Primordial Nucleosynthesis with varying fundamental constants: A semi-analytical approach

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ABSTRACT

Using the semi-analytic method proposed by Esmailzadeh et al. (1991) we calculate the abundances of the light elements produced during primordial nucleosynthesis assuming that the gauge coupling constants of the fundamental interactions may vary. We analyze the dependence of the nucleon masses, nuclear binding energies and cross sections involved in the calculation of the abundances with the fundamental constants assuming the chiral limit of QCD. The abundances of light elements as a function of the fundamental constants are obtained. Finally, using the observational data of D, ^{3}He, ^{4}He and ^{7}Li we estimate constraints on the variation of the fundamental constants between the primordial nucleosynthesis and the present. All observational abundances and the WMAP estimate of the baryon density, can be fitted to the theoretical predictions with varying coupling constants. The possible systematic errors in the observational data, precludes from stronger conclusions.

Subject headings: primordial nucleosynthesis, varying fundamental constants, cosmology

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1. Introduction

Big Bang Nucleosynthesis (BBN) is one of the most important tools to study the early universe. The model is simple and has only one free parameter, the density of baryonic matter, which can be determined by comparison between theoretical calculations and observations of the abundances of the light elements. On the other hand, data on cosmic microwave background (CMB) provide an alternative, independent method for determining $\Omega_B h^2$ (Spergel et al. 2003). Recently, the concordance between both methods has been investigated by many authors (Cyburt et al. 2003; Romano et al. 2003; Cuoco et al. 2004; Cyburt 2004; Coc et al. 2004a,b). From the WMAP baryon density , the predicted abundances are highly consistent with the observed D but not with ^{4}He and ^{7}Li. They are produced more than observed. Such discrepancy is usually ascribed to non reported systematic errors in the observations of ^{4}He and ^{7}Li. Indeed, more realistic determinations of the ^{4}He uncertainty implies a baryon density in line with the WMAP estimate (Cyburt 2004; Olive and Skillman 2004). On the other hand, Richard et al. (2005) have pointed out that a better understanding of turbulent transport in the radiative zones of the stars is needed for a better determination of the ^{7}Li abundance. However, if the systematic errors of ^{4}He and ^{7}Li are correctly estimated, we may have insight into new physics beyond the minimal BBN model, for example: new neutron lifetime (Mathews et al. 2005), super WIMP scenario (Feng et al. 2003), lepton asymmetry (Ichikawa et al. 2004) and varying constants (Bergström et al. 1999; Nollet and Lopez 2002; Ichikawa and Kawasaki 2002, 2004). Therefore, BBN is not only one of the most important tests of the Big Bang theory, but it is also useful to obtain stringent constraints on deviations from standard cosmology and on alternative theories to the Standard Model of fundamental interactions (SM).

Among these theories, there are some in which the gauge coupling constants may vary over cosmological time scales like string derived field theories (Wu and Wang 1986; Maeda 1988; Barr and Mohapatra 1988; Damour and Polyakov 1994; Damour et al. 2002a,b), related brane-world theories (Youm 2001a,b; Palma et al. 2003; Brax et al. 2003), and (related or not) Kaluza-Klein theories (Kaluza 1921; Klein 1926; Weinberg 1983; Gleiser and Taylor 1985; Overduin and Wesson 1997). On the other hand, a theoretical framework in which only the fine structure constant varies was developed by Bekenstein (1982) and improved by Barrow et al. (2002). This model was generalized in order to study the time variation of the strong coupling constant (Chamoun et al. 2001). Different versions of the theories mentioned above predict different time behaviors of the gauge coupling constants. Thus, bounds obtained from astronomical and geophysical data are an important tool to test the validity of these theories.

The experimental research can be grouped into astronomical and local methods. The latter ones include geophysical methods such as the natural nuclear reactor that operated about 1.8 10^9 years ago in Oklo, Gabon (Damour and Dyson 1996; Fujii et al. 2000, 2002), the analysis of natural long-lived β decayers in geological minerals and meteorites (Dyson 1966; Sisterna and Vucetich 1990; Smolliar 1996) and laboratory measurements such as comparisons of rates between clocks with different atomic number (Prestage et al. 1995; Sortais et al. 2000; Marion et al. 2003). The astronomical methods are based mainly in the analysis of spectra form high-redshift quasar absorption systems (Cowie and Songaila 1995; Varshalovich et al. 1996; Webb et al. 1999, 2001; Murphy et al. 2001a,b; Levshakov et al. 2002; Ivanchik et al. 2002; Murphy et al. 2003; Ivanchik et al. 2003; Bahcall et al. 2004). Although, most of the previous mentioned experimental data gave null results, evidence of time variation of the fine structure constant was reported recently from high-redshift quasar absorption systems (Webb et al. 1999, 2001; Murphy et al. 2001a,b, 2003; Ivanchik et al. 2003). However, other recent independent analysis of similar data (Martínez Fiorenzano et al. 2003; Quast et al. 2004; Bahcall et al. 2004; Srianand et al. 2004) found no variation. On the other hand, measurements of molecular hydrogen (Ivanchik et al. 2002, 2003) reported a variation of the proton to electron mass $\mu = \frac{m_p}{m_e}$

The time variation of the gauge coupling constants in the early universe can be constrained using data from the Cosmic Microwave Background (CMB) (Battye et al. 2001; Avelino et al. 2000; Martins et al. 2002; Rocha et al. 2003) and the primordial abundances of light elements (Bergström et al. 1999; Nollet and Lopez 2002; Ichikawa and Kawasaki 2002, 2004).

The prediction of the light elements abundances (^{4}He, D, ^{7}Li) produced during the first minutes of the universe can be calculated using numerical (Wagoner 1973; Kawano) and analytical (Esmailzadeh et al. 1991; Mukhanov 2003) methods. Ichikawa and Kawasaki (2002) modified the public code in order to analyze the BBN scenario with varying gauge coupling constants. They considered a theoretical model taken from string theory where the variation of the coupling constant is related to the expectation values of the dilaton field and compared with observational data. In consequence, the results they obtained are restricted to the validity of this model. Furthermore, numerical calculations of the theoretical abundances of the light elements allowing only a variation of the fine structure constant were performed by different authors (Bergström et al. 1999; Nollet and Lopez 2002; Ichikawa and Kawasaki 2004). On the other hand, an analytical study of ^{4}He abundance including variation of the gauge coupling constants was performed by Müller et al. (2004). Moreover, the change in the abundance of ^{4}He due to variable mass in 5 dimensional theories was analyzed by Anchordoqui et al. (1996). Finally, the effect of considering non extensive thermostatistics has been analyzed by various authors (Torres et al. 1997; Pessah et al. 2001; Pessah and Torres 2001).

In this work, we follow the semi-analytical method proposed by Esmailzadeh et al. (1991) to study the effect of a possible variation of the values of the three gauge coupling constants of the Standard Model of Particles Interactions (SM) between primordial nucleosynthesis and the present. Even though, the semi-analytical method gives results one order of magnitude less accurate that the calculations performed with the numerical code, it is very useful to find out the dependence of the abundances and temperatures with the fundamental constants, which is one of the principal aims of this work.

We will not assume any of the theoretical models for varying constants mentioned above. Motivated by theoretical predictions and observational data, we will study the formation of the light elements in the early universe assuming that the values of the gauge coupling constants of the fundamental interactions (electromagnetic, strong and weak) may be different from their actual value. Thus, our approach is a phenomenological one and our results will be model independent. Furthermore, we assume the chiral limit of QCD to analyze the dependence of nucleon masses, binding energies and cross sections with the strong interaction coupling constants. The gauge coupling constants of $U(1)$, $SU(2)$ and $SU(3)$, namely, α_1, α_2 and α_3 are related with the fine structure constant α, the QCD energy scale Λ_{QCD} and the Fermi coupling constant G_F through the following equations:

$$\alpha^{-1}(E) = \frac{5}{2}\alpha_1^{-1}(E) + \alpha_2^{-1}(E) \tag{1}$$

$$\Lambda_{QCD} = E \exp\left[-\frac{2\pi}{7}\alpha_3^{-1}(E)\right] \tag{2}$$

$$G_F = \frac{\pi\ \alpha_2(M_Z)}{\sqrt{2}M_Z^2} \tag{3}$$

where E refers to the energy scale and M_Z refers to the boson Z mass. Actually, we will study the dependence of the different physical quantities involved in the calculation of the primordial abundances with α, Λ_{QCD} and G_F.

Almost all of the observational and experimental data are consistent with no variation of the constants (Landau and Vucetich 2002). Moreover, the reported variations (Murphy et al. 2003; Ivanchik et al. 2003) are very small ($\frac{\Delta\alpha_i}{\alpha_i} \sim 10^{-5}$). Therefore, in order to find out the dependences of relevant physical quantities with α, Λ_{QCD} and G_F we will perform a Taylor expansion to first order in each case as follows:

$$\begin{aligned}\Delta Q &= \frac{\partial Q}{\partial \alpha}|_{(\alpha^{today},\Lambda_{QCD}^{today},G_F^{today})}\Delta\alpha \\ &+\frac{\partial Q}{\partial \Lambda_{QCD}}|_{(\alpha^{today},\Lambda_{QCD}^{today},G_F^{today})}\Delta\Lambda_{QCD} + \frac{\partial Q}{\partial G_F}|_{(\alpha^{today},\Lambda_{QCD}^{today},G_F^{today})}\Delta G_F\end{aligned} \tag{4}$$

where Q refers to the physical quantities involved in the nucleosynthesis calculation such as nucleon and nucleus masses, nuclear binding energies, cross sections and abundances of the elements.

In the standard picture, the only free parameter of the nucleosynthesis calculation is the density of baryonic matter $\Omega_B h^2$. This quantity has been determined with a great accuracy with data from the CMB provided by WMAP (Spergel et al. 2003). On the other hand, the baryon density can also be estimated using data provided by galaxy surveys (SDSS, 2dF) and x-ray satellites (Chandra, XMM-Newton, ROSAT, ASCA). In appendix A we combine different data to obtain an estimation of $\Omega_B h^2$ independent of the WMAP estimate. Therefore, we shall approach to the problem studying the dependences of all physical quantities and abundances with both the fundamental constants and the baryon density. Thus, we will obtain the uncertainties of the abundances of the light elements as function of the variations of the fundamental constants with respect to their actual value and as function of the variation of $\Omega_B h^2$ with respect to the WMAP estimate (Spergel et al. 2003). On the other hand, we will also compare the predicted theoretical expressions for the abundances with observational data and include independent estimates of the baryon density in the analysis (see section 5).

Furthermore, in section 2, we shall calculate the dependence of the nucleon masses and binding energies with the fundamental constants, and in section 3, the corresponding dependence of the relevant scattering cross sections. We have carried this calculations in some detail, since there are several subtle points in these dependences that will be clearly exhibited in the final results. In section 4 we apply the semi-analytical method proposed by Esmailzadeh et al. (1991) to calculate the abundances of the light elements and their dependence with the fundamental constants. In section 5 we briefly describe the observational data and the results of comparing them with the theoretical predictions calculated in this work. We also discuss our conclusions.

2. Masses and binding energies of light elements

In this section we analyze the dependence of the nucleon masses, nuclear binding energies and nuclei masses of the light elements with the fundamental constants α and Λ_{QCD}. The weak interaction contribution is too small to produce any observable consequences (Haugan and Will 1976; Chamoun and Vucetich 2002).

The dependence of the hadronic masses and nuclear binding energies with the QCD coupling constant α_3 or the QCD scale parameter Λ_{QCD} depends on the model of hadronic interactions considered. However, if we assume that the quark masses are null, an assumption which is called in the literature as chiral limit, there is a only a single parameter in the theory, namely the QCD scale parameter Λ_{QCD}. Even though great efforts (Beane and Savage 2003b,a; Epelbaum et al. 2003; Flambaum and Shuryak 2002, 2003; Dmitriev and Flambaum 2003; Dmitriev et al. 2004; Olive et al. 2002) have been done in order to analyze the dependence of nucleon masses and binding energies with Λ_{QCD} beyond the chiral limit, this task is not trivial and highly model dependent.

On the other hand, from simple dimensional analysis (Stevenson 1981), it follows that in a theory with only one relevant parameter all static observables with dimension of mass must be proportional to this parameter, which in our case is Λ_{QCD}. More precisely, any quantity σ with units of E^D (where E means energy) must satisfy an equation of the form:

$$\sigma = \Lambda_{QCD}^D f\left[\frac{Q}{\Lambda_{QCD}}\right] \tag{5}$$

where Q is a quantity specifying the energy scale. Furthermore, for static quantities such as nucleon masses the previous equation takes the form:

$$\sigma = \Lambda_{QCD}^D f\left[\frac{\sigma}{\Lambda_{QCD}}\right] \tag{6}$$

since the only scale parameter is σ itself. The solution of equation 6 reads:

$$\sigma = \Lambda_{QCD}^D X \tag{7}$$

where X is a dimensionless numerical constant. In such way, all low-energy static quantities will satisfy an equation of the form 7. Moreover, all nucleon masses and energies will have a linear dependence:

$$m_N \sim \epsilon_B \sim \Lambda_{QCD} \tag{8}$$

and all nuclear radii will satisfy:

$$R \sim \Lambda_{QCD}^{-1} \tag{9}$$

since we use units where $\hbar = c = 1$ for this analysis. The chiral limit was previously considered by Sisterna and Vucetich (1990) studying time variation of fundamental constants in planetary phenomena.

The mass of the nucleons can be written as a sum of two contributions: the electromagnetic contribution m_N^C and the strong interaction contribution m_N^S:

$$m_N = m_N^C + m_N^S \tag{10}$$

The electromagnetic contribution depends on the nuclear radius R as follows:

$$\epsilon_C = \frac{Z}{4\pi\epsilon_0}\frac{e^2}{R} \tag{11}$$

Therefore the electromagnetic contribution to the nucleon mass in the chiral limit has the following dependence with Λ_{QCD}:

$$m_N^C \sim \Lambda_{QCD} \tag{12}$$

Cottingham (1963) used perturbation theory to calculate the electromagnetic self energy of a nucleon m_N to first order in α:

$$m_N^C \sim K\alpha \tag{13}$$

where K can be expressed as a function of Sachs form factors $G_{E,M}^N$, which can be calculated from measurements of electron-nucleon scattering. On the other hand, the strong interaction contribution to the mass in the chiral limit is proportional to Λ_{QCD}. Therefore, we can write:

$$m_N^C = m_N^C \frac{\alpha}{\alpha^{today}}\frac{\Lambda_{QCD}}{\Lambda_{QCD}^{today}} \tag{14}$$

$$m_N^S = m_N^S \frac{\Lambda_{QCD}}{\Lambda_{QCD}^{today}} \tag{15}$$

After performing a Taylor expansion to first order, as explained in section 1 and using equations 14 and 15, we obtain the dependence of the nucleon masses with the fundamental constants:

$$\frac{\delta m_N}{m_N} = \frac{m_N^C}{m_N}\frac{\delta\alpha}{\alpha} + \frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} = P\frac{\delta\alpha}{\alpha} + \frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} \tag{16}$$

The values of P are shown in table 1.

Next, we analyze the dependence of the nuclei masses with α and Λ_{QCD}. As we did for nucleons, we perform a Taylor expansion to first order to obtain for a nucleus of mass m_x the following expression:

$$\frac{\delta m_x}{m_x} = (A-Z)\frac{m_n}{m_x}\frac{\delta m_n}{m_n} + Z\frac{m_p}{m_x}\frac{\delta m_p}{m_p} - \frac{\epsilon_x}{m_x}\frac{\delta\epsilon_x}{\epsilon_x} \tag{17}$$

In the more general case, the binding energy (ϵ_x) can be written as a sum of two terms: the electromagnetic contribution (ϵ_C) and the strong interaction contribution (ϵ_S) as follows: $\epsilon_x = \epsilon_C + \epsilon_S$. However, in the cases of nuclei with only one proton (D and T), there is no electromagnetic interaction and therefore the electromagnetic contribution (ϵ_C) is null . On the other hand, the same arguments that were used to obtain equations 14 and 15 can be applied for the binding energy to obtain:

$$\frac{\delta\epsilon_x}{\epsilon_x} = \frac{\epsilon_C}{\epsilon_x}\frac{\delta\alpha}{\alpha} + \frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} \tag{18}$$

Table 1: Dependence of nucleon and nuclei masses with the fundamental constants: $\frac{\delta m_N}{m_N} = P\frac{\delta\alpha}{\alpha} + \frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}}$

Nucleon/Nucleus	$P(\times 10^{-4})$
m_p	6.71
m_n	-1.38
D	2.67
T	1.32
^{3}He	1.05
^{4}He	0.66
^{6}Li	1.50
^{7}Li	1.14
^{7}Be	2.30

Inserting this last expression in equation 17, we obtain the general expression for the dependence of a nucleus mass with α and Λ_{QCD}:

$$\frac{\delta m_x}{m_x} = P\frac{\delta\alpha}{\alpha} + \frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} \tag{19}$$

The values P for different nuclei are shown in table 1.

3. Thermonuclear reaction rates

In this section we calculate the thermonuclear reaction rates as functions of fundamental constants. We also show the dependence of the reaction rates with the baryon density $\rho_B = \Omega_B h^2$. Following Esmailzadeh et al. (1991) we can write the thermonuclear reaction rate as:

$$[ij \to kl] = \rho_B N_A \langle\sigma v\rangle = 0.93 \times 10^{-3}\Omega_B h^2 T_9^3 N_A \langle\sigma v\rangle \frac{1}{\mathrm{seg}} \tag{20}$$

where σ is the cross section, v is the relative velocity, $\rho_B = 0.93 \times 10^{-3}\Omega_B h^2 T_9^3 \frac{\mathrm{g}}{\mathrm{cm}^3}$ is the density of baryonic matter, N_A is Avogadro's number per gram, T_9 is the temperature in units of $10^9 K$.

Using a Maxwell-Boltzmann distribution in velocities, the Boltzmann averaged cross section, $\langle\sigma v\rangle$ can be expressed as follows:

$$\langle\sigma v\rangle = \left(\frac{\mu}{2\pi kT}\right)^{3/2} \int e^{-\frac{\mu v^2}{2kT}} v\sigma(E) d^3v \tag{21}$$

We need to find out $\frac{\delta[ij\to kl]}{[ij\to kl]}$ as a function of the relative variations of the fundamental constants, $\left(\frac{\delta\alpha}{\alpha}, \frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}}, \frac{\delta G_F}{G_F}\right)$ and $\frac{\delta\Omega_B h^2}{\Omega_B h^2}$: the relative variation of the value of the baryon density with respect to WMAP estimate $\Omega_B h^2 = 0.0224$ (Spergel et al. 2003). The temperature does not depend on the values of the fundamental constants, but the final temperature of each stage does and therefore, we can write:

$$\frac{\delta[ij\to kl]}{[ij\to kl]} = \frac{\delta\Omega_B h^2}{\Omega_B h^2} + 3\frac{\delta T_9^f}{T_9^f} + \frac{\delta\langle\sigma v\rangle}{\langle\sigma v\rangle} \tag{22}$$

where $T_9^f = f(\alpha, \Lambda_{QCD}, G_F)$ for all the reaction rates. On the other hand, $\frac{\delta\langle\sigma v\rangle}{\langle\sigma v\rangle}$ depends on the fundamentals constants through the masses of the nucleons and light nuclei and the form factor of the reactions. In the general case, there are not analytic expressions for $\sigma(E)$ derived from "first principles". We suggest several expressions that attempt to fit $\sigma(E)$, according to the elements in the reactions .

3.1. Cross sections for charged particles reactions

The cross section for charged particles reactions is given by Fowler et al. (1967, 1975); Wagoner et al. (1967):

$$\sigma = \frac{S(E)}{E} e^{-2\pi\alpha Z_1 Z_2 \sqrt{\mu c^2/2E}} \tag{23}$$

where Z_i is the charge of the i particle, $\mu = \frac{m_1 m_2}{m_1+m_2}$ is the reduced mass, E is the energy, $S(E)$ is the form factor. The dependence of the cross sections for charged particle reactions have been analyzed previously (Bergström et al. 1999; Nollet and Lopez 2002). In particular, Nollet and Lopez (2002) improved the analysis and studied the form factor as a function of α. In this paper, we use the criteria established by these authors to analyze the dependence of the form factor with α.

Next, we analyze the dependence of the form factor with Λ_{QCD} using dimensional arguments and the chiral limit. The units of the cross section are cm^2 and therefore it follows that in a theory with massless quarks $\sigma \sim \Lambda_{QCD}^{-2}$. The only quantity of eq. 23 that has units is the factor $\frac{S(E)}{E}$ and thus we obtain:

$$S(E) \sim \Lambda_{QCD}^{-1} \tag{24}$$

This is valid for all charged particle reactions. The exact dependence of the form factor $S(E)$ with the energy is unknown. However, as it is usually done in the literature (Fowler et al. 1967, 1975; Wagoner et al. 1967), we can do a MacLaurin expansion:

$$S(E) = S(0)\left(1 + \left(\frac{dS}{dE}\right)_{E=0}\frac{1}{S(0)}E + \frac{1}{2}\left(\frac{d^2S}{dE^2}\right)_{E=0}\frac{1}{S(0)}E^2\right) \tag{25}$$

where $\frac{dS}{dE}$ and $\frac{d^2S}{dE^2}$ are expressed in barn and barn Mev^{-1} respectively. The terms inside the brackets have no dimensions, therefore:

$$S(0) \sim \Lambda_{QCD}^{-1} \tag{26}$$

The dependence of the charged particle cross sections with α has been analyzed by Nollet and Lopez (2002), yielding:

$$S(0) \sim \alpha \tag{27}$$

Furthermore, it follows that all radiative capture rates should be multiplied by a factor $\frac{\alpha}{\alpha^{today}}$, except the reactions T$(\alpha\gamma)^7$Li and ^{3}He$(\alpha\gamma)^7$Be. This cross sections should be multiplied by $f(\alpha) = \sum b_i \left[\frac{\alpha}{\alpha^{today}} - 1\right]$ (see table 2). Finally, in the cases in which the reaction produces two charged particles, the cross section should by multiplied by $1 - b + b\frac{\alpha}{\alpha^{today}}$ (see table 3).

We insert the expression for $\sigma(E)$ into the equation (21) in order to calculate the Boltzmann averaged cross sections:

$$\langle\sigma v\rangle = \sqrt{\frac{8}{\mu\pi}}(kT)^{-1/2}\sum_{i=0}^{2}\frac{(kT)^i}{i!}\left(\frac{d^iS}{dE^i}\right)_{E=0}\int_0^\infty y^i e^{-y-\xi y^{-1/2}}dy \tag{28}$$

where $\xi = 2\pi\alpha Z_1 Z_2\sqrt{\mu c^2/2kT}$ and the masses in kg. The integrals are calculated in Bergström et al. (1999). Tables 4 and 5 show the dependence of charged particles reaction rates with the fundamental constants.

3.1.1. Cutoff factor

The truncated MacLaurin series we have use for $S(E)$ diverges at high energy. Thus, it is important to include a cutoff factor for non-resonant reaction rates so that they can

Table 2: Radiative captures, its dependence on α $\left(f(\alpha) = \sum b_i \left[\frac{\alpha}{\alpha^{today}} - 1\right]\right)$

Reaction	b_0	b_1	b_2	b_3	b_4	b_5
^{3}H$(\alpha,\gamma)^7$Li	1	1.372	0.502	0.183	0.269	-0.218
^{3}He$(\alpha,\gamma)^7$Be	1	2.148	0.669	-5.566	-10.630	-5.730

be used at any energy. The next term in the expansion for $S(E)$ would be proportional to $E^3 \sim T^2$, so as it is proposed in the literature (Fowler et al. 1967, 1975) we consider a cutoff factor: $f_{co} = e^{-(T_9/T_{co})^2}$, where $T_{co} \sim \frac{E_r}{\alpha}$ and E_r is the resonant energy (Fowler et al. 1967, 1975). Therefore we multiply the expression (28) by a factor:

$$f_{co} = e^{-\left(\frac{\alpha T_9}{E_r}\right)^2} \tag{29}$$

This correction is relevant for the following reactions: ^{3}He$(d,p)^4$He, ^{3}H$(d,n)^4$He, ^{6}Li$(p,\alpha)^3$H, ^{6}Li$(\alpha,p)^{10}$Be, ^{7}Li$(p,\alpha)^4$He.

3.1.2. Alternative expression for the form factor

In the MeV range the cross section form factor varies considerably. In this range the truncated MacLaurin series is not satisfactory so that it is convenient (Fowler et al. 1967, 1975) to use for $S(E)$ an expression of the form: $S(E) = S(0)e^{-aE}$. In such way, the cross sections are given by:

$$\sigma(E) = \frac{S(0)}{E} e^{-aE} e^{-2\pi\alpha Z_1 Z_2 \sqrt{\mu c^2/2E}} \tag{30}$$

where a has no dependence on the fundamental constants. The quantities with units in equation 30 are $S(0)$ and E, therefore, in the chiral limit we have $S(0) \sim \Lambda_{QCD}^{-1}$. In such way, the Boltzmann cross section (eq.(21)) yields:

$$\langle \sigma v \rangle = \frac{8}{\sqrt{6}} \left(\frac{\mu}{kT}\right)^{3/2} \frac{S(0)kT}{akT+1} \left(\frac{\xi_a^2}{4}\right)^{1/6} e^{-3\left(\frac{\xi_a^2}{4}\right)^{1/3}} \left[1 + \frac{5}{36}\left(\frac{\xi_a^2}{4}\right)^{-1/3}\right] \frac{\mathrm{cm}^3}{\mathrm{seg}} \tag{31}$$

Table 3: Dependence on α of different kinds of reactions rates

Reaction	Multiplied by
Charged particles reaction rates	$\frac{\alpha}{\alpha^{today}}$
Photon emission	$\frac{\alpha}{\alpha^{today}}$
^{2}H(d,p)	$1 + 0.16 - 0.16\frac{\alpha}{\alpha^{today}}$
^{3}He(n,p)	$1 - 0.30 + 0.30\frac{\alpha}{\alpha^{today}}$
^{3}He(d,p)	$1 + 0.09 - 0.09\frac{\alpha}{\alpha^{today}}$
^{7}Li(p,α)	$1 + 0.18 - 0.18\frac{\alpha}{\alpha^{today}}$
^{7}Be(n,p)	$1 - 0.20 + 0.20\frac{\alpha}{\alpha^{today}}$

where $\xi_a = \xi\sqrt{akT+1} = 2\pi\alpha Z_1 Z_2\sqrt{\frac{\mu c^2}{2kT}(akT+1)}$. This alternative expression for non-resonant reaction rates is relevant for the following reactions: ^{6}Li$(p,\gamma)^7$Be, ^{3}H$(\alpha,\gamma)^7$Li and ^{3}He$(\alpha,\gamma)^7$Be

3.2. Resonant charged particle reaction rates

The expressions for the cross sections vary with the temperature. Moreover, in the range of energies relevant for our calculation there are certain reactions that proceed through many resonances. In this case, we have to include an extra term in the cross section. There are two kinds of resonances: i) Single Resonance, ii) Continuum Resonance. In each case we use the expressions given by Fowler et al. (1967, 1975).

3.2.1. Resonance cross sections

In this case, the following expression provides a good fit to the cross section: Fowler et al. (1975):

$$\sigma(E) = \frac{\pi\hbar^2}{2\mu E}\frac{\omega_r\Gamma_1\Gamma_2}{(E-E_r)^2+\Gamma^2/4} \tag{32}$$

where Γ_i is the partial with for the decay of the resonant state by the reemission of $(i-1)+i$, Γ is the sum over all partial widths (the partial widths are not functions of α), $\omega_r = \frac{(1+\delta_{ab})g_r}{g_a g_b}$ and $g_r = 2J_r+1$, J_r being the spin of the resonant state, μ in kg and E_r is the resonance energy in the center of momentum system and depends on the nuclear radius. Finally, the Boltzmann cross section $\langle\sigma v\rangle$ can be calculated as follows:

$$\langle\sigma v\rangle = \left(\frac{2\pi\hbar^2}{\mu kT}\right)^{3/2}\frac{(\omega\gamma)_r}{\hbar}e^{E_r/kT} \quad \frac{\mathrm{cm}^3}{\mathrm{seg}} \tag{33}$$

where $\gamma_r = \left(\frac{\Gamma_1\Gamma_2}{\Gamma}\right)_r$. Here, the cross section depends on the fundamental constants through the final temperature and the resonance energy. Besides, the resonance width is also a function of the fundamental constants, but the cross section is much less sensitive to this dependence.

In a theory with massless quarks: $E_r \sim \Lambda_{QCD}$. On the other hand, E_r does not depend on α. The dependence of the temperature will be analyzed in section 4. This correction is relevant for the following reactions: ^{2}H$(\alpha,\gamma)^6$Li, ^{6}Li$(p,\alpha)^3$H, ^{6}Li$(\alpha,\gamma)^{10}$Be, ^{7}Be$(p,\gamma)^8$Be and ^{7}Li$(p,\alpha)^4$He.

3.2.2. Continuum resonances

When the temperature scale is of order $T_9 \sim 1$, there are several reactions that proceed through many resonances that are separated by intervals not greater than their widths or that overlap to form a continuum. In this cases, the cross section can be written as Fowler et al. (1975):

$$\sigma(E) = \begin{cases} 2\sigma(2C)\frac{C}{E}\left(\frac{E}{C}-1\right)^{m+1/2} & \text{si} \quad E \geq C \\ 0 & \text{si} \quad E \leq C \end{cases} \tag{34}$$

where m is integer or rational fraction, C is the effective continuum threshold energy and $\sigma(2C)$ is the cross section at $E = 2C$.

After inserting this expression in the integral (21), we obtain:

$$\langle \sigma v \rangle = \Gamma(m+3/2)\sigma(2C)\sqrt{\frac{32C}{\pi\mu}}\left(\frac{kT}{C}\right)^m e^{-C/kT} \tag{35}$$

where $\Gamma(m+3/2)$ is the gamma function. On the other hand, C has units of energy. Therefore, in the chiral limit $C \sim \Lambda_{QCD}$. This correction is relevant for the following reactions: ^{3}He$(d,p)^4$He, ^{3}H$(d,n)^4$He, ^{6}Li$(p,\alpha)^3$H and ^{6}Li$(\alpha,\gamma)^{10}$Be.

3.3. Non charged particles reaction rates

In this case, there is no Coulomb barrier so the cross section cannot be written as the equation (23). Following Fowler et al. (1967) we write:

$$\sigma(E) = \frac{S(E)}{v} \tag{36}$$

where v is the relative velocity. We consider the expression given by Fowler et al. (1967):

$$S(E) = S(0) + \left(\frac{dS}{d\widetilde{E}}\right)_{\widetilde{E}=0} E^{1/2} + 1/2\left(\frac{d^2S}{d\widetilde{E}^2}\right)_{\widetilde{E}=0} E \tag{37}$$

where $\widetilde{E} = E^{1/2}$.

In chiral limit $\sigma \sim \Lambda_{QCD}^{-2}$, and therefore:

$$S(0) \sim \Lambda_{QCD}^{-2} \tag{38}$$

In this way, the expression for the reaction rates (equation 21) yields:

$$\langle \sigma v \rangle = S(0)\left(1 + \frac{2}{\sqrt{\pi}}\frac{S^{'}(0)}{S(0)}(kT)^{1/2} + \frac{3}{4}\frac{S^{''}(0)}{2S(0)}kT\right) \tag{39}$$

where $\frac{dS}{d\tilde{E}}$ is in units of $\frac{\mathrm{cm}^3}{\mathrm{seg}}\mathrm{MeV}^{-1/2}$ and $\frac{d^2S}{d\tilde{E}}$ in in units of $\frac{\mathrm{cm}^3}{\mathrm{seg}}\mathrm{MeV}^{-1}$

For radiative emission reactions, the cross section should be multiplied by a factor $\frac{\alpha}{\alpha^{today}}$. Table 6 shows some reaction rates between a neutron and a nucleus.

In some cases the reaction rates of the inverse reactions are needed. Next, we show the expressions for these reaction rates. For inverse reactions of the form $[BCAn]$, where neither B or C are photons, we use the expression given by Fowler et al. (1967, 1975):

$$[BCAn] = \frac{2(1+\delta_{BC})g_A}{(1+\delta_{An})g_B g_C}\left(\frac{m_A m_n}{m_B m_C}\right)^{3/2} e^{-Q/kT}[AnCB]\frac{1}{\mathrm{seg}} \tag{40}$$

where $Q = m_A + m_n - m_B - m_C$. In this case, the form factor $S(0)$ should be multiplied by $\frac{\alpha}{\alpha^{today}}$ because of the Coulomb barrier.

For inverse reactions of the form [BγnA], we use the expression given by Fowler et al. (1967, 1975):

$$Y_\gamma[B\gamma nA] = \frac{g_A g_n}{(1+\delta_{An})g_B}\left(\frac{m_A m_n}{m_B}\right)^{3/2}\left(\frac{M_U^2 kT}{2\pi\hbar^2}\right)^{3/2} e^{-Q/(kT)}\langle\sigma v\rangle\frac{1}{\mathrm{seg}} \tag{41}$$

where $Q = m_A + m_n - m_B$, $g_n = 2j_n + 1 = 2$ and $M_U = \frac{1}{N_A}$.

In both cases, the additional dependence on the fundamental constants introduced by the inverse reactions proceed from the temperature (see section 4) and the masses (see section 2).

3.4. Neutron lifetime

Neutron β decay is one of the few reactions whose cross section can be explicitly computed from first principles in terms of the fundamental constants. It can be approximated by the one point interaction of neutron, proton, electron and neutrino. The reaction rate for neutron β decay is:

$$n \to p + e^- + \overline{\nu_e} \tag{42}$$

Following Ichikawa and Kawasaki (2002) we write the inverse of neutron lifetime as follows:

$$\frac{1}{\tau} \simeq G_F^2 \int_0^{P_0} d^3p_e d^3p_\nu\, \delta(E_e + E_\nu + m_p - m_n) \tag{43}$$

where, G_F is Fermi coupling constant, E_e and E_ν are the electron and neutrino energy, and p_e and p_ν are the electron and neutrino momenta. After integration we obtain:

$$\frac{1}{\tau} = \frac{16}{60}\pi^2 G_F^2 m_e^5 \left(\sqrt{q^2-1}\,(2q^4 - 9q^2 - 8) + 15q\,\ln\left(q + \sqrt{q^2-1}\right)\right)\frac{1}{\mathrm{seg}} \tag{44}$$

where m_e is the electron mass, $q = \frac{Q}{m_e} = \frac{m_n - m_p}{m_e}$

In such way, we obtain the dependence of the neutron decay rate τ^{-1} on G_F and on the mass difference (which is a function of α and Λ_{QCD}, see section 2):

$$\frac{\delta[n]}{[n]} = -\frac{\delta\tau}{\tau} = 2\frac{\delta G_F}{G_F} + 6.54\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} - 3.839\frac{\delta\alpha}{\alpha} \tag{45}$$

4. Abundances as functions of fundamental constants

In this section we calculate the abundances of light elements and their dependence on fundamental constants. First we obtain the neutron abundance until the freeze-out time of weak interaction. After this time the neutrons decay freely into protons and electrons, so their abundance only changes due to this decay.

The general form of the equations that govern the abundances of the light elements is:

$$\dot{Y}_i = J(t) - \Gamma(t) Y_i \tag{46}$$

where $J(t)$ and $\Gamma(t)$ are time-dependent source and sink terms and the dot corresponds to the time derivative. The time-dependent static solution of this equation is what we will call following Esmailzadeh et al. (1991) the quasi-static equilibrium (QSE) solution:

$$f_i = \frac{J(t)}{\Gamma(t)} \tag{47}$$

To determine the formation of light nuclei we shall solve the following equations using only the most important reactions according to the rates of production and destruction following the criteria established by Esmailzadeh et al. (1991):

$$\begin{aligned}
\dot{Y}_n &= Y_d Y_d[ddn3] + Y_d Y_T[dTn\alpha] + Y_p Y_T[pTn3] + Y_d Y_\gamma[d\gamma np] + \\
&\quad - Y_n Y_p[npd\gamma] - Y_n Y_3[n3Tp] - Y_n[n] && (48)\\
\dot{Y}_d &= Y_n Y_p[npd\gamma] - 2Y_d Y_d\left([ddpT] + [ddn3]\right) - Y_d Y_T[dTn\alpha] + \\
&\quad - Y_d Y_3[d3p\alpha] - Y_d Y_\gamma[d\gamma np] - Y_d Y_p[dp3\gamma] && (49)\\
\dot{Y}_3 &= Y_d Y_p[pd3\gamma] + Y_T Y_p[pTn3] + Y_d Y_d[ddn3] +
\end{aligned}$$

$$
\begin{aligned}
& -Y_d Y_3[d3p\alpha] - Y_n Y_3[n3pT] && (50)\\
\dot{Y_T} &= Y_n Y_3[n3pT] + Y_d Y_d[ddpT] - Y_d Y_T[dTn\alpha] + \\
& -Y_p Y_T[pTn3] - Y_p Y_T[pT\gamma\alpha] && (51)\\
\dot{Y_6} &= Y_d Y_\alpha[d\alpha 6\gamma] - Y_n Y_6[n6\alpha T] - Y_p Y_6[p6T\alpha] && (52)\\
\dot{Y_7} &= Y_n Y_\alpha[n67\gamma] + Y_n Y_B[nBp7] + Y_T Y_\alpha[T\alpha 7\gamma] + \\
& -Y_p Y_7[p7\alpha\alpha] - Y_n Y_7[n78\gamma] && (53)\\
\dot{Y_B} &= Y_p Y_6[p6B\gamma] + Y_3 Y_\alpha[3\alpha B\gamma] - Y_\gamma Y_B[B\gamma 3\alpha] - Y_n Y_B[nBp7] + \\
& -Y_p Y_B[pB\gamma 8] - Y_d Y_B[dB\alpha\alpha p] && (54)\\
\dot{Y_\alpha} &= Y_d Y_3[d3p\alpha] + Y_n Y_3[n3\alpha\gamma] + Y_d Y_T[dTn\alpha] + Y_p Y_T[pT\gamma\alpha] && (55)
\end{aligned}
$$

where n refers to neutron, p to proton, d to deuterium, T to tritium, 3 to ^{3}He, α to ^{4}He, 6 to ^{6}Li, 7 to ^{7}Li, B to ^{7}Be, γ to the photon and $[ijkl]$ is the rate of the reaction $i + j \to k + l$ and Y_i is the abundance of the i element relative to baryons $\left(Y_i = \frac{n_i}{n_B}\right)$. In addition, these equations obey neutron number conservation:

$$\dot{Y_n} + \dot{Y_d} + \dot{Y_3} + 2\dot{Y_T} + 2\dot{Y_\alpha} = -Y_n[n] \tag{56}$$

The method of Esmailzadeh et al. (1991) consists in calculating the different abundances between fixed point or stages. We shall solve equations 48 to 55 only for one element in each stage. For the other elements it is necessary to solve the quasi static equilibrium equation using only the most important rates of production and destruction. On the other hand, we perform the calculation of all final temperatures and abundances and all freeze-out temperatures numerically. Table 7 shows the different stages and the used equation.

The equations that describe the production of n, D, ^{3}He and T are independent to the equations for ^{6}Li, ^{7}Li and ^{7}Be. Therefore, we shall solve the first three using the quasi static equilibrium equation and then we use these results to calculate the other abundances.

To calculate the final abundance of light elements it is necessary to know the freeze-out temperature. The freeze-out of the production of each element happens when the most important destruction reaction rate equals to the expansion rate of the Universe. The dependence of the freeze-out temperatures and final temperature of each stage with the fundamental constants, will be calculated by deriving the equation that determines each temperature.

Each section in this chapter will discuss the calculation of abundances during a certain stage.

4.1. Neutron abundance until the freeze-out of weak interaction, $T > 9.1 \times 10^9$K

For the calculation of neutron abundance we follow the analysis performed by Bernstein et al. (1989). Let $\lambda_{pn}(T)$ be the rate of weak process that convert protons into neutrons and $\lambda_{np}(T)$ the rate of weak process that convert neutrons into protons. The basic rate equation reads:

$$\frac{dX}{dt} = \lambda_{pn}(t)(1 - X(t)) - \lambda_{np}(t)X(t) \tag{57}$$

where t is the time, and X is the ratio of the number of neutrons to the total number of baryons. After changing variables $\left(y = \frac{\Delta m}{T}\right)$, the solution of the last equation can be written as follows:

$$X(y) = X_{eq}(y) + \int_0^y dy' e^{y'} \left[X_{eq}(y')\right]^2 e^{K(y)-K(y')} \tag{58}$$

where

$$\begin{aligned} K(y) &= b\left[\frac{4}{y^3} + \frac{3}{y^2} + \frac{1}{y} + \left(\frac{4}{y^3} + \frac{1}{y^2}\right)e^{-y}\right]; \qquad b = 255\frac{M_{pl}}{\Delta m^2 \tau}\sqrt{\frac{45}{43\pi^3}} \\ X_{eq}(y) &= \frac{1}{1+e^y} \end{aligned} \tag{59}$$

τ is the neutron mean life and $\Delta m = m_n - m_p$. In order to obtain the asymptotic behavior, the limit $T \to 0$ or $y \to \infty$ is taken:

$$X(y = \infty) = \int_0^\infty dy' e^{y'} X_{eq}\left(y'\right)^2 e^{-K\left(y'\right)} = 0.151 \tag{60}$$

In the last equation, only b depends on the fundamental constants through τ and Δm (see sections 2 and 3 for the dependence of these quantities with the fundamental constants). In such way, from equation 60, we obtain:

$$\frac{\delta X(y=\infty)}{X(y=\infty)} = -1.04\frac{\delta G_F}{G_F} - 2.361\frac{\delta \Lambda_{QCD}}{\Lambda_{QCD}} + 1.386\frac{\delta\alpha}{\alpha} \tag{61}$$

4.2. Until the production of ^{4}He becomes efficient, $9.1 \times 10^9\mathrm{K} > \mathrm{T} > 0.93 \times 10^9\mathrm{K}$

After the freeze-out of the weak interactions, the only change in the neutron abundance is due to neutron decay. Therefore, the neutron abundance in this stage reads:

$$Y_n = X(y=\infty)\ e^{-t/\tau} = \ e^{-0.198/T_9^2} \tag{62}$$

In the beginning of this stage there are no nucleus with two or more nucleons, therefore it is a good approximation to consider: $Y_p \simeq 1 - Y_n$. However, as the universe expands, the temperature goes down and light nuclei formation begins. Therefore, at the end of this stage, this expression is no longer valid.

In order to get a consistent solution of equation 56 (Esmailzadeh et al. 1991), it it is necessary to set all the rates equal to zero with the exception of the largest rate which equals to $-2\dot{Y}_\alpha - Y_n[n]$. In such way, the equations to solve in this stage are:

$$\dot{Y}_n = -2\dot{Y}_\alpha - Y_n[n] \tag{63}$$

$$\dot{Y}_d = \dot{Y}_3 = \dot{Y}_T = 0 \tag{64}$$

Table 8 shows the solutions.

When the production of ^{4}He becomes efficient the stage ends. The final temperature is given by is given by setting $\dot{Y}_n = 0$ in equation 63. For this stage, we obtain $T_9^f = 0.93$ and the following results:

$$\begin{array}{lll} Y_p^f = 0.76 & Y_d^f = 4.1 \times 10^{-4} & Y_T^f = 2.0 \times 10^{-5} \\ Y_n^f = 0.12 & Y_3^f = 5.8 \times 10^{-8} & Y_\alpha^f = 0.06 \end{array}$$

where Y_i^f is the final abundance of each nucleus or nucleons on this stage. It follows that the the abundances of D, T and ^{3}He are negligible respect to the abundances of neutrons and ^{4}He. This means:

$$\begin{aligned} Y_p^f &= 1 - Y_n^f - Y_d^f - Y_T^f - 2Y_\alpha^f - 2Y_3^f \\ &\simeq 1 - Y_n^f - 2Y_\alpha^f = 1 - 2Y_n^f \end{aligned} \tag{65}$$

Now, in order to calculate the dependence of the final temperature with the fundamental constants for this stage, we derivate the equation $2\dot{Y}_\alpha = Y_n[n]$ with respect to the fundamental constants and the temperature. In such way, we obtain:

$$\frac{\delta T_9^f}{T_9^f} = 0.068\frac{\delta \Omega_B h^2}{\Omega_B h^2} - 0.053\frac{\delta G_F}{G_F} + 0.063\frac{\delta\alpha}{\alpha} + 0.871\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} \tag{66}$$

where we also considered the dependence with the baryon fraction. Finally, the dependence of the abundance of neutrons on the fundamental constants and $\Omega_B h^2$ yields:

$$\frac{\delta Y_n}{Y_n} = 0.029\frac{\delta \Omega_B h^2}{\Omega_B h^2} - 1.522\frac{\delta G_F}{G_F} + 2.296\frac{\delta\alpha}{\alpha} - 3.459\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} \tag{67}$$

4.3. Final abundance of ^{4}He

The next stage corresponds to the calculation until the rate of production of deuterium dominates over the rate of change of neutrons. However, the freeze-out temperature of ^{4}He ($T = 0.915 \times 10^9$K) is lower than the final temperature of the previous stage but bigger than the final temperature of the next one. Therefore, we calculate now the final abundance of ^{4}He. In this case, the neutron number conservation equation reads:

$$2\dot{Y}_\alpha = Y_n[n] \tag{68}$$

For the others nucleus the quasi static equilibrium equation is valid (see table 8). The production of ^{4}He is dominated by $[dTn\alpha]$ and $[pT\gamma\alpha]$:

$$\dot{Y}_\alpha = Y_d Y_T[dTn\alpha] + Y_p Y_T[pT\gamma\alpha] = \left(Y_n Y_p \frac{[npd\gamma]}{Y_\gamma[d\gamma np]}\right)^2 [ddpT] \tag{69}$$

After solving numerically for T_9 the equation (68), we obtain $T_9^\alpha = 0.915$ and $Y_\alpha^f = 2Y_n = 0.238$. When the rate of ^{4}He production equals to the rate of neutron destruction, there is no more neutron that can form ^{4}He. Since this happens earlier than the usual freeze-out-time, we use equation 68 to calculate the freeze-out temperature. In such way, the dependence of the freeze-out temperature on the fundamental constants and $\Omega_B h^2$ yields:

$$\frac{\delta T_9^\alpha}{T_9^\alpha} = 0.061\frac{\delta\Omega_B h^2}{\Omega_B h^2} - 0.052\frac{\delta G_F}{G_F} + 0.063\frac{\delta\alpha}{\alpha} + 0.869\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} \tag{70}$$

Finally, since $Y_\alpha^c = 2Y_n$, we can express the variation of the final abundance of ^{4}He as a function of fundamental constants and $\Omega_B h^2$:

$$\frac{\delta Y_\alpha^c}{Y_\alpha^c} = 0.029\frac{\delta\Omega_B h^2}{\Omega_B h^2} - 1.538\frac{\delta G_F}{G_F} + 2.324\frac{\delta\alpha}{\alpha} - 3.496\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} \tag{71}$$

4.4. Neutron cooking, $0.93 \times 10^9\mathrm{K} > \mathrm{T} > 0.765 \times 10^9\mathrm{K}$

In this section we shall calculate the deuterium abundance as long as the change of neutron dominates the deuterium production rate. This is valid until the production rate of deuterium dominates the rate of change of neutrons, so this stage is over when $Y_n = Y_d$. In this stage, the neutron number conservation equation reads:

$$\dot{Y}_n = -2\dot{Y}_\alpha \tag{72}$$

For D, T and ^{3}He we solve the quasi-static equilibrium equations. The solutions are shown in table 8. For ^{4}He we solve the complete equation but considering only the largest production term $Y_d Y_T[dTn\alpha]$. Inserting all these solutions in equation 72, we obtain:

$$\dot{Y}_n = -2\left(Y_n Y_p \frac{[npd\gamma]}{Y_\gamma[d\gamma np]}\right)^2 [ddpT] \tag{73}$$

where the initial condition is given by the final values of the previous stage: $Y_n^0 = 0.12$ and $T_9^0 = 0.93$. We can write the solution to the last equation as follows:

$$Y_n = \left(\frac{1}{Y_n^0} + 2\int_{t_{initial}}^{t} \left(Y_p \frac{[npd\gamma]}{Y_\gamma[d\gamma np]}\right)^2 [ddpT] dt\right)^{-1} \tag{74}$$

After changing the integration variable to T_9 we perform the integral numerically as a function of temperature. We also compute the final temperature of this stage using the condition:

$$Y_n = Y_d \tag{75}$$

We obtain:

$$T_9^f = 0.765 \qquad\qquad Y_n = 6.4 \times 10^{-4} = Y_d \tag{76}$$

From 75 we obtain the dependence of the final temperature of this stage with respect to the fundamental constants and $\Omega_B h^2$:

$$\frac{\delta T_9^f}{T_9^f} = 0.031\frac{\delta\Omega_B h^2}{\Omega_B h^2} + 0.015\frac{\delta G_F}{G_F} - 0.023\frac{\delta\alpha}{\alpha} + 1.034\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} \tag{77}$$

Finally, the dependence of the final neutron and deuterium abundance can be obtained from equation (74):

$$\frac{\delta Y_d}{Y_d} = \frac{\delta Y_n}{Y_n} = -1.099\frac{\delta\Omega_B h^2}{\Omega_B h^2} - 0.058\frac{\delta G_F}{G_F} + 1.871\frac{\delta\alpha}{\alpha} - 0.488\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} \tag{78}$$

4.5. Deuterium cooking, $T \to 0$

For temperatures lower than $T_9 = 0.765$, the largest production rate corresponds to deuterium. Therefore, we set all other derivatives to zero in equation 56. Since the largest term for deuterium destruction is tritium production, the equation to solve is:

$$\dot{Y}_d = -2Y_d Y_d [ddpT] \tag{79}$$

with the initial condition $Y_d^0 = 6.4 \times 10^{-4}$ on $T_9^0 = 0.765$. Since this equation has the same form of equation 73, the solution reads:

$$Y_d = \left(\frac{1}{Y_d^0} + 2 \int_{t_{initial}}^{t} [ddpT] dt \right)^{-1} \tag{80}$$

In order to calculate the deuterium final abundance we consider the limit $T \to 0$ ($t \to \infty$). We obtain numerically, the deuterium final abundance $Y_d^f = 2.410 \times 10^{-5}$. On the other hand, the dependence of the deuterium final abundance with the fundamental constants and $\Omega_B h^2$ can be calculated by deriving equation 80:

$$\frac{\delta Y_d^c}{Y_d^c} = -1.072 \frac{\delta \Omega_B h^2}{\Omega_B h^2} - 0.036 \frac{\delta G_F}{G_F} + 2.320 \frac{\delta \alpha}{\alpha} + 0.596 \frac{\delta \Lambda_{QCD}}{\Lambda_{QCD}} \tag{81}$$

4.6. Final abundances

Here we calculate the freeze-out temperature and final abundances of ^{3}He, T, ^{6}Li, ^{7}Be and ^{7}Li and the dependence of these quantities with the fundamental constants. In order to calculate any light element abundance it is necessary to solve the quasi-static equilibrium equation:

$$\dot{Y}_i = 0 \tag{82}$$

We solve these equations considering only the most relevant reactions. In table 9 we show the quasi-static equilibrium solutions.

In order to compute the freeze-out temperature, we set the largest rate of destruction Γ of each equation that governs the abundance of the light elements equal to the universe expansion rate H:

$$\Gamma = H = \frac{1}{356} T_9^2 seg^{-1} \tag{83}$$

Table 10 shows the different freeze-out temperatures and their dependence on fundamental constants which is calculated deriving the previous equation. Using the freeze-out temperature we calculate the final abundance of the different nucleus and their dependence on the fundamental constants and $\Omega_B h^2$. In table 11 we show these results.

5. Results and discussion

In this section we compare the theoretical predictions of the abundances of the light elements obtained in the last section with observational data.

In section 4 we have obtained 7 equations of the form:

$$\frac{\delta Y_i^f}{Y_i^f} = A_i \frac{\delta G_F}{G_F} + B_i \frac{\delta \alpha}{\alpha} + C_i \frac{\delta \Lambda_{QCD}}{\Lambda_{QCD}} + D_i \frac{\delta \Omega_B h^2}{\Omega_B h^2} \tag{84}$$

where A_i, B_i, C_i and D_i are constant coefficients (see table 11), $\frac{\delta Y_i}{Y_i} = \frac{Y_i^{obs} - Y_i^{SBBN}}{Y_i^{SBBN}}$; and Y_i^{SBBN} and Y_i^{obs} are the theoretical and observed abundance respectively.

However, independent observational data are only available for the abundances of D, ^{3}He, ^{4}He and ^{7}Li. In table 13 we show the independent data we consider in this work. For a recent review of all observational available data on primordial abundances see Particle Data Group et al. (2004). On the other hand, recent papers (Coc et al. 2004a; Cyburt 2004) have brought the attention to the errors introduced by the values of the cross sections involved in the calculation of the abundances. Cyburt (2004) has also analyzed the propagation through the theoretical abundances, yielding a "theoretical" percent error of 5%. In the original work of Esmailzadeh et al. (1991), the error introduced by the semi-analytical method is estimated to be of order 5%. Therefore, we will add in order to solve system 84, to the errors of table 13 an error of order 10%.

First we perform a test to check the consistency of the data (Riveros and Vucetich 1986). For each group of data (Y_i) belonging to the same abundance, we calculate the weighted averaged value $\overline{Y}$ and its corresponding error σ_i. Then we compute:

$$\chi^2 = \sum_i \frac{(Y_i - \overline{Y})^2}{\sigma_i^2} \tag{85}$$

If the errors are Gaussian distributed, the expected value of χ^2 is $(k-1)$ where k is the number of data in each group. Furthermore, the corresponding ideogram of each group of data (Particle Data Group 2002), should be a Gaussian. It follows from figure 1 and from the calculation of χ^2 that D and ^{4}He data are not Gaussian distributed. However, since $\Theta = \sqrt{\frac{\chi^2}{k-1}}$ is not that greater than one, we can use the data but increasing the observational error by a factor Θ. The values of Θ are 2.4 for D, 2.33 for ^{4}He.

We assume that any difference between the theoretical abundance and the observational abundance is due to the variation of fundamental constants. In such way, the solution of system (84) gives a constraint to this variation. The solution is given by (Arley and Buch 1968):

$$\frac{\delta \alpha_i}{\alpha_i} = \left[\left(B^t P B \right)^{-1} B^t P \delta \right]_i \pm \sqrt{\left[\left(B^t P B \right)^{-1} \right]_{ii}} s \tag{86}$$

where B is the $n \times 4$ matrix, n is the number of observational data:

$$B = \begin{pmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \\ \vdots & \vdots & \vdots & \vdots \\ A_n & B_n & C_n & D_n \end{pmatrix} \tag{87}$$

δ is the $n \times 1$ matrix:

$$\delta = \begin{pmatrix} \frac{\delta Y_1}{Y_1} \\ \frac{\delta Y_2}{Y_2} \\ \vdots \\ \frac{\delta Y_n}{Y_n} \end{pmatrix} \tag{88}$$

and P is the $n \times n$ matrix of weight:

$$P = \begin{pmatrix} p_1 & 0 & \dots & 0 \\ 0 & p_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & p_n \end{pmatrix} \tag{89}$$

where $p_i = \frac{1}{\sigma_i^2}$ and σ_i are the observational errors.

The most accurate estimation of $\Omega_B h^2$ arrives from constraining parameters with data from the CMB provided by WMAP (Spergel et al. 2003). Fixing the baryon fraction with the WMAP value (i.e. setting $D_i = 0$), the results of solving the system 84 with all data listed in table 13 are shown in table 14. These results are consistent within 3σ with variation of the fundamental constants. On the other hand, the results considering only variation of the fine structure constant are shown in table 15. These results are consistent with no variation of α within 3σ. In order to rule out any systematic error of the data, we computed the solution of system 84 again but excluding one group of data at each time. Again, the results are consistent with variation of the fundamental constants in all cases but in the case where the ^{7}Li data were excluded (see tables 14 and 15).

Even though, the WMAP estimate of the baryon density is the most accurate one, it is still affected by degeneracies with other cosmological parameters (Spergel et al. 2003). Therefore, we added an independent estimation of $\Omega_B h^2$ in our analysis. In appendix A we use data from X-ray measurements, galaxy surveys and cepheids calibration in order to get an independent value of the baryon density. Furthermore, we computed again the results of sections 3 and 4, changing the value of $\Omega_B h^2$ to 0.0223. This value is the weighed mean value between the WMAP estimate and the value of appendix A. However, we found no difference in the value of the coefficients of the variation of fundamental constants and $\Omega_B h^2$. The results obtained solving system 84 including both estimates for the baryon fraction (i. e. $D_i \neq 0$) show no significant difference with respect to the case where only the WMAP value was considered (see tables 16 and 17). Furthermore, in order to check for consistency of our method, we solved again system 84 allowing only for variation of $\Omega_B h^2$ with respect to the weighed mean value (i.e. $A_i = B_i = C_i = 0$). These results are shown in table 18. On the other hand, in order to learn about the degeneracies of the fundamental constants within the BBN model, we computed the correlation coefficients from the error matrix. We find that there is high correlation between α and Λ_{QCD}, α and G_F and Λ_{QCD} and G_F, while the correlation between other pairs of parameters is not significant.

In order to understand the discrepancy of the results obtained with and without the ^{7}Li data, we computed the relative residuals (Arley and Buch 1968), and their respective theoretical and empirical probability in both cases. Figure 2 shows that in the case where both the variation of the fundamental constants and the deviation of $\Omega_B h^2$ from the WMAP estimate is considered, the theoretical and empirical probability distributions are very similar, while in the case where only the deviation of $\Omega_B h^2$ is considered, there is slight difference between the empirical probabilities (both with all data and excluding ^{7}Li data) and the theoretical probability. Including the variation of fundamental constants gives more degrees of freedom to system 84. Therefore, we suspect that the possible non reported systematic uncertainties "hide" under the variation of the fundamental constants. On the other hand, we performed a Kolmogorov-Smirnov (K-S) test, in order to check the goodness of our fit. For the results obtained considering variation of all constants and $\Omega_B h^2$, we obtain a probability of 21% to obtain a worse fit, while excluding the ^{7}Li data the probability lowers to 11%. On the other hand, if we only consider the deviation of $\Omega_B h^2$ with respect to the WMAP data, we obtain a probability of 99% for all data, while excluding the ^{7}Li data gives a 49% of probability to get a worse fit. However, we consider the results of the K-S test only indicative, since even though the data considered are independent the residuals are not.

We mentioned in the introduction the disagreement between the ^{7}Li observational abundances with the D observational abundance and WMAP estimate of the baryon density. Richard et al. (2005) claim that a better understanding of turbulent transport in the stars is necessary to understand this discrepancy. Moreover, Meléndez and Ramírez (2004) have reanalyzed the ^{7}Li data with an improved infrared flux method temperature scale, obtaining values that are marginally consistent with the WMAP estimate. However, solving system 84 with the ^{7}Li abundance taken from their work, does not change in a significant way our results.

We adopt the conservative criterion that the third and fourth column of tables 14 and

16 are the constraints on the variation of the constants we obtain with the method and hypothesis described in this paper. We also consider that more observations of ^{7}Li are needed in order to arrive to stronger conclusions. However, if the all data are correct, this analysis shows that varying coupling constants may solve the concordance problem between BBN and CMB. Our results within 2 σ are consistent with the analysis performed by Ichikawa and Kawasaki (2004), where only the variation of α and a non standard expansion rate.

A. Appendix I

In this appendix, we combine independent astronomical data in order to obtain and independent estimation of the baryon density. From measurements of hot gas in clusters it possible to obtain an estimate of $\frac{\Omega_B}{\Omega_m}h^{3/2}$.

Ettori (2003) has brought the attention to the fact that the contribution from baryons in galaxies and "exotic sources" like intergalactic stars and baryonic dark matter are not considered in the results obtained from measurements of hot gas in clusters. Furthermore, Donahue et al. (2003) have estimated the contribution from the galaxies as follows: $f_{gal} = 0.15h^{3/2}f_{gas}$ while the "exotic" contribution has been estimated in $f_{exotic} = 0.3f_{gal}$ (Ettori 2003). Therefore, we add to the estimation of the baryon fraction done by Donahue et al. (2003) the contribution from galaxies, yielding the following value:

$$\frac{\Omega_B}{\Omega_m}h^{3/2} = 0.0737 \pm 0.0143 \tag{A1}$$

The values of the other estimates (Majerowicz et al. 2002; Castillo-Morales and Schindler 2003) are contained within the error in this estimation.

On the other hand, $\Omega_m h$ has been estimated from large redshift galaxy surveys like Sloan Digital Sky Survey (Pope et al. 2004) and 2dF Galaxy Redshift Survey (Percival et al. 2001), while the most stringent bound on the Hubble constant follows from cepheid calibration (Freedman et al. 2001). Thus, combining all these data (see table 19) , and after propagating errors, we obtain the following value for the baryon density:

$$\Omega_B h^2 = 0.017 \pm 0.007 \tag{A2}$$

This value is less accurate that the estimation done with the data of WMAP (Spergel et al. 2003)

$$\Omega_B h^2 = 0.0224 \pm 0.0009 \tag{A3}$$

but we will consider it in order to have an independent data of this quantity.

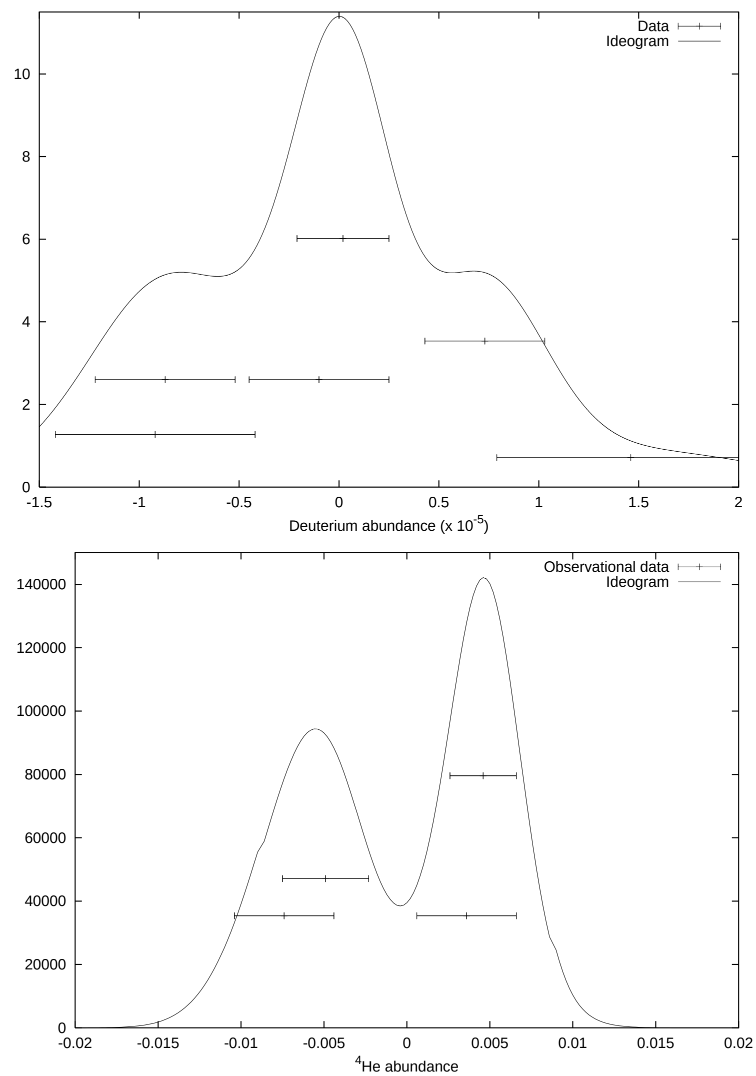

Fig. 1.— Ideograms for D and ^{4}He.

REFERENCES

Anchordoqui, L. A., Torres, D. F., and Vucetich, H.: 1996, *Physics Letters A* **222**, 43

Arley, N. and Buch, R.: 1968, *Introducción a la teoría de probabilidad y de la estadística*, John Wiley and sons, inc.

Avelino, P. P., Martins, C. J. A. P., Rocha, G., and Viana, P.: 2000, *Phys. Rev. D* **62**, 123508

Bahcall, J. N., Steinhardt, C. L., and Schlegel, D.: 2004, *ApJ* **600**, 520

Bania, T. M., Rood, R. T., and Balser, D. S.: 2002, *Nature* **415**, 54

Barr, S. M. and Mohapatra, P. K.: 1988, *Phys. Rev. D* **38**, 3011

Barrow, J. D., Sandvik, H. B., and Magueijo, J.: 2002, *Phys. Rev. D* **65**, 063504

Battye, R. A., Crittenden, R., and Weller, J.: 2001, *Phys. Rev. D* **63**, 043505

Beane, S. R. and Savage, M. J.: 2003a, *Nuclear Physics A* **717**, 91

Beane, S. R. and Savage, M. J.: 2003b, *Nuclear Physics A* **713**, 148

Bekenstein, J. D.: 1982, *Phys. Rev. D* **25**, 1527

Bergström, L., Iguri, S., and Rubinstein, H.: 1999, *Phys. Rev. D* **60**, 45005

Bernstein, J., Brown, L. S., and Feinberg, G.: 1989, *Reviews of Modern Physics* **61**, 25

Bonifacio, P. and Molaro, P.: 1997, *MNRAS* **285**, 847

Bonifacio, P., Molaro, P., and Pasquini, L.: 1997, *MNRAS* **292**, L1

Bonifacio et al: 2002, *Astronomy and Astrophysics* **390**, 91

Brax, P., van de Bruck, C., Davis, A.-C., and Rhodes, C. S.: 2003, *Ap&SS* **283**, 627

Burles, S. and Tytler, D.: 1998a, *Astrophysical Journal* **507**, 732

Burles, S. and Tytler, D.: 1998b, *Astrophysical Journal* **499**, 699

Castillo-Morales, A. and Schindler, S.: 2003, *A&A* **403**, 433

Chamoun, N., Landau, S. J., and Vucetich, H.: 2001, *Physics Letters B* **504**, 1

Chamoun, N. and Vucetich, H.: 2002, *Physics Letters B* **541**, 291

Coc, A., Vangioni-Flam, E., Descouvemont, P., Adahchour, A., and Angulo, C.: 2004a, in *AIP Conf. Proc. 704: Tours Symposium on Nuclear Physics V*, pp 341–350

Coc, A., Vangioni-Flam, E., Descouvemont, P., Adahchour, A., and Angulo, C.: 2004b, *ApJ* **600**, 544

Cottingham, W.: 1963, *Annals of Physics* **25**, 424

Cowie, L. L. and Songaila, A.: 1995, *ApJ* **453**, 596

Crighton, N. H. M., Webb, J. K., Ortiz-Gil, A., and Fernández-Soto, A.: 2004, *MNRAS* **355**, 1042

Cuoco, A., Iocco, F., Mangano, G., Miele, G., Pisanti, O., and Serpico, P. D.: 2004, *International Journal of Modern Physics A* **19**, 4431

Cyburt, R. H.: 2004, *Phys. Rev. D* **70(2)**, 023505

Cyburt, R. H., Fields, B. D., and Olive, K. A.: 2003, *Physics Letters B* **567**, 227

Damour, T. and Dyson, F.: 1996, *Nuclear Physics B* **480**, 37

Damour, T., Piazza, F., and Veneziano, G.: 2002a, *Phys. Rev. Lett.* **89**, 081601

Damour, T., Piazza, F., and Veneziano, G.: 2002b, *Phys. Rev. D* **66**, 046007

Damour, T. and Polyakov, A. M.: 1994, *Nuclear Physics B* **95**, 10347

Dmitriev, V. F. and Flambaum, V. V.: 2003, *Phys. Rev. D* **67(6)**, 063513

Dmitriev, V. F., Flambaum, V. V., and Webb, J. K.: 2004, *Phys. Rev. D* **69(6)**, 063506

Donahue, M., Gaskin, J. A., Patel, S. K., Joy, M., Clowe, D., and Hughes, J. P.: 2003, *ApJ* **598**, 190

Dyson, F.: 1966, *Phys. Rev. Lett.* **19**, 1291

Epelbaum, E., Meißner, U., and Glöckle, W.: 2003, *Nuclear Physics A* **714**, 535

Esmailzadeh, R., Starknam, G. D., and Dimopoulos, S.: 1991, *ApJ* **378**, 504

Ettori, S.: 2003, *MNRAS* **344**, L13

Feng, J. L., Rajaraman, A., and Takayama, F.: 2003, *Phys. Rev. D* **68(6)**, 063504

Flambaum, V. V. and Shuryak, E. V.: 2002, *Phys. Rev. D* **65(10)**, 103503

Flambaum, V. V. and Shuryak, E. V.: 2003, *Phys. Rev. D* **67(8)**, 083507

Fowler, W. A., Caughlan, G. R., and Zimmerman, B. A.: 1967, *ARA&A* **5**, 525

Fowler, W. A., Caughlan, G. R., and Zimmerman, B. A.: 1975, *ARA&A* **13**, 69

Freedman, W. L., Madore, B. F., Gibson, B. K., Ferrarese, L., Kelson, D. D., Sakai, S., Mould, J. R., Kennicutt, R. C., Ford, H. C., Graham, J. A., Huchra, J. P., Hughes, S. M. G., Illingworth, G. D., Macri, L. M., and Stetson, P. B.: 2001, *ApJ* **553**, 47

Fujii, Y., Iwamoto, A., Fukahori, T., Ohnuki, T., Nakagawa, M., Hidaka, H., Oura, Y., and Möller, P.: 2000, *Nuclear Physics B* **573**, 377

Fujii, Y., Iwamoto, A., Fukahori, T., Ohnuki, T., Nakagawa, M., Hidaka, H., Oura, Y., and Möller, P.: 2002, *Nuclear Data in Oklo and Time-Variability of Fundamental Coupling Constants*, hep-th/0205206

Gleiser, M. and Taylor, J. G.: 1985, *Phys. Rev. D* **31**, 1904

Haugan, M. P. and Will, C. M.: 1976, *Phys. Rev. Lett.* **37**, 1

Ichikawa, K. and Kawasaki, M.: 2002, *Phys. Rev.* **D65**, 123511

Ichikawa, K. and Kawasaki, M.: 2004, *Phys. Rev. D* **69(12)**, 123506

Ichikawa, K., Kawasaki, M., and Takahashi, F.: 2004, *Physics Letters B* **597**, 1

Ivanchik, A., Petitjean, P., Rodriguez, E., and Varshalovich, D.: 2002, *Astronomy Letters* 28(423)

Ivanchik, A., Petitjean, P., Rodriguez, E., and Varshalovich, D.: 2003, *Ap&SS* **283**, 583

Izotov, Y. and Thuan, T.: 1998, *Astrophysical Journal* **500**, 188

Izotov, Y., Thuan, T., and Lipovetsky, V. A.: 1997, *Astrophysical Journal Suplement* **108**, 1

Kaluza, T.: 1921, *Sitzungber. Preuss. Akad. Wiss.K* **1**, 966

Kawano, L., FERMILAB-PUB-92-004-A

Kirkman, D., Tytler, D., Suzuki, N., O'Meara, J. M., and Lubin, D.: 2003, *ApJS* **149**, 1

Klein, O.: 1926, *Z. Phys.* **37**, 895

Landau, S. J. and Vucetich, H.: 2002, *ApJ* **570**, 463

Levshakov, S. A., Dessauges-Zavadsky, M., D'Odorico, S., and P., M.: 2002, *MNRAS* **333**, 373

Müller, C. M., Schäfer, G., and Wetterich, C.: 2004, *Phys. Rev. D* **70(8)**, 083504

Maeda, K.: 1988, *Modern Physics. Letters A* **31**, 243

Majerowicz, S., Neumann, D. M., and Reiprich, T. H.: 2002, *A&A* **394**, 77

Marion, H., Pereira Dos Santos, F., Abgrall, M., Zhang, S., Sortais, Y., Bize, S., Maksimovic, I., Calonico, D., Grünert, J., Mandache, C., Lemonde, P., Santarelli, G., Laurent, P., Clairon, A., and Salomon, C.: 2003, *Physical Review Letters* **90**, 150801

Martínez Fiorenzano, A. F., Vladilo, G., and Bonifacio, P.: 2003, *Societa Astronomica Italiana Memorie Supplement* **3**, 252

Martins, C. J., Melchiorri, A., Trotta, R., Bean, R., Rocha, G., Avelino, P. P., and Viana, P. T.: 2002, *Phys. Rev. D* **66**, 023505

Mathews, G. J., Kajino, T., and Shima, T.: 2005, *Phys. Rev. D* **71(2)**, 021302

Meléndez, J. and Ramírez, I.: 2004, *ApJ* **615**, L33

Mukhanov, V.: 2003, *Nucleosynthesis Without a Computer*, astro-ph/0303073

Murphy, M. T., Webb, J. K., and Flambaum, V. V.: 2003, *MNRAS* **345**, 609

Murphy, M. T., Webb, J. K., Flambaum, V. V., Dzuba, V. A., Churchill, C. W., Prochaska, J. X., Barrow, J. D., and Wolfe, A. M.: 2001a, *MNRAS* **327**, 1208

Murphy, M. T., Webb, J. K., Flambaum, V. V., Prochaska, J. X., and Wolfe, A. M.: 2001b, *MNRAS* **327**, 1237

Nollet, K. M. and Lopez, R. E.: 2002, *Phys. Rev.* **D66**, 063507

Olive, K. A., Pospelov, M., Qian, Y., Coc, A., Cassé, M., and Vangioni-Flam, E.: 2002, *Phys. Rev. D* **66(4)**, 045022

Olive, K. A. and Skillman, E. D.: 2004, *ApJ* **617**, 29

Olive, K. A. and Steigman, G.: 1995, *Astrophysical Journal Supplement* **97**, 49

O'Meara, J. M., Tytler, D., Kirkman, D., Suzuki, N., Prochaska, J. X., Lubin, D., and Wolfe, A. M.: 2001, *Astrophysical Journal* **552**, 718

Overduin, J. M. and Wesson, P. S.: 1997, *Phys. Rep.* **283**, 303

Palma, G. A., Brax, P., Davis, A. C., and van de Bruck, C.: 2003, *Phys. Rev. D* **68**, 123519

Particle Data Group: 2002, http://pdg.lbl.gov/

Particle Data Group, Eidelman, S., Hayes, K. G., Olive, K. A., Aguilar-Benitez, M., Amsler, C., Asner, D., Babu, K. S., Barnett, R. M., Beringer, J., Burchat, P. R., Carone, C. D., Caso, S., Conforto, G., Dahl, O., D'Ambrosio, G., Doser, M., Feng, J. L., Gherghetta, T., Gibbons, L., Goodman, M., Grab, C., Groom, D. E., Gurtu, A., Hagiwara, K., Hernández-Rey, J. J., Hikasa, K., Honscheid, K., Jawahery, H., Kolda, C., Kwon, Y., Mangano, M. L., Manohar, A. V., March-Russell, J., Masoni, A., Miquel, R., Mönig, K., Murayama, H., Nakamura, K., Navas, S., Pape, L., Patrignani, C., Piepke, A., Raffelt, G., Roos, M., Tanabashi, M., Terning, J., Törnqvist, N. A., Trippe, T. G., Vogel, P., Wohl, C. G., Workman, R. L., Yao, W.-M., Zyla, P. A., Armstrong, B., Gee, P. S., Harper, G., Lugovsky, K. S., Lugovsky, S. B., Lugovsky, V. S., Rom, A., Artuso, M., Barberio, E., Battaglia, M., Bichsel, H., Biebel, O., Bloch, P., Cahn, R. N., Casper, D., Cattai, A., Chivukula, R. S., Cowan, G., Damour, T., Desler, K., Dobbs, M. A., Drees, M., Edwards, A., Edwards, D. A., Elvira, V. D., Erler, J., Ezhela, V. V., Fetscher, W., Fields, B. D., Foster, B., Froidevaux, D., Fukugita, M., Gaisser, T. K., Garren, L., Gerber, H.-J., Gerbier, G., Gilman, F. J., Haber, H. E., Hagmann, C., Hewett, J., Hinchliffe, I., Hogan, C. J., Höhler, G., Igo-Kemenes, P., Jackson, J. D., Johnson, K. F., Karlen, D., Kayser, B., Kirkby, D., Klein, S. R., Kleinknecht, K., Knowles, I. G., Kreitz, P., Kuyanov, Y. V., Lahav, O., Langacker, P., Liddle, A., Littenberg, L., Manley, D. M., Martin, A. D., Narain, M., Nason, P., Nir, Y., Peacock, J. A., Quinn, H. R., Raby, S., Ratcliff, B. N., Razuvaev, E. A., Renk, B., Rolandi, G., Ronan, M. T., Rosenberg, L. J., Sachrajda, C. T., Sakai, Y., Sanda, A. I., Sarkar, S., Schmitt, M., Schneider, O., Scott, D., Seligman, W. G., Shaevitz, M. H., Sjöstrand, T., Smoot, G. F., Spanier, S., Spieler, H., Spooner, N. J. C., Srednicki, M., Stahl, A., Stanev, T., Suzuki, M., Tkachenko, N. P., Trilling, G. H., Valencia, G., van Bibber, K., Vincter, M. G., Ward, D. R., Webber, B. R., Whalley, M., Wolfenstein, L., Womersley, J., Woody, C. L., Zenin, O. V., and Zhu, R.-Y.: 2004, *Physics Letters B* **592**, 1

Peimbert, M., Peimbert, A., and Ruiz, M.: 2000, *Astrophysical Journal* **541**, 688

Percival, W. J., Baugh, C. M., Bland-Hawthorn, J., Bridges, T., Cannon, R., Cole, S., Colless, M., Collins, C., Couch, W., Dalton, G., De Propris, R., Driver, S. P., Efstathiou, G., Ellis, R. S., Frenk, C. S., Glazebrook, K., Jackson, C., Lahav, O., Lewis, I., Lumsden, S., Maddox, S., Moody, S., Norberg, P., Peacock, J. A., Peterson, B. A., Sutherland, W., and Taylor, K.: 2001, *MNRAS* **327**, 1297

Pessah, M. E. and Torres, D. F.: 2001, *Physica A Statistical Mechanics and its Applications* **297**, 201

Pessah, M. E., Torres, D. F., and Vucetich, H.: 2001, *Physica A Statistical Mechanics and its Applications* **297**, 164

Pettini, M. and Bowen, D. V.: 2001, *Astrophysical Journal* **560**, 41

Pope, A. C., Matsubara, T., Szalay, A. S., Blanton, M. R., Eisenstein, D. J., Gray, J., Jain, B., Bahcall, N. A., Brinkmann, J., Budavari, T., Connolly, A. J., Frieman, J. A., Gunn, J. E., Johnston, D., Kent, S. M., Lupton, R. H., Meiksin, A., Nichol, R. C., Schneider, D. P., Scranton, R., Strauss, M. A., Szapudi, I., Tegmark, M., Vogeley, M. S., Weinberg, D. H., and Zehavi, I.: 2004, *ApJ* **607**, 655

Prestage, J. D., Tjoelker, R. L., and Maleki, L.: 1995, *Phys. Rev. Lett.* **74**, 3511

Quast, R., Reimers, D., and Levshakov, S. A.: 2004, *A&A* **415**, L7

Richard, O., Michaud, G., and Richer, J.: 2005, *ApJ* **619**, 538

Riveros, C. and Vucetich, H.: 1986, *Phys. Rev. D* **34**, 321

Rocha, G., Trotta, R., Martins, C. J. A. P., Melchiorri, A., Avelino, P. P., and Viana, P. T. P.: 2003, *New Astronomy Review* **47**, 863

Romano, D., Tosi, M., Matteucci, F., and Chiappini, C.: 2003, *MNRAS* **346**, 295

Ryan, S., Beers, T., Olive, K., Fields, B. D., and Norris, J. E.: 2000, *Astrophysical Journal* **530**, L57

Sisterna, P. and Vucetich, H.: 1990, *Phys. Rev. D* **41**, 1034

Smolliar, M.: 1996, *Science* **271**, 1099

Sortais, Y., Bize, S., Abgrall, M., Zhang, S., Nicolas, Mandache, C., P, L., Laurent, P., Santarelli, G., Dimarcq, N., Petit, P., Clairon, A., Mann, A., Luiten, A., Chang, S., and Salomon, C.: 2000, *Phys. Scr* **T95**, 50

Spergel, D. N., Verde, L., Peiris, H. V., Komatsu, E., Nolta, M. R., Bennett, C. L., Halpern, M., Hinshaw, G., Jarosik, N., Kogut, A., Limon, M., Meyer, S. S., Page, L., Tucker, G. S., Weiland, J. L., Wollack, E., and Wright, E. L.: 2003, *ApJS* **148**, 175

Srianand, R., Chand, H., Petitjean, P., and Aracil, B.: 2004, *Physical Review Letters* **92**, 121302

Stevenson, P.: 1981, *Annals of Physics* **132**, 383

Torres, D. F., Vucetich, H., and Plastino, A.: 1997, *Physical Review Letters* **79**, 1588

Varshalovich, D. A., Panchuk, V. E., and Ivanchik, A. V.: 1996, *Astronomy Letters* **22**, 6

Wagoner, R. V.: 1973, *Astrophys. J.* **179**, 343

Wagoner, R. V., Fowler, W. A., and Hoyle, F.: 1967, *ApJ* **148**, 3

Webb, J. K., Flambaum, V. V., Churchill, C. W., Drinkwater, M. J., and Barrow, J. D.: 1999, *Phys. Rev. Lett.* **82**, 884

Webb, J. K., Murphy, M. T., Flambaum, V. V., Dzuba, V. A., Barrow, J. D., Churchill, C. W., Prochaska, J. X., and Wolfe, A. M.: 2001, *Phys. Rev. Lett.* **87**, 091301

Weinberg, S.: 1983, *Physics Letters B* **125**, 265

Wu, Y. and Wang, Z.: 1986, *Phys. Rev. Lett.* **57**, 1978

Youm, D.: 2001a, *Phys. Rev. D* **63**, 125011

Youm, D.: 2001b, *Phys. Rev. D* **64**, 085011

Table 4: Charged particles reaction rates $\Theta = \Omega_B h^2 T_9^{7/3} \alpha^{1/3} \mu^{-1/3}$, $\Psi = \mu\alpha^2$, $\Xi(b) = \Omega_B h^2 \mu^{-b}$, $\Sigma(a) = \frac{\alpha}{\alpha^{today}}\left(1 + a - a\frac{\alpha}{\alpha^{today}}\right)$, $P_{IB}(\Psi, T_9, c_1, c_2, c_3, c_4, c_5) = 1 + c_1 \times 10^{-12}\,\Psi^{-1/3}\,T_9^{1/3} + c_2 \times 10^{10}\,\Psi^{1/3}\,T_9^{2/3} + c_3 T_9 + c_4 \times 10^{20}\,\Psi^{2/3}\,T_9^{4/3} + c_5 \times 10^{10}\,\Psi^{1/3}\,T_9^{5/3}$

Reaction	Reaction rate $\left(\frac{1}{\text{seg}}\right)$
$^3\text{H}\,(\text{p},\gamma)^4\,\text{He}$	$1.14 \times 10^{-7}\ \Theta\ \ [\Sigma(0)]^2\, e^{-9.55\times 10^{10}\left(\frac{\Psi}{T_9}\right)^{1/3}} \times$ $P_{IB}\,(\Psi,\ T_9,\ 4.36,\ 4.14,\ 1.26,\ 3.35,\ 2.61)$
$^2\text{H}\,(\text{d},\text{n})^3\,\text{He}$	$2.26 \times 10^{-3}\ \Theta\ \Sigma(0)\ \ e^{-9.55\times 10^{10}\left(\frac{\Psi}{T_9}\right)^{1/3}} \times$ $P_{IB}\,(\Psi,\ T_9,\ 4.36,\ 1.96,\ 0.6,\ -0.206,\ -0.16)$
$^2\text{H}\,(^3\text{He},\text{p})^4\,\text{He}$	$0.39\ \ \Theta\ \ \Sigma(0.09)\ \ e^{-1.52\times 10^{11}\left(\frac{\Psi}{T_9}\right)^{1/3} - (507.36 T_9 \alpha)^2} \times$ $P_{IB}\,(\Psi,\ T_9,\ 2.75,\ -2.16,\ -0.42,\ 13.5,\ 6.58) +$ $+2.63 \times 10^{-8}\ \ \Xi\left(\frac{1}{2}\right)\ \ \Sigma(0.09)\ \ T_9^{5/2}\ \ e^{-1.76 T_9^{-1}}$
$^3\text{H}\,(\text{d},\text{n})^4\,\text{He}$	$0.49\ \ \Theta\ \ \Sigma(0)\ \ e^{-9.55\times 10^{10}\left(\frac{\Psi}{T_9}\right)^{1/3} - (1141.67 T_9 \alpha)^2} \times$ $P_{IB}\,(\Psi,\ T_9,\ 4.36,\ 3.78,\ 1.16,\ 46.8,\ 3.64 \times 10^{11}) +$ $+3.39 \times 10^{-8}\ \ \Xi\left(\frac{1}{2}\right)\ \ \Sigma(0)\ \ T_9^{7/3}\ \ e^{-0.523 T_9^{-1}}$
$^2\text{H}\,(\text{d},\text{p})^3\,\text{H}$	$2.37 \times 10^{-3}\ \ \Theta\ \ \Sigma(0.16)\ \ e^{-9.55\times 10^{10}\left(\frac{\Psi}{T_9}\right)^{1/3}} \times$ $P_{IB}\,(\Psi,\ T_9,\ 4.36,\ 1.16,\ 0.35,\ -0.051,\ -0.04)$
$^2\text{H}\,(\alpha,\gamma)^6\,\text{Li}$	$1.88 \times 10^{-10}\ \ \Theta\ \ [\Sigma(0)]^2\ \ e^{-1.52\times 10^{11}\left(\frac{\Psi}{T_9}\right)^{1/3}} \times$ $P_{IB}\,(\Psi,\ T_9,\ 2.75,\ -9.9,\ 8.85,\ -2.43,\ -1.19) +$ $+8.27 \times 10^{-39}\ \ \Xi\left(\frac{3}{2}\right)\ \ [\Sigma(0)]^2\ \ T_9^{3/2}\ \ e^{-8.228 T_9^{-1}}$
$\text{H}\,(^6\text{Li},\alpha)^3\,\text{H}$	$0.20\ \ \Theta\ \ \Sigma(0)\ \ e^{-1.99\times 10^{11}\left(\frac{\Psi}{T_9}\right)^{1/3} - (24.94 T_9 \alpha)^2} \times$ $P_{IB}\,(\Psi,\ T_9,\ 2.10,\ -0.14,\ -0.02,\ 0.033,\ 0.012) +$ $+4.53 \times 10^{-8}\ \ \Xi\left(\frac{1}{2}\right)\ \ \Sigma(0)\ \ T_9^{2}\ \ e^{-21.82 T_9^{-1}} +$ $+6.68 \times 10^{-34}\ \ \Xi\left(\frac{3}{2}\right)\ \ \Sigma(0)\ \ T_9^{3/2}\ \ e^{-17.76 T_9^{-1}}$

Table 5: Charged particles reaction rates $\Theta = \Omega_B h^2 T_9^{7/3} \alpha^{1/3} \mu^{-1/3}$, $\Psi = \mu\alpha^2$, $\Xi(b) = \Omega_B h^2 \mu^{-b}$, $\Sigma(a) = \frac{\alpha}{\alpha^{today}}\left(1 + a - a\frac{\alpha}{\alpha^{today}}\right)$, $P_{IB}(\Psi, T_9, c_1, c_2, c_3, c_4, c_5) = 1 + c_1 \times 10^{-12}\,\Psi^{-1/3}\,T_9^{1/3} + c_2 \times 10^{10}\,\Psi^{1/3}\,T_9^{2/3} + c_3\,T_9 + c_4 \times 10^{20}\,\Psi^{2/3}\,T_9^{4/3} + c_5 \times 10^{10}\,\Psi^{1/3}\,T_9^{5/3}$, $P_{LN}(x, d_1, d_2, d_3, d_4, d_5) = 1 + d_1\,x + d_2\,x^2 + d_3\,x^3 + d_4\,x^4 + d_5\,x^5$, $T_{9v} = vT_9 + 11.605$

Reaction	Reaction rate $\left(\frac{1}{\text{seg}}\right)$
$^3\mathrm{H}\,(\alpha,\gamma)^7\,\mathrm{Li}$	$\Theta\ \ [\Sigma(0)]^2\ \ P_{LN}\left(\frac{\alpha-\alpha^{today}}{\alpha^{today}}, 3.17, 0.50, 0.18, 0.27, -0.22\right) \times$ $\left\{ 7.47 \times 10^{-6}\ \ e^{-1.52\times 10^{11}\left(\frac{\Psi}{T_9}\right)^{1/3}} \times P_{IB}\,(\Psi,\,T_9,\,2.75,\,-0.76,\,-0.15,\,0.36,\,0.18) + +2.68 \times 10^{-5}\ \ T_{9v}^{-5/6} e^{-6.69\times 10^{10}\left(\frac{\Psi T_{9v}}{T_9}\right)^{1/3}} \right\}$ $v = 1.59$
$^3\mathrm{He}\,(\alpha,\gamma)^7\,\mathrm{Be}$	$\Theta\ [\Sigma(0)]^2\ P_{LN}\left(\frac{\alpha-\alpha^{today}}{\alpha^{today}}, 2.15, 0.67, -5.57, 10.63, -5.73\right) \times$ $\left\{ 3.27 \times 10^{-5}\ \ e^{-2.41\times 10^{11}\left(\frac{\Psi}{T_9}\right)^{1/3}} \times P_{IB}\,(\Psi, T_9, 1.73, -0.0019, -0.00024, -0.00028, -8.8\times 10^{-5}) + +3.12 \times 10^{-4} T_{9v}^{-5/6} e^{-1.06\times 10^{11}\left(\frac{\Psi T_{9v}}{T_9}\right)^{1/3}} \right\}$ $v = 1.24$
$\mathrm{H}\,(^7\mathrm{Li},\alpha)^4\,\mathrm{He}$	$3.33 \times 10^{-3}\ \ \Theta\ \ \Sigma(0)\ \ e^{-1.99\times 10^{11}\left(\frac{\Psi}{T_9}\right)^{1/3} - (0.22\alpha T_9)^2} \times$ $P_{IB}\,(\Psi,\,T_9,\,2.10,\,3.65,\,0.54,\,-5.30,\,-1.98) +$ $+\Xi\left(\frac{2}{3}\right)\ \ \Sigma(0)\ \ T_9^{3/2} \times$ $\left[5.54 \times 10^{-34} e^{-30.44/T_9} + 7.98 \times 10^{-38} e^{-4.479/T_9}\right]$
$\mathrm{H}\,(\mathrm{d},\gamma)^3\,\mathrm{He}$	$1.11 \times 10^{-8}\ \ \Theta\ \ [\Sigma(0)]^2\ \ e^{-9.545\times 10^{10}\left(\frac{\Psi}{T_9}\right)^{1/3}} \times$ $P_{IB}\,(\Psi,\,T_9,\,4.36,\,8.66,\,2.65,\,1.26,\,0.98)$

Table 6: Non charged particles reactions rates, $\Sigma_{nc}(a) = 1 + a - a\frac{\alpha}{\alpha^{today}}$, $P_F(T_9, c_1, c_2) = 1 + c_1 T_9^{1/2} + c_2 T_9$, $q = \frac{m_n - m_p}{m_e}$

Reaction	Reaction rate $\left(\frac{1}{\text{seg}}\right)$
$\mathrm{n}\,(\ ,\mathrm{e}^-)\,\mathrm{H}$	$\frac{16}{60}\pi^2 G_F^2 m_e^5 \left[\sqrt{q^2-1}\,(2q^4 - 9q^2 - 8) + +15q\,\ln\left(q + \sqrt{q^2-1}\right)\right]$
$\mathrm{H}\,(\mathrm{n},\gamma)\,\mathrm{d}$	$40.92\,\Omega_B h^2\,\Sigma_{nc}(-1)\,T_9^3\,P_F(T_9,\,-0.86,\,0.43)$
$\mathrm{Y}_\gamma\,\{\mathrm{H}\,(\gamma,\mathrm{n})\,\mathrm{H}\}$	$2.70\times 10^{49}\,\Sigma_{nc}(-1)\,\left(\frac{m_p m_n}{m_d}\right)^{3/2}\,T_9^{3/2}\,e^{-11.605\epsilon_d/T_9}\times P_F(T_9,\,-0.86,\,0.43)$
$^3\mathrm{He}\,(\mathrm{n},\mathrm{p})^3\,\mathrm{H}$	$6.53\times 10^5\,\Omega_B h^2\,T_9^3\,\Sigma_{nc}(0.3)\,P_F(T_9,\,-0.59,\,0.1832)$
$^3\mathrm{H}\,(\mathrm{p},\mathrm{n})^3\,\mathrm{He}$	$6.53\times 10^5\,\Omega_B h^2\,\Sigma_{nc}(0.3)\,\Sigma_{nc}(-1)\,\left(\frac{m_3 m_n}{m_T m_p}\right)^{3/2}\times P_F(T_9,\,-0.59,\,0.1832)\,e^{-11.605Q_6/T_9}T_9^3$
$^7\mathrm{Be}\,(\mathrm{n},\mathrm{p})^7\,\mathrm{Li}$	$6.27\times 10^6\,\Omega_B h^2\,\Sigma_{nc}(0.2)\,T_9^3\,P_F(T_9,\,-0.903,\,0.215)$

Table 7: Stages and equations

Stage	Equations	Final temperature
Until the weak interaction freeze-out		
Until the production of ^{4}Hebecomes efficient	$\dot{Y}_n = -2\dot{Y}_\alpha - Y_n[n]$ $\dot{Y}_d = \dot{Y}_3 = \dot{Y}_T = 0$	$2\dot{Y}_\alpha \sim Y_n[n]$
Production of deuterium dominates the rate of change of neutrons	$\dot{Y}_n = -2\dot{Y}_\alpha$ $\dot{Y}_d = \dot{Y}_3 = \dot{Y}_T = 0$	$Y_n = Y_d$
Deuterium final abundance	$\dot{Y}_d = -2\dot{Y}_\alpha$ $\dot{Y}_n = \dot{Y}_3 = \dot{Y}_T = 0$	$T_9 \to 0$

Table 8: Solutions of the quasi static equilibrium equations for each stage

T_9	Nucleus	Solution
$9.1 > T_9 > 0.93$	D	$Y_d = Y_n Y_p \frac{[npd\gamma]}{Y_\gamma [d\gamma np]}$
	T	$Y_T = \frac{Y_d Y_p [pd3\gamma] + Y_d Y_d [ddn3] + Y_T Y_p [pTn3]}{Y_d [d3p\alpha] + Y_n [n3pT]}$
	^{3}He	$Y_3 = \frac{Y_n Y_3 [n3pT] + Y_d Y_d [ddpT]}{Y_d [dTn\alpha] + Y_p [pTn3]}$
$0.93 > T_9 > 0.765$	D	$Y_d = Y_n Y_p \frac{[npd\gamma]}{Y_\gamma [d\gamma np]}$
	T	$Y_3 = Y_d \frac{[ddn3]}{[d3p\alpha]}$
	^{3}He	$Y_T = Y_d \frac{[ddpT]}{[dTn\alpha]}$
$T_9 \to 0$	D	$Y_d = Y_n Y_p \frac{[npd\gamma]}{Y_\gamma [d\gamma np]}$
	T	$Y_3 = Y_d \frac{[ddn3]}{[d3p\alpha]}$
	^{3}He	$Y_T = Y_d \frac{[ddpT]}{[dTn\alpha]}$

Table 9: Quasi-static equilibrium solutions

Nucleus	Quasi-static equilibrium solutions
^{3}He	$Y_3 = \frac{Y_d [ddn3]}{[d3p\alpha]}$
T	$Y_T = \frac{Y_d [ddpT]}{[dTn\alpha]}$
^{6}Li	$Y_6 = \frac{Y_d Y_\alpha [d\alpha 6\gamma]}{Y_p [p6T\alpha]}$
^{7}Be	$Y_B = \frac{Y_3 Y_\alpha [3\alpha B\gamma]}{Y_n [nBp7]}$
^{7}Li	$Y_7 = \frac{Y_n Y_B [nBp7] + Y_T Y_\alpha [T\alpha 7\gamma]}{Y_p [p7\alpha\alpha]}$

Table 10: Freeze-Out temperature and their dependence on fundamental constants, $\frac{\delta T_9^i}{T_9^i} = W \frac{\delta G_F}{G_F} + R \frac{\delta\alpha}{\alpha} + T \frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} + J \frac{\delta\Omega_B h^2}{\Omega_B h^2}$

Nucleus	Equation	$T_9^{Freeze-Out}$	W	R	T	J
^{3}He	$Y_d[d3p\alpha] = H$	0.403	0.008	−0.510	1.168	0.016
T	$Y_d[dTn\alpha] = H$	0.105	0.009	0.122	1.181	0.018
^{6}Li	$Y_p[p63\alpha] = H$	0.069	−0.076	1.962	1.118	−0.156
^{7}Be	$Y_n[nBp7] = H$	0.319	0.217	−0.712	1.39	0.350
^{7}Li	$Y_p[p7\alpha\alpha] = H$	0.185	−0.088	1.692	0.946	−0.182

Table 11: Abundances and their dependence on fundamental constants, $\frac{\delta Y_i^f}{Y_i^f} = A\frac{\delta G_F}{G_F} + B\frac{\delta\alpha}{\alpha} + C\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} + D\frac{\delta\Omega_B h^2}{\Omega_B h^2}$

Y_i^f	Abundance	A	B	C	D
^{2}H	2.741×10^{-5}	-0.036	2.320	0.596	-1.072
^3He	6.95×10^{-6}	-0.051	0.983	0.999	-1.102
^{3}H	1.21×10^{-7}	-0.041	0.252	0.941	-1.083
^4He	0.238	-1.538	2.323	-3.497	0.029
^6Li	5.7×10^{-14}	-2.061	7.414	-3.462	-1.047
^7Be	5.60×10^{-10}	-0.172	-9.450	-1.038	2.209
^7Li	2.36×10^{-10}	-0.720	1.824	-3.411	0.068

Table 12: Theoretical abundances in the standard model the WMAP estimate $\Omega_B h^2 = 0.0224$

Nucleus	$Y_i^{SBBN} \pm \delta Y_i^{SBBN}$
^{2}H	$(2.51 \pm 0.37) \times 10^{-5}$
^3He	$(1.05 \pm 0.15) \times 10^{-5}$
^4He	0.2483 ± 0.0012
^7Li	$(5.0 \pm 0.3) \times 10^{-10}$

Table 13: Observational abundances used in this work

Nucleus	$Y_i^{obs} \pm \delta Y_i^{obs}$	Cite
D	$(1.65 \pm 0.35) \times 10^{-5}$	Pettini and Bowen (2001)
D	$(2.54 \pm 0.23) \times 10^{-5}$	O'Meara et al. (2001)
D	$\left(2.42^{+0.35}_{-0.25}\right) \times 10^{-5}$	Kirkman et al. (2003)
D	$(3.25 \pm 0.3) \times 10^{-5}$	Burles and Tytler (1998a)
D	$\left(3.98^{+0.59}_{-0.67}\right) \times 10^{-5}$	Burles and Tytler (1998b)
D	$\left(1.6^{+0.25}_{-0.30}\right) \times 10^{-5}$	Crighton et al. (2004)
^{3}He	$(1.1 \pm 0.2) \times 10^{-5}$	Bania et al. (2002)
^{4}He	0.244 ± 0.002	Izotov and Thuan (1998)
^{4}He	0.243 ± 0.003	Izotov et al. (1997)
^{4}He	0.2345 ± 0.0026	Peimbert et al. (2000)
^{4}He	0.232 ± 0.003	Olive and Steigman (1995)
^{7}Li	$\left(1.23^{+0.68}_{-0.32}\right) \times 10^{-10}$	Ryan et al. (2000)
^{7}Li	$\left(1.58^{+0.24}_{-0.20}\right) \times 10^{-10}$	Bonifacio et al. (1997)
^{7}Li	$(1.73 \pm 0.05) \times 10^{-10}$	Bonifacio and Molaro (1997)
^{7}Li	$\left(2.19^{+0.30}_{-0.26}\right) \times 10^{-10}$	Bonifacio et al (2002)

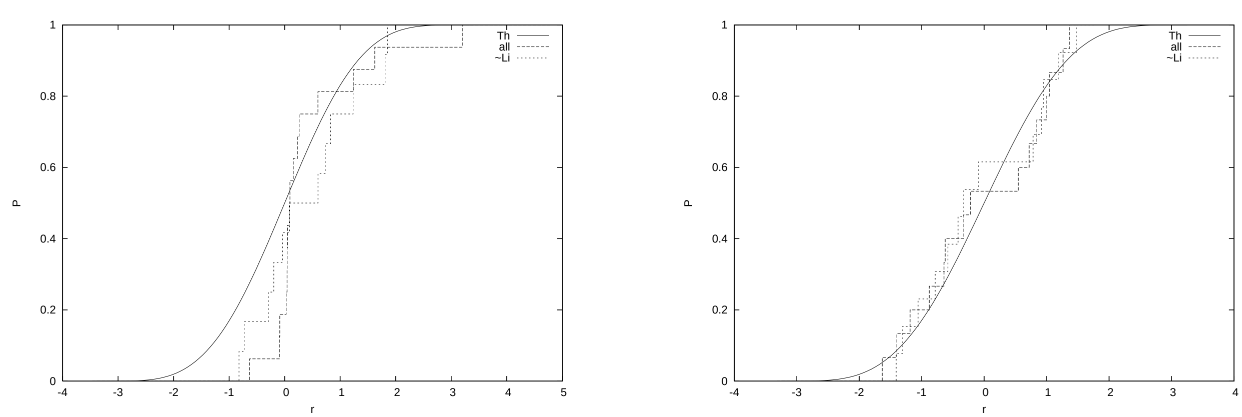

Fig. 2.— The full line shows the theoretical probability of the residuals, the dotted line shows the empirical probability computed with all data and the dotted line shows the empirical probability computed with all data but ^{7}Li. Left: Only deviation of $\Omega_B h^2$ with respect to its mean value is considered; Right: variation of all constants and deviation of $\Omega_B h^2$ from the mean value is considered.

Table 14: Constraints on the variation of fundamental constants ($\Omega_B h^2 = 0.0224$).

	All data		All data but ^{7}Li	
	Value	σ	Value	σ
$\frac{\delta G_F}{G_F}$	-0.886	0.053	-0.257	0.659
$\frac{\delta\alpha}{\alpha}$	-0.136	0.041	-0.054	0.097
$\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}}$	0.309	0.023	0.087	0.233

Table 15: Constraints on on the variation of α ($\Omega_B h^2 = 0.0224$).

	All data		All data but ^{7}Li	
	Valor	σ	Value	σ
$\frac{\delta\alpha}{\alpha}$	-0.041	0.024	-0.015	0.005

Table 16: Constraints on the variation of the fundamental constants using two independent estimates for the baryon fraction.

	All data		All data but ^{7}Li	
	Value	σ	Value	σ
$\frac{\delta\Omega_B h^2}{\Omega_B h^2}$	0.004	0.036	0.0005	0.039
$\frac{\delta G_F}{G_F}$	-0.886	0.050	-0.258	0.64
$\frac{\delta\alpha}{\alpha}$	-0.134	0.044	-0.053	0.095
$\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}}$	0.310	0.023	0.087	0.229

Table 17: Constraints on the variation of α using two independent estimates for the baryon fraction.

	All data		All data but ^{7}Li	
	Value	σ	Value	σ
$\frac{\delta\alpha}{\alpha}$	-0.086	0.034	-0.015	0.005

Table 18: Constraints on the deviations of $\Omega_B h^2$ respect to the mean value considered in this work (0.0223).

	All data		All data but ^{7}Li	
	Value	σ	Value	σ
$\frac{\delta\Omega_B h^2}{\Omega_B h^2}$	-0.085	0.294	-0.014	0.054

Table 19: Observational data used to perform an estimate of the baryon density

$\frac{\Omega_B}{\Omega_m} h^{3/2}$	Cite
0.067 ± 0.03	(Donahue et al. 2003)
0.073 ± 0.013	(Majerowicz et al. 2002)
0.056 ± 0.007	(Castillo-Morales and Schindler 2003)
$\Omega_m h$	Cite
0.20 ± 0.03	(Percival et al. 2001)
0.207 ± 0.030	(Pope et al. 2004)
h	Cite
0.72 ± 0.08	(Freedman et al. 2001)