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G-STRUCTURE ON THE COHOMOLOGY OF HOPF ALGEBRAS

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Abstract. We prove that $\mathrm{Ext}^\bullet_A(k,k)$ is a Gerstenhaber algebra, where A is a Hopf algebra. In case $A = D(H)$ is the Drinfeld double of a finite-dimensional Hopf algebra H, our results imply the existence of a Gerstenhaber bracket on $H^\bullet_{GS}(H,H)$. This fact was conjectured by R. Taillefer. The method consists of identifying $H^\bullet_{GS}(H,H) \cong \mathrm{Ext}^\bullet_A(k,k)$ as a Gerstenhaber subalgebra of $H^\bullet(A,A)$ (the Hochschild cohomology of A).

Introduction

The motivation of this paper is to prove that $H^\bullet_{GS}(H,H)$ has a structure of a G-algebra. The G-algebra structure is, roughly speaking, the existence of two products with compatibilities between them: one is associative graded commutative, and the other is a graded Lie bracket. We prove this result when H is a finite-dimensional Hopf algebra (see Theorem 2.1 and Corollary 2.5). $H^\bullet_{GS}$ is the cohomology theory for Hopf algebras defined by Gerstenhaber and Schack in [4]. In order to obtain commutativity of the cup product we prove a general statement on Ext groups over Hopf algebras (without any finiteness assumption).

When H is finite dimensional, the category of Hopf bimodules is isomorphic to a module category, over an algebra X (also finite dimensional) defined by Cibils and Rosso (see [2]), and this category is also equivalent to the category of Yetter-Drinfeld modules, which is isomorphic to the category of modules over the Hopf algebra $D(H)$ (the Drinfeld double of H). In [10], Taillefer has defined a natural cup product in $H^\bullet_{GS}(H,H) = H^\bullet_b(H,H)$ (see [5] for the definition of $H^\bullet_b$). When H is finite dimensional, she proved that $H^\bullet_b(H,H) \cong \mathrm{Ext}^\bullet_X(H,H)$, and using this isomorphism she showed that it is (graded) commutative. In a later work [11] she extended the result of commutativity of the cup product to arbitrary-dimensional Hopf algebras, and she conjectured the existence (and a formula) of a Gerstenhaber bracket.

Our method for giving a Gerstenhaber bracket is the following: under the equivalence of categories ${}_X\text{-mod} \cong {}_{D(H)}\text{-mod}$, the object H corresponds to $H^{coH} = k$. So $\mathrm{Ext}^\bullet_X(H,H) \cong \mathrm{Ext}^\bullet_{D(H)}(k,k)$ (isomorphism of graded algebras); according to Ştefan [8] one knows that $\mathrm{Ext}^\bullet_{D(H)}(k,k) \cong H^\bullet(D(H),k)$. In Theorem 1.8 we prove

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that, if A is an arbitrary Hopf algebra, then $H^\bullet(A,k)$ is isomorphic to a subalgebra of $H^\bullet(A,A)$—in particular, it is graded commutative—and the morphisms are defined at the complex level. In Theorem 2.1 we prove that the image of $\mathcal{C}^\bullet(A,k)$ in $\mathcal{C}^\bullet(A,A)$ is stable under the brace operation (if M is an A-bimodule, $\mathcal{C}^\bullet(A,M)$ denotes the standard Hochschild complex whose homology is $H^\bullet(A,M)$); in particular, the image of $H^\bullet(A,k)$ is closed under the Gerstenhaber bracket of $H^\bullet(A,A)$. So, the existence of the Gerstenhaber bracket on $H^\bullet_{GS}(H,H)$ follows, at least in the finite-dimensional case, by taking $A = D(H)$. We did not know if this bracket coincides with the formula proposed in [11], but Taillefer, in a personal communication, told us that, using arguments as in [7], one can actually prove that the bracket given by us, in the finite-dimensional case, must agree with the bracket proposed by her. Nevertheless, the argument does not give a proof of existence in the infinite-dimensional case. So the problem, in that generality, remains open.

We also provide a proof that the algebra $\mathrm{Ext}^\bullet_{\mathcal{C}}(k,k)$ is graded commutative when $\mathcal{C}$ is a braided monoidal category satisfying certain exactness hypotheses (see Theorem 1.4). This gives an alternative proof of the commutativity of the cup product in the arbitrary-dimensional case by taking $\mathcal{C} = {}^H_H\mathcal{YD}$, the category of Yetter-Drinfeld modules.

In this paper A will denote a Hopf algebra over a field k.

1. Cup products

This section has two parts. First we prove a generalization of the fact that the cup product on group cohomology $H^\bullet(G,k)$ is graded commutative. The general abstract setting is that of a braided (abelian) category with enough injectives satisfying an exactness condition (see Definition 1.2 below). The other part will concern the relation between self extensions of k and Hochschild cohomology of A with coefficients in k.

Let us recall the definition of a braided category:

Definition 1.1. The data $(\mathcal{C},\otimes,k,c)$ is called a **braided** category with unit element k if

(1) $\mathcal{C}$ is an abelian category.
(2) $-\otimes-$ is a bifunctor, bilinear, associative, and there are natural isomorphisms $k\otimes X\cong X\cong X\otimes k$ for all objects X in $\mathcal{C}$.
(3) For all pair of objects X and Y, $c_{X,Y} : X\otimes Y\to Y\otimes X$ is a natural isomorphism. The isomorphisms $c_{X,k} : X\otimes k\cong k\otimes X$ agree with the isomorphism of the unit axiom, and for all triples X, Y, Z of objects in $\mathcal{C}$, the Yang-Baxter equation is satisfied:

$$(\mathrm{id}_Z\otimes c_{X,Y})\circ(c_{X,Z}\otimes\mathrm{id}_Y)\circ(\mathrm{id}_X\otimes c_{Y,Z}) = (c_{Y,Z}\otimes\mathrm{id}_X)\circ(\mathrm{id}_Y\otimes c_{X,Z})\circ(c_{X,Y}\otimes\mathrm{id}_Z).$$

A data $(\mathcal{C},\otimes,k)$ satisfying axioms 1 and 2, but not necessarily axiom 3 is called a **monoidal** category.

We will use the notion of exact functor for a monoidal structure.

Definition 1.2. Let $(\mathcal{C},\otimes,k)$ be an abelian monoidal category. We say that $\otimes$ is exact if and only if the canonical morphism

$$H_*(X_*,d_X)\otimes H_*(Y_*,d_Y)\to H_*(X_*\otimes Y_*,d_{X\otimes Y})$$

is an isomorphism for all pairs of complexes in $\mathcal{C}$.

Example 1.3. Let H be a Hopf algebra over a field k. Then $\mathcal{C} = {}_H\text{-mod}$ is a monoidal category with $\otimes = \otimes_k$, and this functor is clearly exact.

Theorem 1.4. *Let $(\mathcal{C}, \otimes, k, c)$ be a braided category with enough injectives and exact tensor product. Then $\operatorname{Ext}^\bullet_{\mathcal{C}}(k,k)$ is graded commutative.*

Proof. We proceed as in the proof that $H^\bullet(G,k)$ is graded commutative (see for example [1], page 51, Vol. I). The proof is based on two points: first a definition of a cup product using the bifunctor $\otimes$, and second a lemma relating this construction and the Yoneda product of extensions.

Let $0 \to M \to X_p \to \dots X_1 \to N \to 0$ and $0 \to M' \to X'_q \to \dots X'_1 \to N' \to 0$ be two extensions in $\mathcal{C}$. Then $N_* := (0 \to M \to X_p \to \dots X_1 \to 0)$ and $N'_* := (0 \to M' \to X'_q \to \dots X'_1 \to 0)$ are two complexes, quasi-isomorphic to N and N' respectively. By the Künneth formula, $N_* \otimes N'_*$ is a complex quasi-isomorphic to $N \otimes N'$. So "completing" this complex with $N \otimes N'$ (more precisely considering the mapping cone of the chain map $N_* \otimes N'_* \to N \otimes N'$) one has an extension in $\mathcal{C}$, beginning with $M \otimes M'$ and ending with $N \otimes N'$.

So, we have defined a cup product:

$$\operatorname{Ext}^p_{\mathcal{C}}(N,M) \times \operatorname{Ext}^q_{\mathcal{C}}(N',M') \to \operatorname{Ext}^{p+q}_{\mathcal{C}}(N \otimes N', M \otimes M').$$

We will denote this product by $\otimes$, and the Yoneda product by $\smile$. The lemma relating this product and the Yoneda one is the following:

Lemma 1.5. *If $\eta \in \operatorname{Ext}^p_{\mathcal{C}}(M,N)$ and $\xi \in \operatorname{Ext}^q_{\mathcal{C}}(M',N')$, then*

$$\eta \otimes \xi = (\eta \otimes \mathrm{id}_{N'}) \smile (\mathrm{id}_M \otimes \xi).$$

Proof of the Lemma. Interpreting the elements η and ξ as extensions, it is clear how to define a morphism of complexes $(\eta \otimes \mathrm{id}_{N'}) \smile (\mathrm{id}_M \otimes \xi) \to \eta \otimes \xi$, and by the Künneth formula, it is a quasi-isomorphism.

In the particular case that $M = M' = N = N' = k$, the lemma implies that $\eta \otimes \xi = \eta \smile \xi$ for all η and ξ in $\operatorname{Ext}^\bullet_{\mathcal{C}}(k,k)$. Now the theorem is a consequence of the isomorphism $(X_* \otimes Y_*, d_{X \otimes Y}) \cong (Y_* \otimes X_*, d_{Y \otimes X})$, valid for every pair of complexes in $\mathcal{C}$, defined by

$$(-1)^{pq} c_{X,Y} : X_p \otimes Y_q \to Y_q \otimes X_p.$$

Note that the differentials are morphisms in the category $\mathcal{C}$. So the map defined above commutes with the differentials because of the bifunctoriality of the braiding. □

Example 1.6. Let H be a cocommutative Hopf algebra. Then ${}_H$-mod is braided with c the usual flip. When $H = k[G]$ we recover that $H^\bullet(G,k)$ is graded commutative. The other typical example is $H = \mathcal{U}(\mathfrak{g})$, the enveloping algebra of a Lie algebra $\mathfrak{g}$. It is known that $\operatorname{Ext}_{\mathcal{U}(\mathfrak{g}))}(k,k) = \Lambda^*(\mathfrak{g})$, is graded commutative.

Example 1.7. Let H be an arbitrary Hopf algebra with bijective antipode and $\mathcal{C} = {}^H_H\mathcal{YD}$ the category of Yetter-Drinfeld modules over H. It is well known (see [6], p. 214) that the map $M \otimes N \to N \otimes M$ defined by $m \otimes n \mapsto m_{-1}n \otimes m_0$ is a braiding on ${}^H_H\mathcal{YD}$. So $\operatorname{Ext}_{{}^H_H\mathcal{YD}}(k,k)$ is graded commutative.

Theorem 1.8. *If A is a Hopf algebra, then $\operatorname{Ext}^\bullet_A(k,k) \cong H^\bullet(A,k)$. Moreover, $H^\bullet(A,k)$ is isomorphic to a subalgebra of $H^\bullet(A,A)$.*

Proof. After Ştefan [8], since A is an A-Hopf Galois extension of k, $H^\bullet(A, M) \cong \mathrm{Ext}^\bullet_A(k, M^{\mathrm{ad}})$ for all A-bimodules M.

Here, M^{ad} denotes the left H-module with underlying vector space M, but with structure $h._{\mathrm{ad}}m := h_1 m S(h_2)$. The notation ($S$ for the antipode, and the Sweedler-type summation) is the standard one.

In particular, $H^\bullet(A, k) = \mathrm{Ext}^\bullet_A(k, k)$. But one can give, for this particular case, an explicit morphism at the complex level. In order to do this, we will choose a specific resolution of k as a left A-module. Notice that, in particular, our argument will give an alternative proof of Stefan's result for this case.

Let $C_*(A, b')$ be the standard resolution of A as an A-bimodule, namely $C_n(A, b') = A \otimes A^{\otimes n} \otimes A$ and $b'(a_0 \otimes \ldots \otimes a_{n+1}) = \sum_{i=0}^n (-1)^i a_0 \otimes \ldots \otimes a_i.a_{i+1} \otimes \ldots \otimes a_{n+1}$ ($a_i \in A$). This resolution splits on the right. So $(C_*(A) \otimes_A k, b' \otimes \mathrm{id}_k)$ is a resolution of $A \otimes_A k = k$ as a left A-module. Using this resolution, $\mathrm{Ext}^\bullet_A(k, k)$ is the cohomology of the complex $(\mathrm{Hom}_A(C_*(A) \otimes_A k, k), (b' \otimes_A \mathrm{id}_k)^*) \cong (\mathrm{Hom}(A^{\otimes *}, k), \partial)$. Under this isomorphism, the differential ∂ is given by

$$(\partial f)(a_1 \otimes \ldots \otimes a_n) = \epsilon(a_1) f(a_2 \otimes \ldots \otimes a_n)$$
$$+ \sum_{i=1}^{n-1} (-1)^i f(a_1 \otimes \ldots \otimes a_i.a_{i+1} \otimes \ldots \otimes a_n) + (-1)^n f(a_1 \otimes \ldots \otimes a_{n-1}) \epsilon(a_n),$$

which is precisely the formula of the differential of the standard Hochschild complex computing $H^\bullet(A, k)$.

One can easily check that the cup product on $\mathrm{Ext}^\bullet_A(k, k)$ which, by Lemma 1.5 equals the Yoneda product, corresponds to the cup product on $H^\bullet(A, k)$. So this isomorphism is an algebra isomorphism.

Now we will give two multiplicative maps $H^\bullet(A, k) \to H^\bullet(A, A)$ and $H^\bullet(A, A) \to H^\bullet(A, k)$. Consider the counit $\epsilon : A \to k$. It is an algebra map, and so the induced map $\epsilon_* : H^\bullet(A, A) \to H^\bullet(A, k)$ is multiplicative. We will define a multiplicative section of this map.

Let $f : A^{\otimes p} \to k$ be a Hochschild cocycle, and define $\widehat{f} : A^{\otimes p} \to A$ by the formula

$$\widehat{f}(a^1 \otimes \ldots \otimes a^p) := a^1_1 \ldots a^p_1 . f(a^1_2 \otimes \ldots \otimes a^p_2)$$

where we have used the Sweedler-type notation with summation symbol omitted: $a^i_1 \otimes a^i_2 = \Delta(a^i)$, for $a^i \in A$.

Let us check that $\widehat{f}$ is a Hochschild cocycle with values in A,

$$\partial(\widehat{f})(a^0 \otimes \ldots \otimes a^p) = a^0 \widehat{f}(a^1 \otimes \ldots \otimes a^p)$$
$$+ \sum_{i=0}^{p-1} (-1)^{i+1} \widehat{f}(a^0 \otimes \ldots \otimes a^i.a^{i+1} \otimes \ldots \otimes a^p) + (-1)^{p+1} \widehat{f}(a^0 \otimes \ldots \otimes a^{p-1}) a^p$$
$$= a^0.a^1_1 \ldots a^p_1 . f(a^1_2 \otimes \ldots \otimes a^p_2) + (-1)^{p+1} a^0_1 \ldots a^{p-1}_1 . f(a^0_2 \otimes \ldots \otimes a^{p-1}_2) a^p$$
$$+ \sum_{i=0}^{p-1} (-1)^{i+1} a^0_1 \ldots a^i_1 a^{i+1}_1 \ldots a^p_1 . f(a^0_2 \otimes \ldots \otimes a^i_2 . a^{i+1}_2 \otimes \ldots \otimes a^p_2).$$

Using that f is a Hochschild cocycle with values in k, we know that

$$0 = \epsilon(a^0)f(a^1 \otimes \ldots \otimes a^p) + \sum_{i=0}^{p-1}(-1)^{i+1} f(a^0 \otimes \ldots \otimes a^i.a^{i+1} \otimes \ldots \otimes a^p)$$
$$+ (-1)^{p+1} f(a^0 \otimes \ldots \otimes a^{p-1})\epsilon(a^p).$$

So, the summation term in $\partial(\widehat{f})$ can be replaced using the equality

$$\sum_{i=0}^{p-1}(-1)^{i+1} a_1^0 \ldots a_1^i a_1^{i+1} \ldots a_1^p.f(a_2^0 \otimes \ldots \otimes a_2^i.a_2^{i+1} \otimes \ldots \otimes a_2^p)$$
$$= -a_1^0 \ldots a_1^p. \left(\epsilon(a_2^0) f(a_2^1 \otimes \ldots \otimes a_2^p) + (-1)^{p+1} f(a_2^0 \otimes \ldots \otimes a_2^{p-1})\epsilon(a_2^p)\right)$$
$$= -\left(a^0.a_1^1 \ldots a_1^p.f(a_2^1 \otimes \ldots \otimes a_2^p) + (-1)^{p+1} a_1^0 \ldots a_1^{p-1}.a^p f(a_2^0 \otimes \ldots \otimes a_2^{p-1})\right)$$

and this finishes the computation of $\partial(\widehat{f})$.

Clearly $\epsilon\widehat{f} = f$; so ϵ_* is a split epimorphism. To check that $f \mapsto \widehat{f}$ is multiplicative is straightforward:

Let $g : A^{\otimes q} \to k$ be a cocycle and $\widehat{g} : A^{\otimes q} \to A$ the cocycle with values in A corresponding to g. We can check the following:

$$\begin{aligned} \widehat{f \smile g}(a^1 \otimes \ldots \otimes a^{p+q}) &= a_1^1 \ldots a_1^{p+q}.(f \smile g)(a_2^1 \otimes \ldots \otimes a_2^{p+q}) \\ &= a_1^1 \ldots a_1^{p+q}.f(a_2^1 \otimes \ldots \otimes a_2^p) g(a_2^{p+1} \otimes \ldots \otimes a_2^{p+q}) \\ &= (\widehat{f} \smile \widehat{g})(a^1 \otimes \ldots \otimes a^{p+q}). \end{aligned}$$

□

2. Brace operations

In this section we prove our main theorem, stating that the map $H^\bullet(A, k) \to H^\bullet(A, A)$ is "compatible" with the brace operations, and as a consequence with the Gerstenhaber bracket. Note that the map $H^\bullet(A, k) \to H^\bullet(A, A)$ is defined at the standard complex level. Let us define $\mathcal{C}^p(A, M) := \mathrm{Hom}(A^{\otimes p}, M)$.

Theorem 2.1. *The image of the map $\mathcal{C}^\bullet(A, k) \to \mathcal{C}^\bullet(A, A)$ is stable under the brace operation. Moreover, if $\widehat{f}$ and $\widehat{g}$ are the images in $\mathcal{C}^\bullet(A, A)$ of two elements f and g belonging to $\mathcal{C}^\bullet(A, k)$, then $\widehat{f} \circ_i \widehat{g} = \widehat{f \circ_i \widehat{g}}$.*

Proof. Let us recall the definition of the brace operations (see [3]). If $F : A^{\otimes p} \to M$ and $G : A^{\otimes q} \to A$ and $1 \le i \le p$, then $F \circ_i G : A^{\otimes p+q-1} \to M$ is defined by

$$(F \circ_i G)(a^1 \otimes \ldots \otimes a^i \otimes b^1 \otimes \ldots \otimes b^q \otimes a^{i+1} \otimes \ldots \otimes a^p)$$
$$= F(a^1 \otimes \ldots \otimes a^i \otimes G(b^1 \otimes \ldots \otimes b^q) \otimes a^{i+1} \otimes \ldots \otimes a^p).$$

Assume now that $f : A^{\otimes p} \to k$, $g : A^{\otimes q} \to k$ and $F = \widehat{f}$ and $G = \widehat{g}$, namely

$$F(a^1 \otimes \ldots \otimes a^p) = a_1^1 \ldots a_1^p.f(a_2^1 \otimes \ldots \otimes a_2^p)$$

and similarly for G and g. Then (denoting $(a \otimes b)$ by (a, b)),

$$\begin{aligned}
&(F \circ_i G)(a^1, \dots, a^i, b^1, \dots, b^q, a^{i+1}, \dots, a^p) \\
&= F\left(a^1, \dots, a^i, G(b^1, \dots, b^q), a^{i+1}, \dots, a^p\right) \\
&= F\left(a^1, \dots, a^i, b_1^1 \dots b_1^q . g(b_2^1, \dots, b_2^q), a^{i+1}, \dots, a^p\right) \\
&= a_1^1 \dots a_1^i . b_1^1 \dots b_1^q . a_1^{i+1} \dots a_1^p . f\left(a_2^1, \dots, a_2^i, b_2^1 \dots b_2^q . g(b_3^1, \dots, b_3^q), a_2^{i+1}, \dots, a_2^p\right) \\
&= \widehat{f \circ_i G}(a^1, \dots, a^i, b^1, \dots, b^q, a^{i+1}, \dots, a^p).
\end{aligned}$$

□

Recall that the brace operations define a "composition" operation $F \circ G = \sum_{i=1}^{p} (-1)^{q(i-1)} F \circ_i G$, where $F \in \mathcal{C}^p(A, A)$ and $G \in \mathcal{C}^q(A, A)$. The Gerstenhaber bracket is defined as the commutator of this composition. So we have the desired corollary:

Corollary 2.2. *If A is a Hopf algebra, then $H^\bullet(A, k)$ is a Gerstenhaber subalgebra of $H^\bullet(A, A)$.*

Example 2.3. Let A be a Hopf algebra. Then $\mathrm{Ext}_A^1(k, k) \cong \mathrm{Der}(A, k) = \mathrm{Prim}(A^*)$, where $\mathrm{Prim}(A^*) = \{x \in A^* \text{ such that } m^*(x) = x \otimes 1 + 1 \otimes x\}$. It is easy to check that the Lie bracket given in the above theorem coincides with the commutator of the convolution product, viewing $\mathrm{Der}(A, k)$ as a subset of A^*.

Example 2.4. Let G be a connected affine algebraic group and $\mathfrak{g} := \mathrm{Ker}(\epsilon)/\mathrm{Ker}(\epsilon)^2$ its tangent Lie algebra. One has that $HH^\bullet(\mathcal{O}(G), \mathcal{O}(G)) = \Lambda^\bullet_{\mathcal{O}(G)} \mathrm{Der}(\mathcal{O}(G)) \cong \mathcal{O}(G) \otimes \Lambda^\bullet \mathfrak{g}$, where the Gerstenhaber structure here is the Schouten-Nijenhuis bracket. Also $\mathrm{Ext}^\bullet_{\mathcal{O}(G)}(k, k) = \Lambda^\bullet \mathfrak{g}$, and it is generated (as an algebra) in degree one. So the bracket is determined by its values on $\mathrm{Ext}^1_{\mathcal{O}(G)}(k, k) = \mathfrak{g}$, which is the bracket of $\mathfrak{g}$ as a Lie algebra. This G-algebra structure is also well known.

Consider H a finite-dimensional Hopf algebra and $X = X(H)$ the algebra defined by Cibils and Rosso (see [2]). We can prove, at least in the finite-dimensional case, the conjecture of [11] that $H^\bullet_{GS}(H, H)$ is a Gerstenhaber algebra:

Corollary 2.5. *Let H be a finite-dimensional Hopf algebra. Then $H^\bullet_{GS}(H, H)$ is a Gerstenhaber algebra.*

Proof. The isomorphism $H^\bullet_{GS}(H, H) \cong \mathrm{Ext}^\bullet_X(H, H)$ was proved in [10].

Let A denote $D(H)$, the Drinfeld double of H. One knows that ${}_X\text{-mod} \cong {}_A\text{-mod}$ via $M \mapsto M^{coH}$. Then $\mathrm{Ext}^\bullet_X(H, H) \cong \mathrm{Ext}^\bullet_A(H^{coH}, H^{coH}) = \mathrm{Ext}^\bullet_A(k, k)$, and this a Gerstenhaber subalgebra of $H^\bullet(A, A)$. □

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