PERFILES DE DENSIDAD DE RADÓN EN POROS ALVEOLARES: UNA APLICACIÓN DEL FORMALISMO DE ECUACIÓN INTEGRAL

RADON DENSITY PROFILES IN ALVEOLAR PORES: AN INTEGRAL EQUATION FORMALISM APPLICATION

J. C. Corona-Oran[1], D. Osorio-González[1] y J. Mulia-Rodríguez[*1]

[1]*Laboratorio de Biofísica Molecular. Facultad de Ciencias. Universidad Autónoma del Estado de México. Instituto Literario 100. Colonia Centro, Toluca CP 50000. México.*

Recibido: 26/06/2019; Aceptado: 22/09/2021

En este trabajo, obtuvimos perfiles de densidad para partículas de radón cerca de la superficie de la pared alveolar como resultado del desarrollo de un modelo teórico que describe la interacción entre tales partículas y las células de la pared alveolar. El modelo captura características biológicas y fisicoquímicas relevantes, como el ancho de la pared alveolar y la energía necesaria para que las partículas de radón pasen a través de ella. El sistema satisface las condiciones a considerar en equilibrio termodinámico y en condiciones normales de presión y temperatura. Resolvimos numéricamente las ecuaciones de Ornstein-Zernike derivadas del formalismo de ecuación integral y mostramos los efectos de cambiar el tamaño del alvéolo, el ancho de la pared alveolar y la energía requerida para pasar a través de ellos en los perfiles de densidad de radón. Nuestro método es de suma utilidad en biofísica y otras áreas afines.

Palabras Clave: radón, pared alveolar, poro alveolar, biofísica.

In this work, we obtained density profiles of Radon particles near the alveolar wall surface bythe development of a theoretical model that describes the interaction between such particles and the alveolar wall cells. The model captures relevant biological and physicochemical characteristics such as the width of the alveolar wall and the energy required for the Radon particles to pass through it. The system satisfies the conditions to be considered in thermodynamic equilibrium and under normal pressure and temperature conditions. We numerically solved the Ornstein-Zernike equations derived from the integral-equation formalism and showed the effects of changing the alveolus size, the alveolar wall width, and the energy required to pass through itsRadon density profiles. Our method is a helpful tool in biophysics and other related areas.

Keywords: radon, alveolar wall, alveolar pore, biophysics.

https://doi.org/10.31527/analesafa.2022.33.1.1 ISSN 1850-1168 (online)

I. INTRODUCTION

Alveoli are tiny pores or cavities inside a continuum media which behavior is similar to that of a solid or a gel. The concept is frequently associated with the cavities localized in biological tissues; nevertheless, it has also referred to other materials. The $O_2 - CO_2$ gas exchange occurs between inspired air and the lung circulating blood in the pulmonary alveolus. It has an approximately spherical shape with a mean diameter of 0.25 mm; its walls are covered by the alveolar epithelium followed by the blood capillaries epithelium (also called endothelium). The existence of pores in the alveolar tissue has been proven by scanning electron microscopy, and it has been determined that the associated endothelium membrane size is in the range of 50-60 Å, and the alveolar epithelium size is in the range from 6 to 10 Å[1]. Likewise, Kohn pores are localized in the internal alveolus wall, facilitating the airflow from one alveolus to another. Most of them have a diameter from 1 to 4 μm in multiple mammal species [2].

Rn^{222} is a naturally occurring inert gas whose valence electron shell is complete [3, 4]; thereby, it has a relatively low chemical reactivity and does not form compounds. This gas concentrates in the environment, especially interiors, and effortlessly emanates from the soil to the air. Furthermore, it can get inside the human body by water ingestion and air inhalation. Once it has reached the organism, it is conducted by the respiratory tract to alveoli, where a significant amount is expelled [4]. However, the non-expelled amount decay in short-life disintegration solid products (Po^{218} and Po^{214}) have a high probability of depositing in biological tissues, causing damage to the DNA by the alfa radiation emitted.

Recently [4], a potential function of semi-empirical, smooth, and continuous pairs has been proposed to model molecular interactions between Radon and pulmonary alveolar walls; Molecular Dynamics (MD) is used to determine the distribution of gas in an adjacent alveolar wall and estimate the amount of it diffusing through the alveolar membrane as a concentration function. However, there are currently no studies of Radon density profiles using the formalism of integral equations. The description of Radon gas behavior is a significant challenge in science since it has a very low concentration in the atmosphere, approximately 6×10^{-11} *ppb*. Radon emanations are short-lived due to their isotopic instability; the most stable isotope, Rn^{222}, has a half-life of 3.82 days [5]. This gas was discovered in 1900,

* jmr@uaemex.mx jccoronao@gmail.com

and over the following decades, its thermodynamic properties were determined. They include standard boiling point, vapor pressure, vapor density at standard conditions, vaporization heat, critical temperature, and pressure [6-10]. The critical and liquid densities have not been experimentally measured, and there are still discussions points regarding these properties.

This article aims to describe the density profile of Radon particles near the surface of the alveolar wall. The alveolar pore is assumed a spherical cavity with semipermeable walls whose diameter is more prominent than Radon particles. The permeable wall is characterized by an energy interaction potential at the highest point. The potential interaction between the alveolus wall and Radon particles is characterized by the height of the potential and the half-width of the alveolus membrane; these correspond to the energy required for a Radon particle to pass through the wall and to the thickness of the wall, respectively. The Radon particles can go across the membrane if they possess sufficient energy. We propose a theoretical model that captures the main characteristics of the interaction between a semipermeable membrane and a Radon particle.

II. METHOD

We assumed a system composed of two species: the alveolar pore and the Radon particles. We were interested in describing the Radon concentration profile concerning the wall of a single alveolus. Since we considered a diluted alveoli system, we did not consider the alveolus- alveolus interaction. The difference in sizes between the alveolar cavity and the Radon particles is such that we considered the alveolar wall as a flat wall without loss of generality. The concentrations of Rn^{222}, usually found in the environment, were assumed under average body temperature and atmospheric pressure conditions. We assumed that Radon particles are in a highly dilute phase and can cross through the alveolar walls in both directions. Under this set of assumptions, it is helpful to consider the Lennard-Jones potential for Radon-Radon interactions. The values of the potential parameters that we used were presented in our previous works [4] (Table 1).

TABLE 1: Show the Lennard-Jones parameters.

ε / k_B	σ / A
292.0	4.145

The Lennard-Jones potential has the mathematical form:

$$\beta u_{RR}(r) = 4\frac{\varepsilon}{k_B T}\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right] \tag{1}$$

The parameter values of σ_R and ε/k_B for Rn^{222} that we chose were previously calculated [8]. The subscripts *RR* correspond to *Radon-Radon* interaction. Density and temperature correspond to the gas phase and ambient conditions, approximately. The energy units are $k_B T$, where k_B ($\approx 1.38\times10^{23}$ Kg m^2 s^{-2} K^{-1}) is the Boltzmann constant, and T is the temperature in Kelvin. The energy parameter ε^* is defined by the dimensionless expression:

$$\varepsilon^* = \frac{\varepsilon}{k_B T} \tag{2}$$

Radon particles under previously mentioned thermodynamic conditions of pressure, temperature and volume, can be considered a simple fluid, characterized only by the effective diameter of the particle σ_R. Radon particles can go throughout from the inside to the outside of the alveoli and vice versa. The mathematical expression for the interaction of the alveolar membrane with the Radon particles has been reported by several authors [11-13].

$$\beta u_{Ra}(r^*)) = 4\varepsilon^*\left[\frac{u_0 - (u_0+\varepsilon)r^{*6}}{4\varepsilon^* + (u_0+\varepsilon)r^{*12}}\right], \tag{3}$$

with

$$r^* = \frac{r-\sigma_{Ra}}{\omega}, \tag{3a}$$

$$\sigma_{Ra} = \frac{(\sigma_R + \sigma_a)}{2}. \tag{3b}$$

In the above definition, ω is the half-width of the potential, the parameter u_0 is the height, ε is the depth of potential, and σ_{Ra} means the arithmetic average of the diameters of Radon σ_R and Alveoli σ_a respectively. The fluid particles can cross the alveoli membrane and occupy both sides of the membrane. A graphic description of the potential can be seen in Fig. 1.

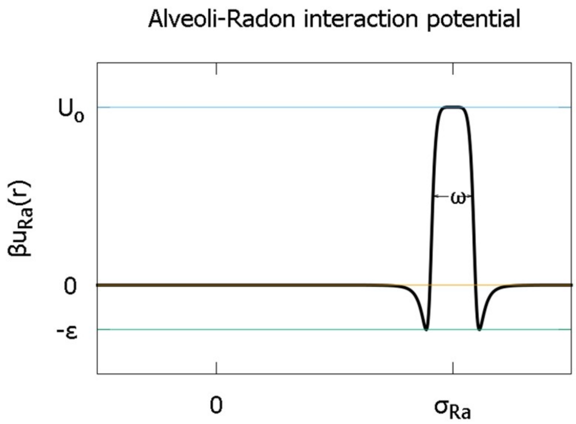

FIG. 1: Radon-alveoli interaction. U_0 and ω are tunable parameters for the potential model.

We used the integral-equation formalism from liquid theory to describe the local density distributions functions

$$g^{(n)}(r_1,r_2,...,r_n) = \frac{1}{\rho^n}\rho^{(n)}(r_1,r_2,...,r_n) = \frac{V^n N!}{Z_n N^n (N-n)!}\int dr_{n+1}...r_n exp(-\beta u(r_1,r_2,...,r_n)) \tag{4}$$

where Z is the partition function in the canonical ensemble, V is the system volume, and N is the number of particles. The Eq. (4) is the probability to find n particles in a particular configuration $(r_1,r_2,...,r_n)$. In the case of $n=2$, we have the radial distribution function (RDF)

$$g_N^2(r_1,r_2) = \frac{\rho_N^2(r_1,r_2)}{\rho_N^2(r_1)\rho_N^2(r_2)}. \quad (5)$$

The geometric interpretation of the RDF is given in Fig. 2.

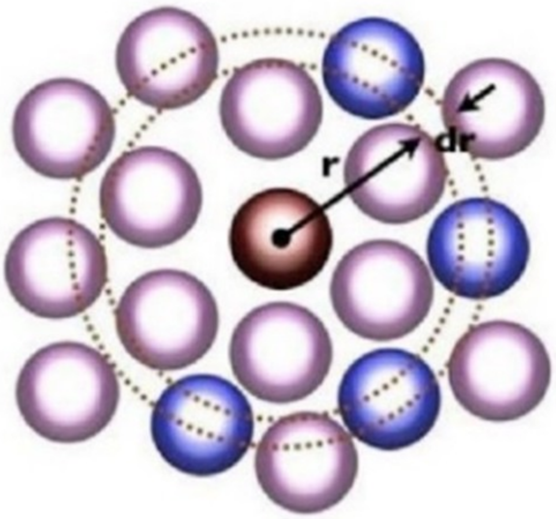

FIG. 2: The radial distribution function is interpreted as the conditional probability of finding a particle at a distance r_2 given that another is in the position r_1. In the figure, we define $r = r_2 - r_1$.

The relationship between the radial distribution function and the integral equations formalism is given by the Ornstein-Zernike equation (OZ). The OZ equation for a simple fluid can be written as

$$h(r) = c(r) + \rho \int h(r)c(r-s)ds. \quad (6)$$

Here, $h(r) = g(r) - 1$ and $c(r)$ are the *total* and *direct* correlation functions, respectively. Another essential equation establishes the relationship between the potential interaction and the pair correlation function

$$g(r) = exp[-\beta u(r) + h(r) - c(r) + b(r)]. \quad (7)$$

We introduced another correlation function named bridge function, $b(r)$, and assumed an approximation to it, called the closure equation. We used the Hypernetted Chain closure, which mathematically is written like

$$b(r) = 0. \quad (8)$$

The relationship between correlation functions and thermodynamics is given by the static structure factor, $S(q)$, which can be obtained by light scattering experiments. The mathematical expression is

$$S(q) = 1 + 4\pi\rho \int g(r)\frac{sin(qr)}{qr}r^2dr. \quad (9)$$

The matrix formulation was used to generalize the formalism to mixtures. The OZ in matrix formulation has the following form

$$h_{\alpha\beta}(r) = c_{\alpha\beta}(r) + \rho \int h_{\alpha\beta}(r)c_{\alpha\beta}(r-s)ds, \quad (10)$$

$$g_{\alpha\beta}(r) = exp[-\beta u_{\alpha\beta}(r) + h_{\alpha\beta}(r) - c_{\alpha\beta}(r) + b_{\alpha\beta}(r)]. \quad (11)$$

The importance of correlation functions is that thermodynamic relationships can be obtained directly [14].

The internal energy and the pressure can be calculated through the expressions

$$U = \frac{3}{2}NkT + 2\pi\rho \int u(r)g(r)r^2dr, \quad (12)$$

$$P = \frac{NkT}{V} - \frac{2}{3}\pi\rho^2 \int u'(r)g(r)r^2dr. \quad (13)$$

This analysis shows the Radon concentration profile in the vicinity of the alveolar membranes and allows us to estimate the number of molecules superficially adsorbed. The thermodynamic properties of the adsorption of Radon over the alveoli membrane surface comprise work for future publications.

In the following section, we show the results obtained from the numerical solution of the Ornstein-Zernike equations systems.

III. RESULTS

In Figs. 3-6, we can observe a structural profile over the surface of the alveolar membrane. We show the results for different sizes of alveoli.As we can see, the highest point reached is symmetrically on both sides of the membrane. The profile density over the surface is three times the bulk density. There is no relationship between the width membrane and the highest point. The increase of local density in the region corresponding to the surface of the membrane allows us to affirm that the Radon molecules are being grouped preferentially in that region. The thickness of the Radon layer adsorbed to the surface of the membrane is approximately ten Radon atoms; beyond this distance, the Radon density tends to its density in gas. The thickness of this layer depends on the width of the membrane (w), as we can

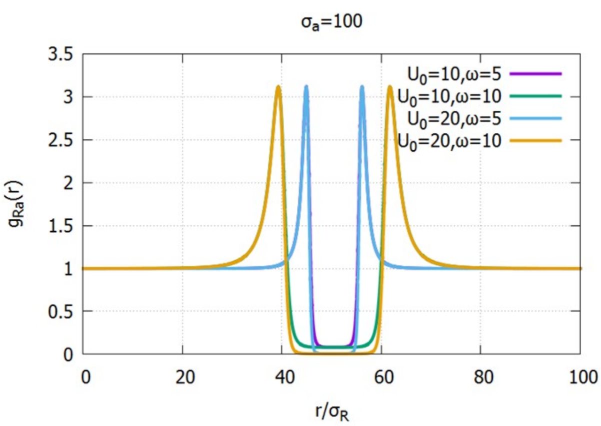

FIG. 3: The radial distribution function of Radon particles.

see in the following figures. However, Radon layer thickness does not depend on the intensity of the energy barrier; therefore, the width barrier role is as important as potential width. We can observe that Radon particles exist inside the membrane; such particles are trapped within the membrane even though the potential barrier expels them. The Radon atoms must cross an additional energy barrier created by those Radon atoms adsorbed on the membrane to emerge from it.

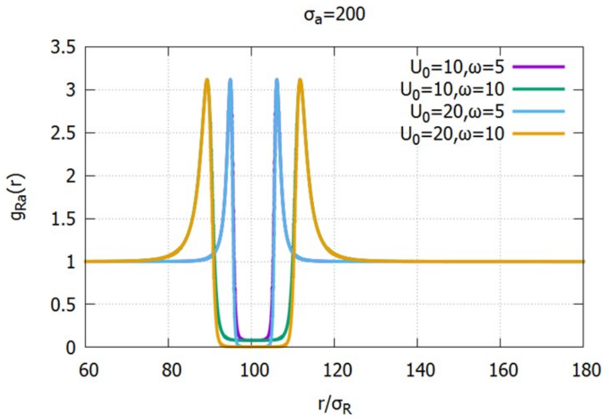

FIG. 4: The radial distribution function of Radon particles.

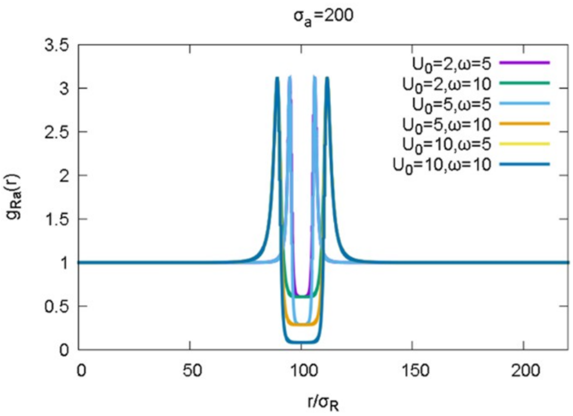

FIG. 5: The radial distribution function of Radon particles.

IV. DISCUSSION AND CONCLUSIONS

We have developed a mathematical model that qualitatively describes the Radon concentration profile near the surface of the alveolar cavities. The proposed model captures globally the observable parameters that characterize biological membranes, such as the thickness and the necessary energy that a particle requires to cross it. With these two parameters, we made a systematic study within the thermodynamic parameters of physicochemical interest that we proposed, specifically, the local density profile. The description of the density profiles on the surfaces gives us a measure of the local concentration of particles adsorbed on the alveolar wall. The results show us that the surface concentrations increase independently of the diameter of the alveolus, while the surface concentrations of Radon on the alveolar membrane are approximately three times the bulk density (away from the walls) in all cases. When analyzing the results for different values of the alveolar wall thickness, controlled by the parameter ω, we observed a fraction of Radon particles contained within the alveolar membrane. Our results show that the thickness of the membrane dominates the surface profile; however, it is remarkable that there are Radon particles trapped inside the membrane. These results give us an estimate of the quantitative behavior of the concentration profiles on the alveolar walls. Let us remember that it is precisely through the alveoli that the processes of physicochemical exchanges of CO_2 and O_2 take place. Our results agree with experimental reports where it has been shown that for attached particles, the activity median aerodynamic diameter is approximately $100-200$ nm, and deposition fractions of inhaled Radon progeny can be estimated from models of particle deposition in the human respiratory tract. It has been reported that such deposition fraction is approximately 20˘40 %, with most of the deposition occurring in the alveolar region [15].

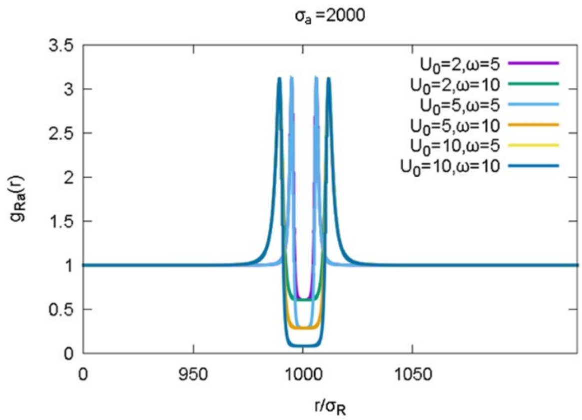

FIG. 6: The radial distribution function of Radon particles.

REFERENCES

[1] ICRP. Human Respiratory Tract Model for Radiological Protection. ICRP Publication 66. **24** (1994).

[2] C. Desplechain, B. Foliguet, E. Barrat, G. Grignon y F. Touati. The pores of Kohn in pulmonary alveoli. Bulletin europeen de physiopathologie respiratoire **19,** 59-68 (1983).

[3] A. D. Griffiths, S. D. Chambers, A. G. Williams y S. Werczynski. Increasing the accuracy and temporal resolution of two-filter radon–222 measurements by correcting for the instrument response. Atmos. Meas. Tech. **9,** 2689-2707 (2016).

[4] J. Corona, F. Zaldivar, L. Mandujano-Rosas, F. Méndez, J. Mulia y D. Osorio-Gonzalez. Theoretical Model to Estimate the Distribution of Radon in Alveolar Membrane Neighborhood. J. Nucl. Phy. Mat. Sci. Rad. A. **4,** 59-68 (2016).

[5] V. M. Rosenoer, M. Oratz y M. A. Rothschild. *Albumin: Structure, function and uses* (Pergamon Press, 1977).

[6] C. Fry y M. Thoennessen. Discovery of the astatine, radon, francium, and radium isotopes. At. Data Nucl. Data Tables **99,** 497-519 (2013).

[7] G. Rudorf. Über einige numerische Konstanten der Radiumemanation und deren Beziehung zu denen der Edelgase. Zeitschrift für Elektrochemie und angewandte physikalische Chemie **15,** 748-749 (1909).

[8] R. W. Gray y W. Ramsay. CXXII.—Some physical properties of radium emanation. J. Chem. Soc., Trans. **95,** 1073-1085 (1909).

[9] W. Herreman. Calculation of the density of liquid radon. Cryogenics **20,** 133-134 (1980).

[10] J. P. Hansen e I. R. McDonald. *Theory of Simple Liquid* (Academic Press London, 2006).

[11] B. Hribar-Lee y M. Lukšič. Replica Ornstein–Zernike Theory Applied for Studying the Equilibrium Distribution of Electrolytes across Model Membranes. J. Phys. Chem. B **122,** 5500-5507 (2018).

[12] B. Hribar-Lee. The application of the replica ornstein-zernike methodology for studying ionic membrane equilibria. Acta Chimica Slovenica **59,** 528-535 (2012).

[13] L. I. Bracamontes, O. Pizio, S. Sokolowski y A. Trokhymchuk. Adsorption and the structure of a hard sphere fluid in disordered quenched microporous matrices of permeable species. Mol. Phys. **96,** 1341-1348 (1999).

[14] E. Rutherford. The Boiling Point of the Radium Emanation. Nature **79,** 457-458 (1909).

[15] G. Butterweck, C. Schuler, G. Vessù, R. Müller, J. W. Marsh, S. Thrift y A. Birchall. Experimental Determination of the Absorption Rate of Unattached Radon Progeny from Respiratory Tract to Blood. Radiat. Prot. Dosim. **102,** 343-348 (2002).