{j!}{k!\beta!}\delta_{\alpha\beta}\alpha! = \sum_{k\geqslant 0}\sum_{|\alpha|=k} a_\alpha z^\alpha = h(z). \qquad \square$$

In order to extend these formulas to an infinite-dimensional Banach space setting, suppose X is a separable Banach space and consider the abstract Wiener space $\iota : H \hookrightarrow X$ given by Gross' theorem.

We will need to define the following transformation for holomorphic functions $F : H \to C$. Recall [5] that F can be expressed locally by its Taylor series expansion $F = \sum F_k$ where F_k are continuous k-homogeneous polynomials, i.e., $F_k(x) = \phi_k(x, \dots, x)$, with ϕ_k continuous symmetric k-linear functionals. Consider $x, \dots, y \in H$. Then

$$\phi_k(x, \dots, y) = \phi_k\left(\sum_{i_1} x_{i_1} e_{i_1}, \dots, \sum_{i_k} y_{i_k} e_{i_k}\right)$$
$$= \sum_{i_1} \dots \sum_{i_k} x_{i_1} \cdots y_{i_k} \phi_k(e_{i_1}, \dots, e_{i_k}).$$

We define $\phi_k^\sharp(x, \dots, y) = \sum_{i_1} \dots \sum_{i_k} x_{i_1} \cdots y_{i_k} \overline{\phi_k(e_{i_1}, \dots, e_{i_k})}$. Note that if * is the involution defined in the previous section,

$$\phi_k\left(x^*, \dots, y^*\right) = \overline{\phi_k^\sharp(x, \dots, y)}.$$

Define $F_k^\sharp(x) = \phi_k^\sharp(x, \dots, x)$ and $F^\sharp = \sum_k F_k^\sharp$, and note that

$$F\left(x^*\right) = \overline{F^\sharp(x)}.$$

$F^\sharp : H \to C$ is just as holomorphic as F; its 'coefficients' have been conjugated.

Denote by $\|.\|_0$ the norm of the space X_0 in Gross' theorem. For the following theorem we will consider functions of the following type. Let $F : H \to C$ be holomorphic and such that $F = \tilde{F} \circ \iota$, with $\tilde{F} : X \to C$ some $L^p(W)$-integrable function $(p > 1)$ which is $\|.\|_0$-continuous on X_0. We will also require that $\tilde{F} \circ P_n$ (P_n, the extended projections in Gross' theorem) be almost surely bounded by $g \in L^p(W)$. We will give examples of such functions in the next section. Note that $\overline{\widetilde{F^\sharp}} = \tilde{F} \circ \iota \circ *$, so $\widetilde{F^\sharp}$ is in $L^p(W)$ when $\tilde{F}$ is.

Set $f = F \circ I \circ \iota^* : X' \to C$. Note that f is holomorphic on X' and denote $f^\sharp = F^\sharp \circ I \circ \iota^*$. Thus on X

$$f^\sharp = F^\sharp \circ I \circ \iota^* = \widetilde{F^\sharp} \circ \iota \circ I \circ \iota^* = \widetilde{F^\sharp} \circ A,$$

where A is the covariance operator of the Wiener measure W. Recall that T is the densely-defined unbounded inverse of the covariance operator A, so

$$f^\sharp \circ T = \widetilde{F^\sharp} \circ A \circ T = \widetilde{F^\sharp}$$

on the dense subspace $\operatorname{Im} A$. We denote with $f^\sharp \circ T$ the class of $\widetilde{F^\sharp}$ in $L^p(W)$. We then have the following theorem.

Theorem 2.4. *For f as above and $z \in X'$,*

$$f(z) = \int_X e^{z(\gamma)} \overline{(f^\sharp \circ T)}(\gamma)\, dW(\gamma). \tag{A}$$

Proof. We use, throughout, the notation introduced in Gross' theorem. The composition $F \circ p_n$ is a holomorphic cylinder function on H, thus by Lemma 2.2 for any $x \in H$ we have, since $\langle p_n x, y\rangle = \langle x, p_n y\rangle$,

$$(F \circ p_n)(x) = \int_H e^{\langle x, p_n y\rangle} (F \circ p_n)(y)\, d\Gamma(y).$$

The left-hand side of course converges to $F(x)$, which for $x = I\iota^*(z)$ is $F(I\iota^*(z)) = f(z)$. Also,

$$\begin{aligned}\langle x, p_n y\rangle &= \langle I\iota^*(z), p_n y\rangle = \overline{\langle p_n y, I\iota^*(z)\rangle} = \overline{\iota^*(z)(p_n y)} = \overline{(z \circ \iota)^*(p_n y)} \\ &= (z \circ \iota)(p_n y^*) = z(\iota(p_n y^*)),\end{aligned}$$

so the integral above is

$$\int_H e^{z(\iota(p_n y^*))} (F \circ p_n)(y)\, d\Gamma(y).$$

Since the involution * is Γ-preserving and $F(x) = \overline{F^\sharp(x^*)}$, we have

$$\begin{aligned}\int_H e^{z(\iota(p_n y^*))}\, \overline{F^\sharp(p_n y^*)}\, d\Gamma(y^*) &= \int_H e^{z(\iota(p_n y))}\, \overline{F^\sharp(p_n y)}\, d\Gamma(y) \\ &= \int_{X_0} e^{z(P_n\gamma)}\, \overline{\widetilde{F^\sharp}(P_n\gamma)}\, dW(\gamma).\end{aligned}$$

Recall that P_n converge to the identity on X_0, a set of W-measure one, so the integrands converge almost surely to $e^{z(\gamma)}\, \overline{\widetilde{F^\sharp}}(\gamma)$. Also, we have the bounds

$$\left|e^{z(P_n\gamma)}\, \overline{\widetilde{F^\sharp}(P_n\gamma)}\right| \leqslant e^{\|z\| \|P_n\gamma\|_0} \left|(\widetilde{F^\sharp} \circ P_n)(\gamma)\right| \leqslant e^{c\|z\|\|\gamma\|_0} g(\gamma),$$

an integrable function, since $g \in L^p(W)$ for some $p > 1$ and $e^{c\|z\|\|\gamma\|_0}$ is in $L^q(W)$ for all $q < \infty$. Indeed,

$$\left(e^{c\|z\|\|\gamma\|_0}\right)^q = e^{qc\|z\|\|\gamma\|_0} \leqslant M e^{\varepsilon\|\gamma\|_0^2}$$

for small ε and large enough M, and the latter function is integrable by Fernique's theorem. Thus, applying the Lebesgue dominated convergence theorem, the integrals above converge to

$$\int_{X_0} e^{z(\gamma)}\, \overline{\widetilde{F^\sharp}(\gamma)}\, dW(\gamma),$$

which is

$$\int_X e^{z(\gamma)}\, \overline{(f^\sharp \circ T)}(\gamma)\, dW(\gamma). \qquad \square$$

Recall [5] that the Taylor series expansion $f = \sum_k f_k$ of a holomorphic function in infinite dimensions converges uniformly in some neighborhood around the point of expansion, but the radius of uniform convergence need not be as large as the distance to the complement of the domain of f. A case in point is the function $f : c_0 \to C$ defined by

$$f(x) = \sum_k x_1 \cdots x_k$$

which is entire, but with radius of uniform convergence equal to one. The radius of uniform convergence may be calculated as

$$r = \frac{1}{\limsup \|f_k\|^{1/k}},$$

where $\|f_k\|$ is the norm of the k-homogeneous polynomial f_k (i.e., the infimum of the numbers c such that $|f_k(x)| \leqslant c\|x\|^k$). Functions whose Taylor series have infinite radius of uniform convergence are bounded on bounded subsets. Such functions are said to be of bounded type.

Now for the extension of formula (B), let B_H° denote the open unit ball of H, and $F : B_H^\circ \to C$ a holomorphic function defined on B_H°, whose Taylor series expansion about 0, $F = \sum_k F_k$, has radius of uniform convergence at least one. Define

$$F^\diamond = \sum_k \frac{1}{k!} F_k.$$

$F^\diamond$ is then holomorphic on all of H, indeed, its Taylor series expansion about 0 has infinite radius of convergence, so $F^\diamond$ is of bounded type. Set $f = F \circ I \circ \iota^*$. The function f is holomorphic on the open unit ball of X', and the series $f = \sum_k F_k \circ I \circ \iota^*$, has radius of uniform convergence at least one. Then $f^\diamond = F^\diamond \circ I \circ \iota^* = \sum_k \frac{1}{k!}(F_k \circ I \circ \iota^*)$ is a holomorphic function of bounded type on X'. We will use the notations $f^{\diamond\sharp}$, $\widetilde{F^\diamond}$, etc. as above. Also we will require of $\widetilde{F^\diamond}$ that it be in $L^1(W)$, $\|.\|_0$-continuous on X_0, and that $\widetilde{F^\diamond}(\delta\|P_n\gamma\|_X P_n\gamma)$ be almost surely bounded by $g \in L^1(W)$ for some $\delta > 0$. We then have the following.

Theorem 2.5. *For f as above and any $z \in \delta B_{X'}^\circ$,*

$$f(z) = \int_X \frac{1}{1 - z\left(\frac{\gamma}{\delta\|\gamma\|}\right)} \overline{(f^{\diamond\sharp} \circ T)}(\delta\|\gamma\|\gamma)\, dW(\gamma). \tag{B}$$

Proof. The proof is analogous to that of the previous theorem. Use Lemma 2.3 on $p_n H$ with $N(p_n y) = \delta\|\iota p_n y\|_X$ to obtain

$$(F \circ p_n)\left(I\iota^* z\right) = \int_H \frac{1}{1 - z\left(\frac{\iota p_n y}{\delta\|\iota p_n y\|_X}\right)} \overline{F^{\sharp\diamond}(\delta\|\iota p_n y\|_X p_n y)}\, d\Gamma(y),$$

which is

$$\int_{X_0} \frac{1}{1 - z\left(\frac{P_n\gamma}{\delta\|P_n\gamma\|_X}\right)} \overline{\widetilde{F^{\sharp\diamond}}(\delta\|P_n\gamma\|_X P_n\gamma)}\, dW(\gamma).$$

The integrands converge to $\frac{1}{1-z(\frac{\gamma}{\delta\|\gamma\|_X})}\overline{\widetilde{F^{\sharp\diamond}}(\delta\|\gamma\|_X\gamma)}$ on X_0. Also, one has the bounds

$$\left|\frac{1}{1-z\left(\frac{P_n\gamma}{\delta\|P_n\gamma\|_X}\right)}\overline{\widetilde{F^{\sharp\diamond}}(\delta\|P_n\gamma\|_X P_n\gamma)}\right| \leqslant \frac{1}{1-\frac{\|z\|}{\delta}}\left|\widetilde{F^{\sharp\diamond}}\big(\delta\|P_n\gamma\|_X P_n\gamma\big)\right| \leqslant \frac{1}{1-\frac{\|z\|}{\delta}} g(\gamma) \in L^1(W).$$

The rest of the proof proceeds as before. □

The theorem could have been written with $\delta = 1$, given the conditions imposed on $\widetilde{F^{\diamond}}$. The role of δ will become clearer in the next section, and is related to the following fact. In the n-dimensional setting the function $e^{a|\omega|^2}$ is G-integrable for any $a < 1$, and the integral is $\frac{1}{(1-a)^n}$. In the infinite-dimensional setting, however, Fernique's theorem assures the integrability of $e^{\varepsilon\|\gamma\|^2}$ only for some sufficiently small ε.

3. The functions

We have proven the integral formulas (A) and (B) for holomorphic functions $f : X' \to C$ which 'extend' holomorphically to $F : H \to C$, in the sense that $f = F \circ I \circ \iota^* = \tilde{F} \circ A$, provided that $\tilde{F} \circ P_n$ are almost surely bounded by $g \in L^p(W)$.

The fact that $f \circ T = \tilde{F}$ on Im A, with $\tilde{F} \in L^p(W)$, can be viewed as an integrability condition, while $f \circ T \circ \iota = F$, with F holomorphic, is essentially a continuity condition. In this section we wish to study classes of functions verifying such conditions but which are more tractable.

The first of these classes is related to growth conditions on the function f. Note that since the covariance operator A is one-to-one, $\|.\|_A = \|A(.)\|$ is a norm on X' (stronger than its usual norm). We will denote with r_A the radii of uniform convergence calculated with this norm,

$$r_A = \frac{1}{\limsup \|f_k\|_A^{1/k}}.$$

Note that $r_A \leqslant r$, the usual radius of convergence.

Definition 1. We will say that $f : X' \to C$ is of *A-exponential type* if there are positive constants c and σ such that for all $z \in X'$,

$$\left|f(z)\right| \leqslant c e^{\sigma\|z\|_A}.$$

Proposition 3.1. *Let f be a Gateaux-holomorphic function on X'. Then the following are equivalent*:

(i) *f is of A-exponential type.*

(ii) *For each* $a \in X'$, $f(z) = \sum_k f_{k,a}(z-a)$ *with* $f_{k,a}$ k*-homogeneous polynomials which are continuous for the norm* $\|.\|_A$ *and*

$$\|f_{k,a}\|_A \leqslant C_{\|a\|_A} \frac{\sigma^k}{k!}.$$

(iii) $f(z) = \sum_k f_k(z)$ *with* f_k k*-homogeneous polynomials which are continuous for the norm* $\|.\|_A$ *and*

$$\|f_k\|_A \leqslant C \frac{\sigma^k}{k!}.$$

Thus if (i), (ii), *or* (iii) *hold,* f *is Fréchet-holomorphic,* $r_A = \infty$ *and* f *is uniformly* $\|.\|_A$*-continuous on* $\|.\|_A$*-bounded sets.*

Proof. (i) $\Rightarrow$ (ii): For each $a \in X'$ there are unique k-homogeneous polynomials $f_{k,a}$ such that $f(z) = \sum_{k \geqslant 0} f_{k,a}(z-a)$, and the $f_{k,a}$ may be expressed

$$f_{k,a}(z) = \frac{1}{2\pi i} \int\limits_{|\lambda|=r} \frac{f(a+\lambda z)}{\lambda^{k+1}}\, d\lambda$$

(see [5]). Thus for all $r > 0$,

$$|f_{k,a}(z)| \leqslant \frac{1}{2\pi} \int\limits_{|\lambda|=r} \frac{|f(a+\lambda z)|}{r^{k+1}} |d\lambda| \leqslant \frac{1}{2\pi} \int\limits_{|\lambda|=r} \frac{ce^{\sigma(\|a\|_A + r\|z\|_A)}}{r^{k+1}} |d\lambda|$$

$$= \frac{ce^{\sigma\|a\|_A}}{2\pi} \int\limits_0^{2\pi} \frac{e^{\sigma\|z\|_A r}}{r^{k+1}} r\, dt = ce^{\sigma\|a\|_A} \frac{e^{r\sigma\|z\|_A}}{r^k}.$$

From where, setting $\|z\|_A \leqslant 1$, the $f_{k,a}$ are $\|.\|_A$-continuous and

$$\|f_{k,a}\|_A \leqslant c'_{\|a\|_A} \frac{e^{\sigma r}}{r^k} \quad \text{for all } r > 0.$$

But the minimum of the function $r \mapsto \frac{e^{\sigma r}}{r^k}$ is attained for $r_k = \frac{k}{\sigma}$. Thus

$$\|f_{k,a}\|_A \leqslant c'_{\|a\|_A} \frac{e^{\sigma r}}{r^k} \leqslant c'_{\|a\|_A} \frac{e^k}{k^k} \sigma^k = c'_{\|a\|_A} \frac{e^k k!}{k^k} \frac{\sigma^k}{k!} \leqslant c_{\|a\|_A} \frac{\sigma^k}{k!},$$

for some $c_{\|a\|_A}$, by Stirling's formula. $c_{\|a\|_A}$ is independent of k.

(ii) $\Rightarrow$ (iii) is clear.

(iii) $\Rightarrow$ (i): We have, for any $z \in X'$,

$$\left|f(z)\right| \leqslant \sum_{k \geqslant 0} \left|f_k(z)\right| \leqslant \sum_{k \geqslant 0} \|f_k\|_A \|z\|_A^k \leqslant \sum_{k \geqslant 0} C \frac{(\sigma\|z\|_A)^k}{k!} = Ce^{\sigma\|z\|_A},$$

so f is of A-exponential type.

Note that given any of the three equivalent conditions,

$$\|f_{k,a}\|_A^{1/k} \leqslant c_{\|a\|_A}^{1/k} \frac{\sigma}{(k!)^{1/k}},$$

so $\limsup_k \|f_{k,a}\|_A^{1/k} = 0$, and $r_A = \infty$. Also,

$$|f(z) - f(a)| \leqslant \sum_{k\geqslant 1} \left| f_{k,a}(z-a) \right| \leqslant \sum_{k\geqslant 1} c_{\|a\|_A} \frac{(\sigma \|z-a\|_A)^k}{k!} = c_{\|a\|_A} \left(e^{\sigma \|z-a\|_A} - 1 \right),$$

so f is uniformly $\|.\|_A$-continuous on $\|.\|_A$-bounded sets. □

If f is of A-exponential type, it verifies the conditions for formula (A) to hold. Indeed, since f is $\|.\|_A$-continuous, $f \circ T$ is continuous on $\operatorname{Im} A$, a dense subspace of X. Extend $f \circ T$ by continuity to all of X ($f \circ T$ will also be $\|.\|_0$-continuous), and set $F = f \circ T \circ \iota$. F is continuous and Gateaux-holomorphic, thus holomorphic on H. Also, one has the bounds

$$\left| (f \circ T \circ P_n)(\gamma) \right|^p \leqslant C^p e^{p\sigma \|P_n(\gamma)\|_0} \leqslant C^p e^{p\sigma c \|\gamma\|_0} \leqslant M e^{\varepsilon \|\gamma\|_0^2},$$

integrable by Fernique's theorem.

For the case of formula (B) we define f to be of A-*harmonic type* when $r_A \geqslant 1$. For such an f, $f^{\diamond}$ is of A-exponential type. Let f be holomorphic on the open unit ball of X', with $r_A \geqslant 1$. Then formula (B) is applicable to f for some $\delta > 0$. Indeed, $f^{\diamond\sharp} \circ T$ is continuous on $\operatorname{Im} A$; extend continuously to all X, and take $\widetilde{F^{\diamond\sharp}} = f^{\diamond\sharp} \circ T$. Then $|(f^{\diamond\sharp} \circ T)(\delta \|P_n\gamma\| P_n\gamma)|$ is bounded by an $L^1(W)$-function for sufficiently small δ:

$$\begin{aligned} \left| \left(f^{\diamond\sharp} \circ T\right)\left(\delta \|P_n\gamma\| P_n\gamma\right) \right| &\leqslant \sum_k \frac{1}{k!} \left| f_k\left(T\left(\delta \|P_n\gamma\| P_n\gamma\right)\right) \right| \\ &\leqslant \sum_k \frac{1}{k!} \|f_k\|_A \delta^k \|P_n\gamma\|^{2k} \leqslant \sum_k \frac{1}{k!} \|f_k\|_A \delta^k \|P_n\gamma\|_0^{2k} \\ &\leqslant \sum_k \frac{1}{k!} \|f_k\|_A \delta^k c^k \|\gamma\|_0^{2k} \\ &\leqslant M \sum_k \frac{(\delta c \|\gamma\|_0^2)^k}{k!} = M e^{\delta c \|\gamma\|_0^2}, \end{aligned}$$

integrable for small δ by Fernique's theorem. Note that δ does not depend on the particular function f, so any function of A-harmonic type can be represented by formula (B) on the same ball $\delta B^{\circ}_{X'}$.

A second class of functions to which our formulas apply is related to an analog of Hardy space of holomorphic functions on the unit ball of H. H.-H. Kuo defined k-linear functionals of Hilbert–Schmidt type in [8]. O. Lopushansky and A. Zagorodnyuk define and study, in [9], the space of Hilbertian k-homogeneous polynomials $P_h({}^kH)$ over H and their ℓ^2-sum, the Hardy space $\mathcal{H}^2$. These are Hilbert spaces, and $\mathcal{H}^2$ is dual to the symmetric Fock space, which plays an important role in quantum mechanics. Note that for $P \in P_h({}^kH)$, the usual polynomial norm is bounded by the Hilbertian norm: $\|P\| \leqslant \|P\|_h$. Also, for $\alpha = (\alpha_1, \ldots, \alpha_n, \ldots) \in \{0, 1, \ldots, k\}^{(N)}$ with $|\alpha| = \sum_i \alpha_i = k$, consider $P_\alpha : H \to C$, defined by $P_\alpha(x) = x^\alpha = x_1^{\alpha_1} \cdots x_n^{\alpha_n} \cdots$. The polynomials $\sqrt{\frac{|\alpha|!}{\alpha!}} P_\alpha$ form an orthonormal basis for $P_h({}^kH)$.

Now let $S(P_\alpha) = \frac{1}{\sqrt{|\alpha|!}} P_\alpha$, and extend linearly to obtain $S : \mathcal{H}^2 \to \mathcal{H}(H)$. We will denote by $\mathcal{F}$ the space of functions

$$\mathcal{F} = \{S(h)\colon\ h \in \mathcal{H}^2\},$$

which, if we set $\langle S(h_1), S(h_2)\rangle_{\mathcal{F}} = \langle h_1, h_2\rangle_{\mathcal{H}^2}$, is a Hilbert space isometric to $\mathcal{H}^2$. Note however that functions in $\mathcal{F}$ are holomorphic on all of H, and in fact are functions of bounded type, i.e., bounded on bounded subsets of H. Indeed, if $h = \sum_k h_k$ is the Taylor series expansion of h at 0, $\|S(h)_k\| \leqslant \|S(h)_k\|_h \leqslant \frac{1}{\sqrt{k!}} \|h_k\|_h$, so

$$\|S(h)_k\|^{1/k} \leqslant \left(\frac{\|h_k\|_h}{\sqrt{k!}}\right)^{1/k},$$

which tends to zero, since $(\|h_k\|_h) \in \ell^2$. Thus the Taylor series $h = \sum_k h_k$ has infinite radius of convergence.

Recall that $(z_k) \subset X'$ are such that $\iota^*(z_k) = e'_k$, and set $z^\alpha(\gamma) = z_1(\gamma)^{\alpha_1} \cdots z_n(\gamma)^{\alpha_n} \cdots$. Since by Lemma 3.1 $(\frac{1}{\sqrt{\alpha!}} z^\alpha)$ is an orthonormal set in $L^2(W)$, the linear map

$$J : \mathcal{F} \to L^2(W) \quad \text{such that} \quad J(x^\alpha) = z^\alpha$$

is isometric. Let $F(x) = \frac{1}{\sqrt{\alpha!}} x^\alpha$ and $\tilde{F}(\gamma) = \frac{1}{\sqrt{\alpha!}} z^\alpha(\gamma)$, and note that $\tilde{F}$ is continuous on X. Applying Theorem 2.4, formula (A) is valid for $f(z) = F \circ I \circ \iota^*(z)$. Thus for a dense subset of $\mathcal{F}$ (all finite sums of $\frac{1}{\sqrt{|\alpha|!}} P_\alpha$) we have

$$f(z) = F \circ I \circ \iota^*(z) = \int_X e^{z(\gamma)} \overline{(f^\sharp \circ T)}(\gamma)\, dW(\gamma) = \int_X e^{z(\gamma)} \overline{J(F^\sharp)}(\gamma)\, dW(\gamma).$$

Now fix $z \in X'$. Using the Cauchy-Schwarz inequality we see that this integral is continuous over $\mathcal{F}$. Thus to obtain formula (A) for $f = F \circ I \circ \iota^*$ with any $F \in \mathcal{F}$ we need only see that

$$F \mapsto F \circ I \circ \iota^*(z) = f(z)$$

is continuous on $\mathcal{F}$. But this is true, for

$$\begin{aligned}
&\left|F \circ I \circ \iota^*(z) - G \circ I \circ \iota^*(z)\right| \\
&\quad \leqslant \sum_{k\geqslant 0} \left|(F_k - G_k)\big(I \circ \iota^*(z)\big)\right| \leqslant \sum_{k\geqslant 0} \|F_k - G_k\|_{P_h({}^k H)} \left\|I \circ \iota^*(z)\right\|^k \\
&\quad \leqslant \sum_{k\geqslant 0} \sqrt{k!} \|F_k - G_k\|_{P_h({}^k H)} \frac{\|I \circ \iota^*(z)\|^k}{\sqrt{k!}} \\
&\quad \leqslant \left(\sum_{k\geqslant 0} k! \|F_k - G_k\|^2_{P_h({}^k H)}\right)^{1/2} \left(\sum_{k\geqslant 0} \frac{\|I \circ \iota^*(z)\|^{2k}}{k!}\right)^{1/2} = C(z)\|F - G\|_{\mathcal{F}}.
\end{aligned}$$

We remark that if $f : X' \to C$ is a function representable as

$$f(z) = \int_X e^{z(\gamma)} h(\gamma)\, dW(\gamma) \quad \text{with } h \in L^2(W),$$

then there exists an $F \in \mathcal{F}$ for which $f(z) = F \circ I \circ \iota^*(z)$.

Similarly, if we set $S(P_\alpha) = \sqrt{|\alpha|!}P_\alpha$ and extend linearly, we obtain a space of functions

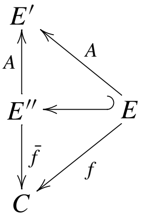
$$\mathcal{G} = \{S(h)\colon h \in \mathcal{H}^2\},$$

for which formula (B) is valid.

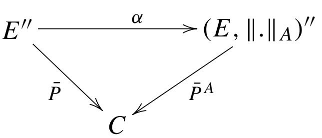
Finally, we want to be able to apply our formulas (A) and (B) to functions which are holomorphic on a Banach space E, even if E is not a dual space. This can be done through the Aron–Berner extension of the function to the bidual E'' of E. The Aron–Berner construction may be seen in [1,5,11]. We need to recall only the following. If $f = \sum_k f_k$ is the Taylor series expansion of f (about 0, say). Then each k-homogeneous polynomial $f_k : E \to C$ may be canonically extended to the bidual: $\overline{f_k} : E'' \to C$, and the Aron–Berner extension of f is defined to be

$$\bar{f} = \sum_k \overline{f_k}.$$

A result of Davie and Gamelin [2] says that the Aron–Berner extension preserves the norms of homogeneous polynomials, so the radius of uniform convergence of the Taylor series of f and $\bar{f}$ coincide. If f is a holomorphic function on E, consider



where we still denote with A the restriction of A to E. We want to prove that if f is of A-exponential type (respectively of A-harmonic type), then $\bar{f}$ is of A-exponential type (respectively of A-harmonic type). By Proposition 3.1, we need only see that for any $\|.\|_A$-continuous k-homogeneous polynomial P over E, $\|\bar{P}\|_A = \|P\|_A$. To check this, note that the identity mapping $E \to (E, \|.\|_A)$ is continuous and of norm less than or equal to one. Call $\alpha : E'' \to (E, \|.\|_A)''$ its bitranspose and consider the Aron–Berner extensions of P to both spaces:



Note that by the Davie–Gamelin theorem $\|\bar{P}^A\|_A = \|P\|_A$. Now for any $z \in E''$ we have

$$\left|\bar{P}(z)\right| = \left|\bar{P}^A\big(\alpha(z)\big)\right| \leqslant \left\|\bar{P}^A\right\|_A \left\|\alpha(z)\right\|_A^k = \|P\|_A \left\|\alpha(z)\right\|_A^k \leqslant \|P\|_A \|z\|_A^k.$$

Thus $\|\bar{P}\|_A \leqslant \|P\|_A$. The opposite inequality is trivial, so $\|\bar{P}\|_A = \|P\|_A$.

We have then the following variations of Theorems 2.4 and 2.5, where W is Wiener measure on E' and $A : E'' \to E'$ its covariance operator.

Theorem 3.2. *Suppose E has a separable dual, and let $f : E \to C$ be a holomorphic function of A-exponential type. If $\bar{f}$ is the Aron–Berner extension of f, and $z \in E''$,*

$$\bar{f}(z) = \int_{E'} e^{z(\gamma)} \overline{\left(\bar{f}^{\sharp} \circ T\right)}(\gamma)\, dW(\gamma). \tag{A}$$

Theorem 3.3. *Suppose E has a separable dual. There is a $\delta > 0$ such that: if f is a holomorphic function of A-harmonic type on E, and $\bar{f}$ its Aron–Berner extension, then for $z \in \delta B^{\circ}_{E''}$,*

$$\bar{f}(z) = \int_{E'} \frac{1}{1 - z\left(\frac{\gamma}{\delta\|\gamma\|}\right)} \overline{\left(\bar{f}^{\diamond\sharp} \circ T\right)}\left(\delta\|\gamma\|\gamma\right) dW(\gamma). \tag{B}$$

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