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Extension of vector-valued integral polynomials

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To Richard Aron on his sixtieth birthday

Abstract

We study the extendibility of integral vector-valued polynomials on Banach spaces. We prove that an X-valued Pietsch-integral polynomial on E extends to an X-valued Pietsch-integral polynomial on any space F containing E, with the same integral norm. This is not the case for Grothendieck-integral polynomials: they do not always extend to X-valued Grothendieck-integral polynomials. However, they are extendible to X-valued polynomials. The Aron–Berner extension of an integral polynomial is also studied. A canonical integral representation is given for domains not containing ℓ_1.

Keywords: Integral polynomials; Extendibility

Introduction

In this note we study extendibility properties of Pietsch and Grothendieck integral polynomials. Generally, polynomials on Banach spaces do not extend to larger spaces, even in the scalar valued case [20]. In other words, there is no Hahn–Banach extension theorem

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for polynomials. However, since the symmetric injective tensor product respects subspaces, scalar-valued integral polynomials are extendible. For vector-valued polynomials, the word "extendible" needs to be properly defined. We say that a polynomial $P : E \to X$ is extendible if for any Banach space F containing E, there exists $\tilde{P} : F \to X$ extending P ([20], see also [5]). The problem of extending polynomials (and multilinear mappings) has been studied by many authors (see, for example, [4,8,9,17,18,21,26]). It is important to remark that in the definition, the extension of P must be X-valued. Another consideration to take into account regarding extendibility is the preservation of the norm. Even when there are extensions of P, the norm of P may not be preserved by any of these extensions. Moreover, the infimum of the extension norms might be strictly greater than the norm of $\|P\|$ (see [21] for a concrete finite-dimensional example). Since we focus on Grothendieck and Pietsch integral polynomials, we discuss the preservation of the respective integral norms.

In order to extend holomorphic functions of bounded type, Aron and Berner showed in [4] how to extend a continuous homogeneous polynomial defined on a Banach space E to a polynomial on E'', the bidual of E (see also [3]). For X-valued mappings, the Aron–Berner extension may take values in X'' (and therefore it would not be actually an extension). An important feature of the Aron–Berner extension (even when it is not X-valued) is that it preserves the norm [5,12,17].

The paper is organized as follows. In the first section we state some general results about integral polynomials. In the second one, we prove that a Pietsch-integral polynomial $P : E \to X$ extends to an X-valued Pietsch-integral polynomial over any $F \supset E$, with the same integral norm. This is not the case for Grothendieck-integral polynomials: if a Grothendieck-integral polynomial $P : E \to X$ extends to an X-valued Grothendieck-integral polynomial over any $F \supset E$, P turns out to be Pietsch-integral. What is possible to obtain is an X''-valued Grothendieck-integral extension of P, but this is not an extension in the proper sense. However, we show that Grothendieck-integral polynomials are extendible: they extend to (non-integral) X-valued polynomials. The third section deals with the Aron–Berner extension of a (Pietsch or Grothendieck) integral polynomial. We show that this extension is also integral, with the same integral norm. We also present a canonical expression for this extension in the case that E does not contain an isomorphic copy of ℓ_1.

We refer to [14,22] for notation and results regarding polynomials in general, to [13, 16,23,24] for tensor products of Banach spaces and to [1,2,13,15] for integral operators, polynomials and multilinear mappings.

1. Definitions and general results

Throughout, E, F, and X will be Banach spaces. The space of continuous n-homogeneous polynomials from E into X will be denoted by $\mathcal{P}(^nE, X)$. This is a Banach space endowed with the norm $\|P\| = \sup\{\|P(x)\|\colon \|x\| \leqslant 1\}$. If $P \in \mathcal{P}(^nE, X)$, $\check{P} : E \times \cdots \times E \to X$ and $L_P : \bigotimes_s^n E \to X$ will denote, respectively, the continuous symmetric n-linear form and the linear operator associated with P.

Following [20], we will say that a polynomial $P : E \to X$ is *extendible* if for any Banach space F containing E there exists $\tilde{P} \in \mathcal{P}({}^n F, X)$ an extension of P. We will denote the space of all such polynomials by $\mathcal{P}_e({}^n E, X)$. For $P \in \mathcal{P}_e({}^n E, X)$, its extendible norm is given by

$$\|P\|_e = \inf\{c > 0: \text{ for all } F \supseteq E \text{ there is an extension of } P \text{ to } F \text{ with norm} \leqslant c\}.$$

In order to study extendibility, the natural (isometric) inclusions $E \hookrightarrow C(B_{E'}, w^*)$ and $E \hookrightarrow \ell_\infty(B_{E'})$ are useful. It was shown in [5, Theorem 3.1] that a polynomial $P : E \to X$ is extendible if and only if P extends to $C(B_{E'}, w^*)$, whenever X is a $\mathcal{C}_l$ space. This is not true for arbitrary spaces: without conditions on X, a polynomial $P : E \to X$ is extendible if and only if P extends to $\ell_\infty(B_{E'})$ [5, Theorem 3.2].

If (Ω, μ) is a finite measure space, $L_\infty(\Omega, \mu)$ has the metric extension property, which means that $L_\infty(\Omega, \mu)$ is complemented in any larger space with a norm-one projection. Consequently, any polynomial defined on this space is extendible and the extendible and usual norms coincide. This fact and [5, Theorem 3.4] enable us to ensure that any polynomial that factors through some L_∞ is extendible.

A polynomial $P \in \mathcal{P}({}^n E, X)$ is *Pietsch-integral* (P-integral for short) if there exists a regular X-valued Borel measure G, of bounded variation on $(B_{E'}, w^*)$ such that

$$P(x) = \int_{B_{E'}} \gamma(x)^n \, dG(\gamma)$$

for all $x \in E$. The space of n-homogeneous Pietsch-integral polynomials is denoted by $\mathcal{P}_{\mathrm{PI}}({}^n E, X)$ and the integral norm of a polynomial $P \in \mathcal{P}_{\mathrm{PI}}({}^n E, X)$ is defined as

$$\|P\|_{\mathrm{PI}} = \inf\big\{|G|(B_{E'})\big\},$$

where the infimum is taken over all measures G representing P.

The definition of *Grothendieck-integral* (G-integral for short) polynomials is analogous, but taking the measure G to be X''-valued. The space of Grothendieck-integral polynomials is denoted by $\mathcal{P}_{\mathrm{GI}}({}^n E, X)$.

Following [16], we will write ϵ_s for the injective symmetric tensor norm on $\bigotimes_s^n E$. Consequently, $\bigotimes_{s,\epsilon_s}^n E$ will stand for the symmetric tensor product $\bigotimes_s^n E$ endowed with the injective symmetric tensor norm.

In [10, Proposition 2.5] and [25, Corollary 2.8], the authors show that there is a correspondence between (G and P)-integral polynomials from E to X and (G and P)-integral operators from $\bigotimes_{s,\epsilon_s}^n E$ to X. In [7, Proposition 2.10] we show that this correspondence is actually an isometric isomorphism for P-integral polynomials. Next proposition states the analogous isometric result for G-integral polynomials. Although it could be deduced from [7], we give a direct proof for the sake of completeness.

Proposition 1. *The spaces $\mathcal{P}_{\mathrm{GI}}({}^n E, X)$ and $\mathcal{L}_{\mathrm{GI}}(\bigotimes_{s,\epsilon_s}^n E, X)$ are isometrically isomorphic.*

Proof. For $P \in \mathcal{P}_{\mathrm{GI}}({}^n E, X)$, let G be a X''-valued measure on $B_{E'}$ representing P and set $\mu = |G|$. Define $R : \bigotimes_{s,\epsilon_s}^n E \to L_\infty(\mu)$ by $R(x^{(n)}) = \hat{x}^n$, where $\hat{x}^n(\gamma) = \gamma(x)^n$ for

$\gamma \in B_{E'}$. Clearly, $\|R\| \leqslant 1$. If L_P is the linearization of P, we have the following diagram:

$$\begin{array}{ccccc} \bigotimes_{s,\epsilon_s}^n E & \xrightarrow{L_P} & X & \longrightarrow & X'' \\ \downarrow R & & & \nearrow S & \\ L_\infty(\mu) & \xrightarrow{j} & L_1(\mu), & & \end{array} \tag{1}$$

where j is the natural inclusion and $S(f) = \int_{B_{E'}} f\,dG$ for $f \in L_1(\mu)$. This factorization shows that L_P is G-integral. Since $\|j\| \leqslant |G|$, $\|R\| \leqslant 1$, $\|S\| \leqslant 1$ and this holds for any measure G representing P, we have $\|L_P\|_{\mathrm{GI}} \leqslant \|P\|_{\mathrm{GI}}$.

Conversely, suppose that $T \in \mathcal{L}_{\mathrm{GI}}(\bigotimes_{s,\epsilon_s}^n E, X)$. Then, T admits a factorization as the one in diagram (1), with T instead of L_P, and with $\|S\| = 1$, $\|j\| = \|T\|_{\mathrm{GI}}$ and $\|R\| = 1$.

We choose $G \in \mathcal{M}(B_{E'}; X'')$ a representing measure for the integral operator $S \circ j$, so that $S \circ j(f) = \int_{B_{E'}} f\,dG$ and $|G| = \|S \circ j\|_{\mathrm{GI}} \leqslant \|T\|_{\mathrm{GI}}$. Therefore, P, the polynomial associated to T, can be written as

$$P(x) = \int_{B_{E'}} \gamma(x)^n\,dG(\gamma).$$

This means that P is G-integral and $\|P\|_{\mathrm{GI}} \leqslant |G| \leqslant \|T\|_{\mathrm{GI}} + \varepsilon$. This holds for any $\varepsilon > 0$ and the isometry follows. □

Any G-integral operator $T : E \to X$ identifies with a linear form on $E \otimes_\epsilon X'$ with norm $\|T\|_{\mathrm{GI}}$ (in fact, this can be taken as the definition of G-integral operators). Now, the previous proposition allows us to identify a G-integral polynomial with a linear form on $(\bigotimes_{s,\epsilon_s}^n E \otimes_\epsilon X')$ with norm $\|P\|_{\mathrm{GI}}$. On the other hand, if we consider G-integral mappings with range in a dual space Y', there is an isometric isomorphism between $\mathcal{L}_{\mathrm{GI}}(E, Y')$ and $(E \otimes_\epsilon Y)'$ [13, Proposition 10.1]. From Proposition 1 we extend this to n-homogeneous G-integral polynomials. Since G-integral operators with range in a dual space are automatically P-integral [15, Corollary VIII.2.10], we have:

Corollary 2.

(a) $\mathcal{P}_{\mathrm{GI}}({}^n E, X) \hookrightarrow (\bigotimes_{s,\epsilon_s}^n E \otimes_\epsilon X')'$ *isometrically.*
(b) $\mathcal{P}_{\mathrm{GI}}({}^n E, Y') = \mathcal{P}_{\mathrm{PI}}({}^n E, Y') = (\bigotimes_{s,\epsilon_s}^n E \otimes_\epsilon Y)'$ *isometrically.*

In [10], integral polynomials are defined as those which can be identified with continuous linear functionals on $\bigotimes_{s,\epsilon_s}^n E \otimes_\epsilon X'$. Therefore, we have shown that the definition in [10] is equivalent to the one given above for G-integral polynomials and also that the G-integral norm of the polynomial coincides with the norm of the linear functional.

In [11, Theorem 3], the authors show that whenever E' has the approximation property and the Radon–Nikodým property, the spaces of Grothendieck integral and nuclear n-homogeneous polynomials from E to any Banach space are isomorphic. Proposition 1 allows us to show that the isomorphism is in fact an isometry. Indeed, it follows from

[7, Proposition 2.10], Proposition 1, and [15, Theorem VIII.4.6] that if E' has the approximation property and the Radon–Nikodým property, then the spaces of n-homogeneous Grothendieck and Pietsch integral polynomials are isometrically isomorphic. Now, the result follows from [6, Theorem 1.4]. So, we have:

Corollary 3. *Let E be a Banach space such that E' has the approximation property and the Radon–Nikodým property. Then, for any Banach space F, the spaces $\mathcal{P}_{\mathrm{GI}}({}^nE, F)$ and $\mathcal{P}_N({}^nE, F)$ are isometrically isomorphic.*

2. Extension of integral polynomials

We have mentioned that L_∞ spaces play a crucial role when extending polynomials. Therefore, we start this section by showing a natural example of integral polynomial on these spaces.

Lemma 4. *Let (Ω, Σ, μ) be a finite measure space and $G : \Sigma \to X$ a vector measure which is absolutely continuous with respect to μ. Then*

$$P_0(f) = \int_\Omega f^n(w)\, dG(w) \tag{2}$$

is a Pietsch-integral n-homogeneous polynomial on $L_\infty(\Omega, \mu)$ with $\|P_0\|_{\mathrm{PI}} \leqslant |G|$.

Also, for any compact Hausdorff space K and any regular, Borel measure G on K, the polynomial on $C(K)$ given in (2) *is Piestch-integral with $\|P_0\|_{\mathrm{PI}} \leqslant |G|$.*

Proof. For the first statement, by [7, Proposition 2.10] it is enough to prove that L_{P_0}, the linearization of P_0, belongs to $\mathcal{L}_{\mathrm{PI}}(\bigotimes^n_{s,\epsilon_s} L_\infty(\mu), X)$.

Define the linear operator $R : \bigotimes^n_{s,\epsilon_s} L_\infty(\mu) \to L_\infty(\mu)$ by $R(f^{(n)}) = f^n$. As a consequence of Maharam's theorem [13, B.7], R has norm one. Now, if we define $S(f) = \int_\Omega f\, dG$ for all $f \in L_1(\mu)$ and if $j : L_\infty(\mu) \to L_1(\mu)$ is the natural inclusion, we have the commutative diagram:

$$\begin{array}{ccc} \bigotimes^n_{s,\epsilon_s} L_\infty(\mu) & \xrightarrow{L_{P_0}} & X \\ {\scriptstyle R}\downarrow & & \uparrow{\scriptstyle S} \\ L_\infty(\mu) & \xrightarrow{\ j\ } & L_1(\mu). \end{array}$$

Therefore, L_{P_0} is P-integral. Since $\|j\| \leqslant |G|$, by the isometry given in [7, Proposition 2.10], we have $\|P_0\|_{\mathrm{PI}} = \|L_{P_0}\|_{\mathrm{PI}} \leqslant |G|$.

The statement for $C(K)$ can be proved analogously. Also, it can be seen as a consequence of the first result. Indeed, just take $\mu = |G|$ and factor P_0 via the natural mapping $C(K) \to L_\infty(\mu)$. □

A scalar-valued integral polynomial P on a Banach space E can be extended to any larger space F, in such a way that the extension $\tilde{P}$ is also integral and P and $\tilde{P}$ have the

same integral norm. This follows from the fact that the symmetric injective tensor product respects subspaces and the Hahn–Banach theorem applied to the linearization of P (see, for example, [8]). Next proposition states a similar result for Pietsch-integral vector-valued polynomials.

Theorem 5. *Let F be a Banach space containing E. Any $P \in \mathcal{P}_{\mathrm{PI}}({}^nE, X)$ can be extended to $\tilde{P} \in \mathcal{P}_{\mathrm{PI}}({}^nF, X)$, with $\|P\|_{\mathrm{PI}} = \|\tilde{P}\|_{\mathrm{PI}}$. As a consequence, $\|P\|_e \leqslant \|P\|_{\mathrm{PI}}$.*

Proof. Let $P \in \mathcal{P}_{\mathrm{PI}}({}^nE, X)$, let G be a measure representing P and consider $\mu = |G|$. We write $P = P_0 \circ i$, where $i : E \to L_\infty(B_{E'}, \mu)$ is the natural inclusion and $P_0 : L_\infty(B_{E'}, \mu) \to X$ is the polynomial,

$$P_0(f) = \int_\Omega f^n(w)\, dG(w).$$

Since $L_\infty(B_{E'}, \mu)$ has the metric extension property, we have $\tilde{i} : F \to L_\infty(B_{E'}, \mu)$ a norm one extension of i. Therefore, $\tilde{P} = P_0 \circ \tilde{i}$ extends P. By Lemma 4, P_0 is P-integral and therefore $\tilde{P}$ is P-integral, with $\|\tilde{P}\|_{\mathrm{PI}} \leqslant \|P_0\|_{\mathrm{PI}}\|\tilde{i}\|^n \leqslant |G|$. This holds for any measure G representing P and then $\|\tilde{P}\|_{\mathrm{PI}} \leqslant \|P\|_{\mathrm{PI}}$. The other inequality holds since $\tilde{P}$ is an extension on P. The inequality $\|P\|_e \leqslant \|P\|_{\mathrm{PI}}$ is a straightforward consequence of the definition of the extendible norm and the inequality $\|\tilde{P}\| \leqslant \|\tilde{P}\|_{\mathrm{PI}} = \|P\|_{\mathrm{PI}}$. □

If $E = C(K)$ or $E = L_\infty(\mu)$, Grothendieck and Pietsch integral operators on E coincide [13, D.6]. We show that the result remains true for homogeneous polynomials.

Remark 6. Let P be in $\mathcal{P}({}^nE, X)$, for $E = C(K)$ or $E = L_\infty(\mu)$. Then, P is Grothendieck-integral if and only if P is Pietsch-integral.

Proof. Since $L_\infty(\mu)$ is isomorphic to $C(K)$ for some compact Hausdorff space K, we assume $E = C(K)$. The symmetric multilinear mapping $\check{P}$ associated to a G-integral polynomial P is also G-integral and defines a G-integral linear operator $L_{\check{P}}$ on the (full) injective tensor product (see [25]). The n-fold injective tensor product of $C(K)$ is isomorphic to $C(K \times \cdots \times K)$. Thus, $L_{\check{P}}$ is P-integral and so is P. □

Any G-integral polynomial $P : E \to X$ is a P-integral polynomial considered with values in X''. Theorem 5 gives us a P-integral extension of P, $\tilde{P}$ with values in X'', which is also a G-integral X''-valued extension of P. Another way to obtain this extension is to identify P with a continuous linear functional on $\bigotimes^n_{s,\epsilon_s} E \otimes_\epsilon X'$, and extend it to $\bigotimes^n_{s,\epsilon_s} F \otimes_\epsilon X'$ by Hahn–Banach theorem. This extension identifies with a G-integral polynomial from F to X'' extending P (and which is, by the way, also P-integral).

A natural question arises: is it possible to obtain a Grothendieck-integral X-valued extension of P to any larger space? We answer that question by the negative: suppose we can extend P to a G-integral polynomial on $C(B_{E'})$. By Remark 6, this extension is P-integral and therefore, so is P. Since there are G-integral polynomials that are not P-integral (see [1] and [13, Proposition D9]), the conclusion follows.

Consequently, a G-integral polynomial $P : E \to X$ cannot in general be extended to an X-valued integral polynomial. However G-integral polynomials are extendible: they can be extended to (non-integral) X-valued polynomials to any larger space.

Proposition 7. *Any Grothendieck-integral polynomial $P : E \to X$ is extendible* (*to X-valued polynomials*) *and $\|P\|_e \leqslant \|P\|_{\mathrm{GI}}$.*

Proof. If $P : E \to X$ is a G-integral polynomial, by Proposition 1, $L_p : \bigotimes^n_{s,\epsilon_s} E \to X$ is a G-integral operator with the same integral norm. Consider the inclusion $E \subset \ell_\infty(B_{E'})$. Since L_P is G-integral, it is absolutely 2-summing with $\|L_P\|_{2-\mathrm{sum}} \leqslant \|L_P\|_{\mathrm{GI}}$. We have that $\bigotimes^n_{s,\epsilon_s} E$ is isometrically a subspace of $\bigotimes^n_{s,\epsilon_s} \ell_\infty(I)$ and therefore L_P extends to an (absolutely 2-summing) operator $\tilde{L} : \bigotimes^n_{s,\epsilon_s} \ell_\infty(I) \to X$ with $\|\tilde{L}\| \leqslant \|L_P\|_{2-\mathrm{sum}} \leqslant \|P\|_{\mathrm{GI}}$. We can define $\tilde{P} : \ell_\infty(I) \to X$ as $\tilde{P}(a) = \tilde{L}(a^{(n)})$. $\tilde{P}$ extends P and $\|\tilde{P}\| \leqslant \|P\|_{\mathrm{GI}}$. An appeal to [5] completes the proof. □

3. The Aron–Berner extension of an integral polynomial

In [7] it is shown that the Aron–Berner extension of a P-integral polynomial $P : E \to X$ is a P-integral polynomial from E'' to X, with the same integral norm. This statement involves two facts. On the one hand, the Aron–Berner extension is X-valued. On the other hand, it is integral when considered with range in X. This is not immediate, since P-integral polynomials are not a regular ideal. An analogous result for G-integral polynomials can be obtained from the Pietsch-integral case. However, for G-integral polynomials is easy to give a direct proof. We denote by $AB(P)$ the Aron–Berner extension of P.

Proposition 8. *If $P \in \mathcal{P}_{\mathrm{GI}}({}^n E, X)$, then $AB(P) \in \mathcal{P}_{\mathrm{GI}}({}^n E'', X)$ and $\|AB(P)\|_{\mathrm{GI}} = \|P\|_{\mathrm{GI}}$.*

Proof. Let $P : E \to X$ be a G-integral polynomial. By Proposition 1, its linearization $L_P : \bigotimes^n_{s,\epsilon_s} E \to X$ is G-integral and has the same integral norm. Thus, L''_P is a G-integral operator from E'' to X'' (with the same norm). Moreover, since L_P is weakly compact, L''_P takes its values in X and, by [13, 10.2] L''_P is G-integral from E'' to X, with the same norm. Now, the linearization of $AB(P)$ is $L''_P \circ i$, where the map $i : \bigotimes^n_{s,\epsilon_s} E'' \hookrightarrow (\bigotimes^n_{s,\epsilon_s} E)''$ is the (norm one) inclusion via the identification given in [8]. Therefore, $AB(P)$ is G-integral from E'' to X with the same G-integral norm as P. □

We turn our attention to the validity of a canonical integral representation for the Aron–Berner extension of an integral polynomial. If $P : E \to X$ is an integral polynomial and G is a representing measure for P (X or X''-valued), we want to know if the Aron–Berner extension of P can be written as

$$AB(P)(z) = \int_{B_{E'}} z(\gamma)^n \, dG(\gamma). \tag{3}$$

For scalar-valued polynomials, the validity of this expression is equivalent to E not containing an isomorphic copy of ℓ_1. We show that this remains true for vector-valued poly-

nomials. In fact, if E contains ℓ_1, the function $\gamma \mapsto z(\gamma)$ is not μ-measurable for some measure μ, so expression (3) cannot hold.

Theorem 9. *Suppose E does not contain isomorphic copies of ℓ_1. If P is a* (*Grothendieck or Pietsch*)*-integral polynomial with representing measure G, then*

$$AB(P)(z) = \int_{B_{E'}} z(\gamma)^n \, dG(\gamma).$$

Proof. If E does not contain ℓ_1, the function $\gamma \mapsto z(\gamma)$ is Borel-measurable on $(B_{E'}, w^*)$ and we can define the polynomial

$$Q(z) = \int_{B_{E'}} z(\gamma)^n \, dG(\gamma).$$

Let us see that $Q = AB(P)$. The symmetric n-linear mapping associated to Q is given by

$$\check{Q}(z_1, \ldots, z_n) = \int_{B_{E'}} z_1(\gamma) \cdots z_n(\gamma) \, dG(\gamma).$$

We are done if we show that for fixed $z_1, \ldots, z_{n-1}$, the mapping $z \mapsto \check{Q}(z_1, \ldots, z_{n-1}, z)$ is w^* to w^* continuous from E'' to X''. We fix $\varphi \in X'$.

$$\check{Q}(z_1, \ldots, z_{n-1}, z)(\varphi) = \int_{B_{E'}} z(\gamma) z_1(\gamma) \cdots z_{n-1}(\gamma) \, d\varphi \circ G(\gamma) = \int_{B_{E'}} z(\gamma) \, d\mu,$$

where μ is the scalar measure given by $d\mu = z_1(\gamma) \cdots z_{n-1}(\gamma) \, d\varphi \circ G(\gamma)$. The measure μ can be written as a linear combination of probability measures. Since E does not contain ℓ_1, each $z \in E''$ satisfies the barycentric calculus [19]. Therefore, if $\gamma_0 \in E'$ is the corresponding linear combination of the barycenters of the probability measures, we have $\int_{B_{E'}} z(\gamma) \, d\mu = z(\gamma_0)$, which is w^*-continuous in z. □

If E contains an isomorphic copy of ℓ_1, expression (3) does not hold. However, by Proposition 8 (and the analogous result for Pietsch-integral polynomials in [7]) $AB(P)$ is an integral polynomial if P is. It is natural to ask if $AB(P)$ admits an integral expression involving the measures that represent P. In [8] such an expression is shown for scalar-valued polynomials. The same expression holds for vector-valued polynomials, and the proof of it is essentially contained in the proof of the previous theorem. If $P : E \to X$ is an integral polynomial with representation $P(x) = \int_{B_{E'}} \gamma(x)^n \, dG(\gamma)$, we define $S : L_1(|G|) \to E'$ as $S(f)(x) = \int_{B_{E'}} f(\gamma)\gamma(x) \, d|G|(\gamma)$. With this notation, we have:

Proposition 10. *The Aron–Berner extension of P may be written as*

$$AB(P)(z) = \int_{B_{E'}} \big(S'(z)(\gamma)\big)^n \, dG(\gamma).$$

Note that Proposition 8 can be seen as a corollary of the previous proposition, Lemma 4, and the ideal property of integral polynomials.

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